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HYPER E-BANHATTI INDICES OF CERTAIN NETWORKS

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ABSTRACT

We introduce the first and second hyper E-Banhatti indices and their corresponding polynomials of a graph. In this paper, we compute these newly defined hyper E-Banhatti indices of some standard classes of graphs. We also determine the first and second hyper E-Banhatti indices and their corresponding polynomials for wheel graphs, friendship graphs, silicate and honeycomb networks.

Mathematics Subject Classification: *05C05, 05C12, 05C35.*

Keywords: *E-Banhatti index, hyper E-Banhatti index, hyper E-Banhatti polynomial, network.*

1. INTRODUCTION

Throughout this paper, we consider simple graphs which are finite, connected, undirected graphs without loops and multiple edges. Let G be such a graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. The edge e connecting the vertices u and v is denoted by uv. If $e=uv$ is an edge of G, then the vertex u and edge e are incident as are v and e. Let $d_G(e)$ denote the degree of an edge e in G, which is defined by $d_G(e) = d_G(u) + d_G(v) – 2$ with $e=uv$. For term and concept not given here, we refer [1].

A molecular graph is a simple graph, representing the carbon atom skeleton of an organic molecule of the hydrocarbon. Therefore the vertices of a molecular graph represent the carbon atoms and its edges the carbon-carbon bonds. Chemical Graph Theory is a branch of Mathematical Chemistry which has an important effect on the development of Chemical Sciences. Several graph indices [2] have found some applications in Chemistry, especially in QSPR/QSAR research [3, 4, 5].

In [6], Kulli defined the Banhatti degree of a vertex u of a graph G as

$$B(u)=\frac{d_G(e)}{n-d_G(u)},$$

where n is the number of vertices of G and the vertex u and edge e are incident in G.

In [6], Kulli proposed the first and second E-Banhatti indices of a graph G and they are defined as

$$EB_1(G)=\sum_{uv\in E(G)}\left[B(u)+B(v)\right],$$

$$EB_2(G)=\sum_{uv\in E(G)}B(u)B(v).$$

We now introduce the first and second hyper E-Banhatti indices of a graph G and they are defined as

$$HEB_1(G)=\sum_{uv\in E(G)}\left[B(u)+B(v)\right]^2,$$

$$HEB_2(G)=\sum_{uv\in E(G)}\left[B(u)B(v)\right]^2.$$

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Considering the first and second hyper E-Banhatti indices, we define the first and second hyper E-Banhatti polynomials of a graph G as

$$HEB_1(G,x) = \sum_{uv \in E(G)} x^{[B(u)+B(v)]^2},$$

$$HEB_2(G,x) = \sum_{uv \in E(G)} x^{[B(u)B(v)]^2}.$$

In Graph Index Theory, several graph indices were introduced and studied such as the Wiener index [7, 8, 9, 10], the Zagreb indices [11, 12, 13, 14], the Revan indices [15, 16, 17, 18], the reverse indices [19, 20, 21, 22], the Banhatti indices [23, 24, 25, 26], and the Gourava indices [27, 28, 29, 30, 31].

In this paper, we compute the first and second hyper E-Banhatti indices and their corresponding polynomials for wheel graphs, friendship graphs, silicate networks and honeycomb networks.

2. RESULTS FOR SOME STANDARD GRAPHS

2.1. First Hyper E-Banhatti Index

Proposition 1: If G is an r-regular graph with n vertices and $r \geq 2$, then

$$HEB_1(G) = \frac{8nr(r-1)^2}{(n-r)^2}.$$

Proof: Let G be an r-regular graph with n vertices and $r \geq 2$. Then G has $\frac{nr}{2}$ edges. For any edge $uv = e$ in G, $d_G(e) = d_G(u) + d_G(u) - 2 = 2r - 2$.
From definition we have

$$HEB_1(G) = \sum_{uv \in E(G)} [B(u)+B(v)]^2 = \frac{nr}{2}\left[\frac{2r-2}{n-r} + \frac{2r-2}{n-r}\right]^2 = \frac{8nr(r-1)^2}{(n-r)^2}.$$

