

ON NEW MAPPINGS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper a new class of homeomorphism called minimal Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism and maximal Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism are introduced and investigated and during this process some properties of the new concepts have been studied.

**Keywords:** Maximal open set, Minimal open set, Maximal Homeomorphism, Minimal homeomorphism.

**Mathematics subject classification (2000):** 54A05.

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1. INTRODUCTION

In the year 2001 and 2003, F.Nakaoka and N.oda [1] [2] [3] introduced and studied minimal open [resp. minimal closed] sets which are subclass of open [resp. closed sets]. The family of all minimal open [minimal closed] in a topological space  $X$  is denoted by  $m_o(X)$  [ $m_c(X)$ ]. Similarly the family of all maximal open [maximal closed] sets in a topological space  $X$  is denoted by  $M_aO(X)$  [ $M_aC(X)$ ]. The complements of minimal open sets and maximal open sets are called maximal closed sets and minimal closed sets respectively. In the year 2000, M.Sheik john [4] introduced and studied weakly homeomorphism in topological spaces and in the year 2008 B.M.Ittanagi [5] introduced and studied minimal open sets and maps in topological spaces and minimal homeomorphism and maximal homeomorphism in topological spaces. In the year 2014 R.S.Wali and Vivekananda Dembre [6] [7] introduced and studied minimal weakly open sets and maximal weakly closed sets and maximal weakly open sets and minimal weakly closed sets in topological spaces. In the year 2014 [8] Vivekananda Dembre, Manjunath Gowda and Jeetendra Gurjar introduced and studied minimal weakly and maximal weakly continuous functions in topological spaces. In the year 2014 [9] Vivekananda Dembre and Jeetendra Gurjar introduced and studied minimal weakly and maximal weakly open maps in topological spaces.

**Definition 1.1[1]:** A proper non-empty open subset  $U$  of a topological space  $X$  is said to be minimal open set if any open set which is contained in  $U$  is  $\varphi$  or  $U$ .

**Definition 1.2[2]:** A proper non-empty open subset  $U$  of a topological space  $X$  is said to be maximal open set if any open set which is contained in  $U$  is  $X$  or  $U$ .

**Definition 1.3[3]:** A proper non-empty closed subset  $F$  of a topological space  $X$  is said to be minimal closed set if any closed set which is contained in  $F$  is  $\varphi$  or  $F$ .

**Definition 1.4[3]:** A proper non-empty closed subset  $F$  of a topological space  $X$  is said to be maximal closed set if any closed set which is contained in  $F$  is  $X$  or  $F$ .

**Definition 1.5[4]:** Let  $X$  and  $Y$  be the topological spaces. A bijective function  $f: X \rightarrow Y$  is called weakly homeomorphism if both  $f$  and  $f^{-1}$  are weakly continuous.

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**Definition 1.6[5]:** Let  $X$  and  $Y$  be the topological spaces. A bijective function  $f : X \rightarrow Y$  is called

- (i) Minimal homeomorphism if both  $f$  and  $f^{-1}$  are minimal continuous maps.
- (ii) Maximal homeomorphism if both  $f$  and  $f^{-1}$  are maximal continuous maps.

**Definition 1.7 [5]:** Let  $X$  and  $Y$  be the topological spaces. A map  $f : X \rightarrow Y$  is called

- (i) Minimal continuous function if for every minimal open set  $N$  in  $Y$ ,  $f^{-1}(N)$  is an open set in  $X$ .
- (ii) Maximal continuous function if for every maximal open set  $N$  in  $Y$ ,  $f^{-1}(N)$  is an open set in  $X$ .
- (iii) Minimal open map for every open set  $U$  of  $X$ ,  $f(U)$  is minimal open set in  $Y$ .
- (iv) Maximal open map for every open set  $U$  of  $X$ ,  $f(U)$  is maximal open set in  $Y$ .
- (v) Minimal closed map for every closed set  $F$  of  $X$ ,  $f(F)$  is minimal closed set in  $Y$ .
- (vi) Maximal closed map for every closed set  $F$  of  $X$ ,  $f(F)$  is maximal closed set in  $Y$ .
- (vii) Minimal-Maximal open map for every minimal open set  $N$  of  $X$ ,  $f(N)$  is maximal open set in  $Y$ .
- (viii) Maximal-Minimal openmap for every maximal open set  $N$  of  $X$ ,  $f(N)$  is minimal open set in  $Y$ .
- (ix) Minimal-Maximal continuous if for every minimal open set  $N$  in  $Y$ ,  $f^{-1}(N)$  is a maximal open set in  $X$ .
- (x) Maximal-Minimal continuous if for every maximal open set  $N$  in  $Y$ ,  $f^{-1}(N)$  is a minimal open set in  $X$ .