Corollary 1.1: Let C_n be a cycle with $n \geq 3$ vertices. Then

$$HEB_1(C_n) = \frac{16n}{(n-2)^2}.$$

Corollary 1.2: Let K_n be a complete graph with $n \geq 3$ vertices. Then

$$HEB_1(K_n) = 8n(n-1)(n-2)^2.$$

Proposition 2: Let P_n be a path with $n \geq 3$ vertices. Then

$$HEB_1(P_n) = 2\left[\frac{1}{n-1} + \frac{2}{n-2}\right]^2 + (n-3)\left[\frac{2}{n-2} + \frac{2}{n-2}\right]^2$$
$$= \frac{2(3n-4)^2}{(n-1)^2(n-2)^2} + \frac{16(n-3)}{(n-2)^2}.$$

Proposition 3: Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$ *and* $n \geq 2$. Then

$$HEB_1(K_{m,n}) = \frac{1}{mn}[(m+n)(m+n-2)]^2.$$

Proof: Let $K_{m,n}$ be a complete bipartite m n graph with $m + n$ vertices and mn *edges* such that $|V_1| = m$, $|V_2| = n$, $V(K_{r,s}) = V_1 \cup V_2$ for $1 \leq m \leq n$, and $n \geq 2$. Every vertex of V_1 is incident with n edges and every vertex of V_2 is incident with m edges. Then $d_G(e) = d_G(u) + d_G(v) - 2 = m + n - 2$.

$$HEB_1(K_{m,n}) = \sum_{uv \in E(G)} [B(u)+B(v)]^2 = mn\left[\frac{m+n-2}{m+n-n} + \frac{m+n-2}{m+n-m}\right]^2$$
$$= \frac{1}{mn}[(m+n)(m+n-2)]^2.$$

Corollary 3.1: Let $K_{n,n}$ be a complete bipartite graph with $n \geq 2$. Then

$$HEB_1\left(K_{n,n}\right) = 16(n-1)^2.$$

Corollary 3.2: Let $K_{1,n}$ be a star with $n \geq 2$. Then

$$HEB_1\left(K_{1,n}\right) = \frac{1}{n}\left(n^2-1\right)^2.$$

2.2. Second Hyper E-Banhatti Index

Proposition 4: If G is an r-regular graph with n vertices and $r \geq 2$, then

$$HEB_2(G) = \frac{8nr(r-1)^4}{(n-r)^4}.$$

Proof: Let G be an r-regular graph with n vertices and $r \geq 2$. Then G has $\frac{nr}{2}$ edges. For any edge $uv = e$ in G, $d_G(e) = d_G(u) + d_G(u) - 2 = 2r - 2$.

From definition we have

$$HEB_2(G) = \sum_{uv \in E(G)} \left[B(u) \times B(v)\right]^2 = \frac{nr}{2}\left[\frac{2r-2}{n-r} \times \frac{2r-2}{n-r}\right]^2 = \frac{8nr(r-1)^4}{(n-r)^4}.$$

Corollary 4.1: Let C_n be a cycle with $n \geq 3$ vertices. Then

$$HEB_2\left(C_n\right) = \frac{16n}{(n-2)^4}.$$

Corollary 4.2: Let K_n be a complete graph with $n \geq 3$ vertices. Then

$$HEB_2\left(K_n\right) = 8n(n-1)(n-2)^4.$$

Proposition 5: Let P_n be a path with $n \geq 3$ vertices. Then

$$HEB_2\left(P_n\right) = 2\left[\frac{1}{n-1} \times \frac{2}{n-2}\right]^2 + (n-3)\left[\frac{2}{n-2} \times \frac{2}{n-2}\right]^2$$

$$= \frac{2(3n-4)^2}{(n-1)^2(n-2)^2} + \frac{16(n-3)}{(n-2)^2}.$$

Proposition 6: Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$ *and* $n \geq 2$. Then

$$HEB_2\left(K_{m,n}\right) = \frac{(m+n-2)^4}{mn}.$$

Proof: Let $K_{m,n}$ be a complete bipartite graph with $m + n$ vertices and mn *edges* such that $|V_1| = m$, $|V_2| = n$, $V(K_{r,s}) = V_1 \cup V_2$ for $1 \leq m \leq n$, and $n \geq 2$. Every vertex of V_1 is incident with n edges and every vertex of V_2 is incident with m edges. Then $d_G(e) = d_G(u) + d_G(v) - 2 = m + n - 2$.