**Definition 1.8 [6]:** Let  $X$  and  $Y$  be the topological spaces. A map  $f : X \rightarrow Y$  is called

- (i) Minimal weakly continuous if for every minimal weakly open set  $N$  in  $Y$ ,  $f^{-1}(N)$  is an open set in  $X$ .
- (ii) Maximal weakly continuous if for every maximal weakly open set  $N$  in  $Y$ ,  $f^{-1}(N)$  is an open set in  $X$ .

**Definition 1.9 [7]:** A proper non-empty weakly open subset  $U$  of  $X$  is said to be minimal weakly open set if any weakly open set which is contained in  $U$  is  $\emptyset$  or  $U$ .

**Definition 1.10 [8]:** A proper non-empty weakly closed subset  $U$  of  $X$  is said to be maximal weakly open set if any weakly open set which is contained in  $U$  is  $X$  or  $U$ .

**Definition 1.11 [9]:** Let  $X$  and  $Y$  be topological spaces.

- (i) A map  $f : X \rightarrow Y$  is called minimal weakly open map for every open set  $U$  of  $X$ ,  $f(U)$  is minimal weakly open set in  $Y$ .
- (ii) A map  $f : X \rightarrow Y$  is called maximal weakly open map for every open set  $U$  of  $X$ ,  $f(U)$  is maximal weakly open set in  $Y$ .
- (iii) A map  $f : X \rightarrow Y$  is called minimal weakly closed map for every closed set  $F$  of  $X$ ,  $f(F)$  is minimal weakly closed set in  $Y$ .
- (iv) A map  $f : X \rightarrow Y$  is called maximal weakly closed map for every closed set  $F$  of  $X$ ,  $f(F)$  is maximal weakly closed set in  $Y$ .

## 2. MINIMAL PRE GENERALIZED PRE REGULAR WEAKLY HOMEOMORPHISM IN TOPOLOGICAL SPACES AND MAXIMAL REGULAR WEAKLY HOMEOMORPHISM.

**Definition 2.1:** A bijection function  $f : X \rightarrow Y$  is called

- (i) Minimal Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism if both  $f$  and  $f^{-1}$  are minimal regular weakly continuous maps.
- (ii) Maximal Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism if both  $f$  and  $f^{-1}$  are maximal Pre generalized pre regular weakly homeomorphism in topological spaces continuous maps.

**Theorem 2.2:** Every homeomorphism is minimal Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism but not conversely.

**Proof:** Let  $f : X \rightarrow Y$  be a homeomorphism. Now  $f$  and  $f^{-1}$  are continuous maps then  $f$  and  $f^{-1}$  are minimal Pre generalized pre regular weakly homeomorphism in topological spaces continuous as every continuous map is minimal Pre generalized pre regular weakly homeomorphism in topological spaces continuous. Hence  $f$  is a minimal Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism.

**Example 2.3 :** Let  $X=Y=\{a,b,c\}$  be with  $\tau = \{X, \emptyset, \{a\}, \{a,b\}\}$  and  $\mu = \{Y, \emptyset, \{a\}, \{b\}, \{a,b\}\}$  then  $f : X \rightarrow Y$  be a function defined by  $f(a) = a$ ,  $f(b) = a$  and  $f(c) = c$ , then  $f$  and  $f^{-1}$  are minimal Pre generalized pre regular weakly homeomorphism in topological spaces continuous maps then  $f$  is a minimal Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism but it is not a homeomorphism. Since  $f$  is not continuous map for the open set  $\{b\}$  in  $Y$ ;  $f^{-1}(\{b\}) = b$  which is not open set in  $X$ .