$$HEB_2\left(K_{m,n}\right) = \sum_{uv \in E(G)} \left[B(u) \times B(v)\right]^2$$

$$= mn\left[\frac{m+n-2}{m+n-n} \times \frac{m+n-2}{m+n-m}\right]^2 = \frac{(m+n-2)^4}{mn}.$$

Corollary 6.1: Let $K_{n,n}$ be a complete bipartite graph with $n \geq 2$. Then

$$HEB_2\left(K_{n,n}\right) = \frac{16(n-1)^4}{n^2}.$$

Corollary 6.2: Let $K_{1,n}$ be a star with $n \geq 2$. Then

$$HEB_2\left(K_{1,n}\right) = \frac{(n-1)^4}{n}.$$

3. RESULTS FOR FRIENDSHIP GRAPHS

A friendship graph F_n, $n \geq 2$, is a graph that can be constructed by joining n copies of C_3 with a common vertex. A graph F_4 is shown in Figure 1.

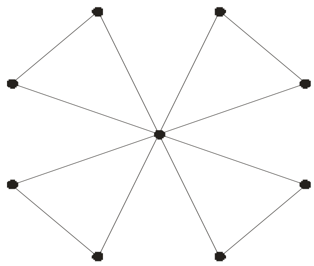

Figure-1: Friendship graph F_4

Let F_n be a friendship graph with $2n$+1 vertices and $3n$ edges. By calculation, we obtain that there are two types of edges as follows:

$$E_1 = \left\{uv \in E\left(F_n\right) \mid d_{F_n}(u) = d_{F_n}(v) = 2\right\}, \qquad |E_1| = n.$$

$$E_2 = \left\{uv \in E\left(F_n\right) \mid d_{F_n}(u) = 2, d_{F_n}(v) = 2n\right\}, \quad |E_2| = 2n.$$

Therefore, in F_n, there are two types of Banhatti edges based on Banhatti degrees of end vertices of each edge follow:

$$BE_1 = \{uv \in E(F_n) \mid B(u) = B(v) = \frac{2}{2n-1}\}, \qquad |BE_1| = n.$$

$$BE_2 = \{uv \in E(F_n) \mid B(u) = \frac{2n}{2n-1}, B(v) = 2n\}, \quad |BE_2| = 2n.$$

We now compute the first hyper E-Banhatti index of a friendship graph F_n.

Theorem 1: Let F_n be a friendship graph with $2n$ + 1 vertices and $3n$ edges. Then

$$HEB_1\left(F_n\right) = \frac{32n^5 + 16n}{(2n-1)^2}.$$

Proof: From definition and by cardinalities of the Banhatti edge partition of F_n, we obtain

$$HEB_1\left(F_n\right) = \sum_{uv \in E(F_n)} \left[B(u) + B(v)\right]^2 = n\left(\frac{2}{2n-1} + \frac{2}{2n-1}\right)^2 + 2n\left(\frac{2n}{2n-1} + 2n\right)^2$$

$$= \frac{32n^5 + 16n}{(2n-1)^2}.$$

In the following theorem, we obtain the second hyper E-Banhatti index of a friendship graph F_n.

Theorem 2: Let F_n be a friendship graph with $2n$ + 1 vertices and $3n$ edges. Then

$$HEB_2\left(F_n\right) = \frac{16n}{(2n-1)^4} + \frac{32n^5}{(2n-1)^2}.$$

Proof: From definition and by cardinalities of the Banhatti edge partition of F_n, we obtain

$$HEB_2\left(F_n\right) = \sum_{uv \in E(F_n)} \left[B(u)B(v)\right]^2 = n\left(\frac{2}{2n-1} \times \frac{2}{2n-1}\right)^2 + 2n\left(\frac{2n}{2n-1} \times 2n\right)^2$$

$$= \frac{16n}{(2n-1)^4} + \frac{32n^5}{(2n-1)^2}.$$

By using definitions and by cardinalities of the Banhatti edge partition of F_n, we obtain the first and second hyper E-Banhatti polynomials of F_n.