**Theorem 2.4:** Every homeomorphism is maximal Pre generalized pre regular weakly homeomorphism in topological spaceshomeomorphism but not conversely.

**Proof:** Let  $f : X \rightarrow Y$  be a homeomorphism. Now  $f$  and  $f^{-1}$  are continuous maps then  $f$  and  $f^{-1}$  are maximal Pre generalized pre regular weakly homeomorphism in topological spacescontinuous as every continuous map is minimal Pre generalized pre regular weakly homeomorphism in topological spacescontinuous. Hence  $f$  is a minimal Pre generalized pre regular weakly homeomorphism in topological spaceshomeomorphism.

**Example 2.5:** Let  $X = Y = \{a,b,c\}$  be with  $\tau = \{X, \varphi, \{a,b\}\}$  and  $\mu = \{Y, \varphi, \{a\},\{b\},\{a,b\}\}$  then  $f : X \rightarrow Y$  be a identity function then  $f$  and  $f^{-1}$  are maximal Pre generalized pre regular weakly homeomorphism in topological spacescontinuous maps then  $f$  is a maximal Pre generalized pre regular weakly homeomorphism in topological spaceshomeomorphism but it is not a homeomorphism. Since  $f$  is not a continuous map for the open set  $\{b\}$  in  $Y$ ;  $f^{-1}(\{b\}) = b$  which is not open set in  $X$ .

**Remark 2.6:** Minimal Pre generalized pre regular weakly homeomorphism in topological spaceshomeomorphism and maximal Pre generalized pre regular weakly homeomorphism in topological spaceshomeomorphism are independent of each other.

**Example 2.7:** In example 2.3  $f$  is minimal Pre generalized pre regular weakly homeomorphism in topological spaceshomeomorphism but it is not maximal Pre generalized pre regular weakly homeomorphism in topological spaceshomeomorphism. In example 2.5  $f$  is maximal Pre generalized pre regular weakly homeomorphism in topological spaceshomeomorphism but it is not minimal Pre generalized pre regular weakly homeomorphism in topological spaceshomeomorphism.

**Theorem 2.8:** Let  $f: X \rightarrow Y$  be a bijective and minimal Pre generalized pre regular weakly homeomorphism in topological spacescontinuous then the following statements are equivalent.

- (i)  $f: X \rightarrow Y$  is minimal Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism.
- (ii)  $f$  is minimal Pre generalized pre regular weakly homeomorphism in topological spacesopen map.
- (iii)  $f$  is maximal Pre generalized pre regular weakly homeomorphism in topological spacesclosed map.

**Proof :**

**(i)  $\rightarrow$  (ii):** Let  $N$  be any minimal Pre generalized pre regular weakly homeomorphism in topological spacesopen set in  $X$ ; by assumption  $(f^{-1})^{-1}(N)$  is an open set in  $Y$ . But  $(f^{-1})^{-1}(N) = f(N)$  is an open set in  $Y$ ; therefore  $f$  is a minimal Pre generalized pre regular weakly homeomorphism in topological spacesopen map.

**(ii)  $\rightarrow$  (iii):** Let  $F$  be any maximal Pre generalized pre regular weakly homeomorphism in topological spacesclosed set in  $X$ ; then  $X-F$  is a minimal Pre generalized pre regular weakly homeomorphism in topological spacesopen set in  $X$ ; by assumption  $f(X-F)$  is an open set in  $Y$ . But  $f(X-F) = Y - f(F)$  is an open set in  $Y$ ; therefore  $f(F)$  is a closed set in  $Y$ . Hence  $f$  is a maximal Pre generalized pre regular weakly homeomorphism in topological spacesclosed map.

**(iii)  $\rightarrow$  (i):** Let  $N$  be any minimal Pre generalized pre regular weakly homeomorphism in topological spacesopen set in  $X$ ; then  $X-N$  is a maximal Pre generalized pre regular weakly homeomorphism in topological spacesclosed set in  $X$ ; by assumption  $f(X-N)$  is a closed set in  $Y$ . But  $f(X-N) = (f^{-1})^{-1}(X - N) = Y - (f^{-1})^{-1}(N)$  is closed set in  $Y$ ; therefore  $(f^{-1})^{-1}(N)$  is an open set in  $Y$ . Hence  $f^{-1} : Y \rightarrow X$  is a minimal Pre generalized pre regular weakly homeomorphism in topological spaceshomeomorphism and similarly  $f$  is minimal Pre generalized pre regular weakly homeomorphism in topological spaceshomeomorphism.

**Theorem 2.9:** Let  $f : X \rightarrow Y$  be a bijective and maximal Pre generalized pre regular weakly homeomorphism in topological spacescontinuous then the following statements are equivalent.

- (i)  $F^{-1} : X \rightarrow Y$  is maximal Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism.
- (ii)  $f$  is maximal Pre generalized pre regular weakly homeomorphism in topological spacesopen map.
- (iii)  $f$  is minimal Pre generalized pre regular weakly homeomorphism in topological spacesclosed map.

**Proof :**

**(i)  $\rightarrow$  (ii):** Let  $N$  be any maximal Pre generalized pre regular weakly homeomorphism in topological spacesopen set in  $X$ ; by assumption  $(f^{-1})^{-1}(N)$  is an open set in  $Y$ . But  $(f^{-1})^{-1}(N) = f(N)$  is an open set in  $Y$ ; therefore  $f$  is a minimal Pre generalized pre regular weakly homeomorphism in topological spacesopen map.

(ii)  $\rightarrow$  (iii): Let  $F$  be any minimal Pre generalized pre regular weakly homeomorphism in topological spaces closed set in  $X$ ; then  $X-F$  is a maximal Pre generalized pre regular weakly homeomorphism in topological spaces open set in  $X$ ; by assumption  $f(X-F)$  is an open set in  $Y$ . But  $f(X-F) = Y - f(F)$  is an open set in  $Y$ ; therefore  $f(F)$  is a closed set in  $Y$ . Hence  $f$  is a maximal Pre generalized pre regular weakly homeomorphism in topological spaces closed map.

(iii)  $\rightarrow$  (i): Let  $N$  be any maximal Pre generalized pre regular weakly homeomorphism in topological spaces open set in  $X$ ; then  $X-N$  is a minimal Pre generalized pre regular weakly homeomorphism in topological spaces closed set in  $X$ ; by assumption  $f(X-N)$  is a closed set in  $Y$ . But  $f(X-N) = (f^{-1})(X-N) = Y - (f^{-1})(N)$  is a closed set in  $Y$ ; therefore  $(f^{-1})(N)$  is an open set in  $Y$ . Hence  $f^{-1}: Y \rightarrow X$  is a minimal Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism and similarly  $f$  is minimal Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism.

**Definition 2.10:** Let  $X$  and  $Y$  be the topological spaces. A map  $f: X \rightarrow Y$  is called

- (i) Minimal-Maximal Pre generalized pre regular weakly homeomorphism in topological spaces continuous if for every minimal Pre generalized pre regular weakly homeomorphism in topological spaces open set  $N$  in  $Y$ ,  $f^{-1}(N)$  is a maximal Pre generalized pre regular weakly homeomorphism in topological spaces open set in  $X$ .
- (ii) Maximal-Minimal Pre generalized pre regular weakly homeomorphism in topological spaces continuous if for every maximal Pre generalized pre regular weakly homeomorphism in topological spaces open set  $N$  in  $Y$ ,  $f^{-1}(N)$  is a minimal Pre generalized pre regular weakly homeomorphism in topological spaces open set in  $X$ .
- (iii) Minimal-Maximal Pre generalized pre regular weakly homeomorphism in topological spaces open map for every minimal Pre generalized pre regular weakly homeomorphism in topological spaces open set  $N$  of  $X$ ,  $f(N)$  is maximal Pre generalized pre regular weakly homeomorphism in topological spaces open set in  $Y$ .
- (iv) Maximal-Minimal Pre generalized pre regular weakly homeomorphism in topological spaces open map for every maximal Pre generalized pre regular weakly homeomorphism in topological spaces open set  $N$  of  $X$ ,  $f(N)$  is minimal Pre generalized pre regular weakly homeomorphism in topological spaces open set in  $Y$ .
- (v) Minimal-Maximal Pre generalized pre regular weakly homeomorphism in topological spaces closed map for every minimal Pre generalized pre regular weakly homeomorphism in topological spaces closed set  $F$  of  $X$ ,  $f(F)$  is maximal Pre generalized pre regular weakly homeomorphism in topological spaces closed set in  $Y$ .
- (vi) Maximal-Minimal Pre generalized pre regular weakly homeomorphism in topological spaces closed map for every maximal regular weakly closed set  $F$  of  $X$ ,  $f(F)$  is minimal Pre generalized pre regular weakly homeomorphism in topological spaces closed set in  $Y$ .