Theorem 3: The first hyper E-Banhatti polynomial of F_n is given by

$$HEB_1(F_n, x) = nx^{\left(\frac{4}{2n-1}\right)^2} + 2nx^{\left(\frac{4n^2}{2n-1}\right)^2}.$$

Theorem 4: The second hyper E-Banhatti polynomial of F_n is given by

$$HEB_2(F_n, x) = nx^{\left(\frac{2}{2n-1}\right)^4} + 2nx^{\left(\frac{4n^2}{2n-1}\right)^2}.$$

4. RESULTS FOR WHEEL GRAPHS

A wheel graph W_n is the join of C_n and K_1. Then W_n has n+1vertices and 2n edges. A graph W_n is presented in Figure 2.

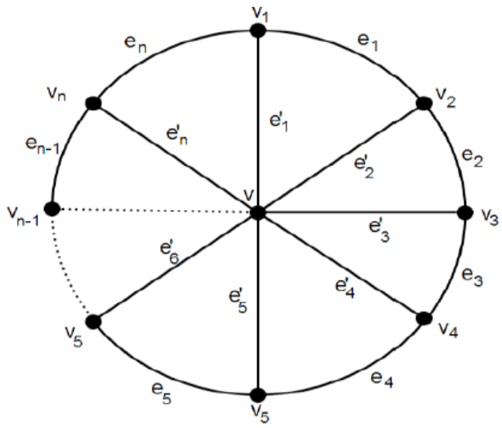

Figure-2: Wheel graph W_n

In W_n, there are two types of edges as follows:

$E_1 = \{uv \in E(W_n) \mid d(u) = d(v) = 3\}$, $|E_1| = n$.

$E_2 = \{uv \in E(W_n) \mid d(u) = 3, d(v) = n\}$, $|E_2| = n$.

Therefore, in W_n, there are two types of Banhatti edges based on Banhatti degrees of end vertices of each edge follow:

$$BE_1 = \left\{uv \in E(W_n) \mid B(u) = B(v) = \frac{4}{(n-2)}\right\}, \qquad |BE_1| = n.$$

$$BE_2 = \left\{uv \in E(W_n) \mid B(u) = \frac{n+1}{n-2}, B(v) = n+1\right\}, \qquad |BE_2| = n.$$

We determine the first hyper E-Banhatti index of a wheel graph W_n.

Theorem 5: Let W_n be a wheel graph with n + 1 vertices and 2n edges. Then

$$HEB_1(W_n) = \frac{64n}{(n-2)^2} + \frac{n(n^2-1)^2}{(n-2)^2}.$$

Proof: From definition and by cardinalities of the Banhatti edge partition of W_n, we obtain

$$HEB_1(W_n) = \sum_{uv \in E(W_n)} [B(u) + B(v)]^2 = n\left(\frac{4}{n-2} + \frac{4}{n-2}\right)^2 + n\left(\frac{n+1}{n-2} + n + 1\right)^2$$

$$= \frac{64n}{(n-2)^2} + \frac{n(n^2-1)^2}{(n-2)^2}.$$

In the next theorem, we compute the second hyper E-Banhatti index of a wheel graph W_n.

Theorem 6: Let W_n be a wheel graph with n + 1 vertices and 2n edges. Then

$$HEB_2(W_n) = \frac{256n}{(n-2)^4} + \frac{n(n+1)^2}{(n-2)^2}.$$

Proof: From definition and by cardinalities of the Banhatti edge partition of W_n, we obtain

$$HEB_2(W_n) = \sum_{uv\in E(W_n)} [B(u)B(v)]^2 = n\left(\frac{4}{n-2}\times\frac{4}{n-2}\right)^2 + n\left(\frac{n+1}{n-2}\times(n+1)\right)^2$$

$$= \frac{256n}{(n-2)^4} + \frac{n(n+1)^2}{(n-2)^2}.$$

By using definitions and by cardinalities of the Banhatti edge partition of W_n, we obtain the first and second hyper E-Banhatti polynomials of W_n.