**Definition 2.11:** A bijection  $f: X \rightarrow Y$  is called

- (i) min - max Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism if both  $f$  and  $f^{-1}$  are min - max Pre generalized pre regular weakly homeomorphism in topological spaces continuous maps.
- (ii) max - min Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism if both  $f$  and  $f^{-1}$  are max - min Pre generalized pre regular weakly homeomorphism in topological spaces continuous maps.

**Theorem 2.12:** Every min-max Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism is minimal Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism but not conversely.

**Proof:** Let  $f: X \rightarrow Y$  be a homeomorphism. Now  $f$  and  $f^{-1}$  are continuous maps then  $f$  and  $f^{-1}$  are min-max Pre generalized pre regular weakly homeomorphism in topological spaces continuous as every continuous map is min-max Pre generalized pre regular weakly homeomorphism in topological spaces continuous. Hence  $f$  is a min-max Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism.

**Example 2.13:** In example 2.3  $f$  is a minimal Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism but it is not a min-max Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism. Since  $f$  is not a min-max Pre generalized pre regular weakly homeomorphism in topological spaces continuous map for the minimal Pre generalized pre regular weakly homeomorphism in topological spaces open set  $\{b\}$  in  $Y$ ,  $f^{-1}(\{b\}) = \{b\}$  which is not a maximal Pre generalized pre regular weakly homeomorphism in topological spaces open set in  $X$ .

**Theorem 2.14:** Every max-min Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism is maximal Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism but not conversely.

**Proof :** Let  $f: X \rightarrow Y$  be a homeomorphism. Now  $f$  and  $f^{-1}$  are continuous maps then  $f$  and  $f^{-1}$  are max-min Pre generalized pre regular weakly homeomorphism in topological spaces continuous as every continuous map is max-min

Pre generalized pre regular weakly homeomorphism in topological spaces continuous. Hence  $f$  is a min-max Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism.

**Theorem 2.15:** Let  $f : X \rightarrow Y$  be a bijective and min-max Pre generalized pre regular weakly homeomorphism in topological spaces continuous map then the following statements are equivalent.

- (i)  $f$  is minimal - maximal Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism.
- (ii)  $f$  is minimal - maximal Pre generalized pre regular weakly homeomorphism in topological spaces open map.
- (iii)  $f$  is maximal - minimal Pre generalized pre regular weakly homeomorphism in topological spaces closed map.

**Proof :**

**(i)  $\rightarrow$  (ii):** Let  $N$  be any minimal Pre generalized pre regular weakly homeomorphism in topological spaces open set in  $X$ ; by assumption  $(f^{-1})^{-1}(N)$  is an open set in  $Y$ . But  $(f^{-1})^{-1}(N) = f(N)$  is a maximal Pre generalized pre regular weakly homeomorphism in topological spaces open set in  $Y$ ; therefore  $f$  is a min-max Pre generalized pre regular weakly homeomorphism in topological spaces open map.

**(ii)  $\rightarrow$  (iii):** Let  $F$  be any maximal Pre generalized pre regular weakly homeomorphism in topological spaces closed set in  $X$ ; then  $X-F$  is a minimal Pre generalized pre regular weakly homeomorphism in topological spaces open set in  $X$ ; by assumption  $f(X-F)$  is a maximal Pre generalized pre regular weakly homeomorphism in topological spaces open set in  $Y$ . But  $f(X-F) = Y - f(F)$  is a maximal Pre generalized pre regular weakly homeomorphism in topological spaces open set in  $Y$ ; therefore  $f(F)$  is a minimal Pre generalized pre regular weakly homeomorphism in topological spaces closed set in  $Y$ . Hence  $f$  is a max-min Pre generalized pre regular weakly homeomorphism in topological spaces closed map.

**(iii)  $\rightarrow$  (i):** Let  $N$  be any minimal Pre generalized pre regular weakly homeomorphism in topological spaces open set in  $X$ ; then  $X-N$  is a maximal Pre generalized pre regular weakly homeomorphism in topological spaces closed set in  $X$ ; by assumption  $f(X-N)$  is a minimal Pre generalized pre regular weakly homeomorphism in topological spaces closed set in  $Y$ .

But  $f(X-N) = (f^{-1})(X - N) = Y - (f^{-1})(N)$  is a minimal Pre generalized pre regular weakly homeomorphism in topological spaces open set in  $Y$ ; therefore  $(f^{-1})(N)$  is a maximal regular weakly open set in  $Y$ . Hence  $f^{-1}: Y \rightarrow X$  is a minimal-maximal Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism and similarly  $f$  is min-max Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism.

**Theorem 2.16:** Let  $f : X \rightarrow Y$  be a bijective and min-max Pre generalized pre regular weakly homeomorphism in topological spaces continuous map then the following statements are equivalent.

- (i)  $f$  is max - min Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism.
- (ii)  $f$  is max - min Pre generalized pre regular weakly homeomorphism in topological spaces open map.
- (iii)  $f$  is min - max Pre generalized pre regular weakly homeomorphism in topological spaces closed map.

**Proof:**

**(i)  $\rightarrow$  (ii):** Let  $N$  be any maximal Pre generalized pre regular weakly homeomorphism in topological spaces open set in  $X$ ; by assumption  $(f^{-1})^{-1}(N)$  is an open set in  $Y$ . But  $(f^{-1})^{-1}(N) = f(N)$  is a minimal Pre generalized pre regular weakly homeomorphism in topological spaces open set in  $Y$ ; therefore  $f$  is a max - min Pre generalized pre regular weakly homeomorphism in topological spaces open map.

**(ii)  $\rightarrow$  (iii):** Let  $F$  be any minimal Pre generalized pre regular weakly homeomorphism in topological spaces closed set in  $X$ ; then  $X-F$  is a maximal Pre generalized pre regular weakly homeomorphism in topological spaces open set in  $X$ ; by assumption  $f(X-F)$  is a minimal Pre generalized pre regular weakly homeomorphism in topological spaces open set in  $Y$ .

But  $f(X-F) = Y - f(F)$  is a minimal Pre generalized pre regular weakly homeomorphism in topological spaces open set in  $Y$ ; therefore  $f(F)$  is a minimal Pre generalized pre regular weakly homeomorphism in topological spaces closed set in  $Y$ . Hence  $f$  is a max-min Pre generalized pre regular weakly homeomorphism in topological spaces closed map.

**(iii)  $\rightarrow$  (i):** Let  $N$  be any minimal Pre generalized pre regular weakly homeomorphism in topological spaces open set in  $X$ ; then  $X-N$  is a maximal Pre generalized pre regular weakly homeomorphism in topological spaces closed set in  $X$ ; by assumption  $f(X-N)$  is a minimal Pre generalized pre regular weakly homeomorphism in topological spaces closed set in  $Y$ .

But  $f(X-N) = (f^{-1})(X - N) = Y - (f^{-1})(N)$  is a minimal Pre generalized pre regular weakly homeomorphism in topological spaces open set in  $Y$ ; therefore  $(f^{-1})(N)$  is a maximal regular weakly open set in  $Y$ . Hence  $f^{-1}: Y \rightarrow X$  is a minimal-maximal Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism and similarly  $f$  is min-max Pre generalized pre regular weakly homeomorphism in topological spaces homeomorphism.

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