Theorem 7: The first hyper E-Banhatti polynomial of W_n is given by

$$HEB_1(W_n, x) = nx^{\left(\frac{8}{n-2}\right)^2} + nx^{\left(\frac{n^2-1}{n-2}\right)^2}.$$

Theorem 8: The second hyper E-Banhatti polynomial of W_n is given by

$$HEB_2(W_n, x) = nx^{\frac{256}{(n-2)^4}} + nx^{\frac{(n+1)^2}{(n-2)^2}}.$$

5. RESULTS FOR SILICATE NETWORKS

Silicates are obtained by fusing metal oxide or metal carbonates with sand. A silicate network is symbolized by SL_n, where n is the number of hexagons between the center and boundary of SL_n. A 2-dimensional silicate network is depicted in Figure 3.

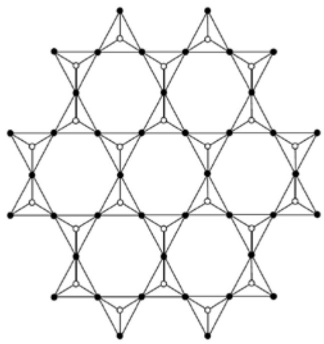

Figure-3: A 2-dimensional silicate network

Let G be the graph of a silicate network SL_n. By calculation, we obtain that G has $15n^2 + 3n$ vertices and $36n^2$ edges. In G, by calculation, there are three types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv \in E(SL_n) \mid d_G(u) = d_G(v) = 3\}$, $|E_1| = 6n$.
$E_2 = \{uv \in E(SL_n) \mid d_G(u) = 3, d_G(v) = 6\}$, $|E_2| = 18n^2 + 6n$.
$E_3 = \{uv \in E(SL_n) \mid d_G(u) = d_G(v) = 6\}$, $|E_3| = 18n^2 - 12n$.

Therefore, in SL_n, there are three types of Banhatti edges based on Banhatti degrees of end vertices of each edge as follow:

$$BE_1 = \left\{uv \in E(SL_n) \mid B(u) = B(v) = \frac{4}{(15n^2+3n-3)}\right\}, \quad |BE_1| = 6n.$$

$$BE_2 = \left\{uv \in E(SL_n) \mid B(u) = \frac{7}{15n^2+3n-3}.B(v) = \frac{7}{15n^2+3n-6}\right\}, \quad |BE_2| = 18n^2 + 6n.$$

$$BE_3 = \left\{uv \in E(SL_n) \mid B(u) = B(v) = \frac{10}{15n^2+3n-6}\right\}, \quad |BE_2| = 18n^2 - 12n.$$

In Theorem 9, we establish the first hyper E-Banhatti index of a silicate network SL_n.

Theorem 9: Let SL_n be a silicate network. Then

$$HEB_1(SL_n) = 6n\left(\frac{8}{15n^2+3n-3}\right)^2 + (18n^2 + 6n)\left(\frac{7}{15n^2+3n-3}+\frac{7}{15n^2+3n-6}\right)^2$$
$$+(18n^2 = 12n)\left(\frac{20}{15n^2+3n-6}\right)^2.$$

Proof: From definition and by cardinalities of the Banhatti edge partition of SL_n, we obtain

$$HEB_1(SL_n) = \sum_{uv\in E(SL_n)} [B(u)+B(v)]^2$$
$$= 6n\left(\frac{4}{15n^2+3n-3}+\frac{4}{15n^2+3n-3}\right)^2 + (18n^2 + 6n)\left(\frac{7}{15n^2+3n-3}+\frac{7}{15n^2+3n-6}\right)^2$$
$$+(18n^2 = 12n)\left(\frac{10}{15n^2+3n-6}+\frac{10}{15n^2+3n-6}\right)^2.$$

After simplification, we get the desired result.

In the following theorem, we obtain the second hyper E-Banhatti index of a silicate network SL_n.

Theorem 10: Let SL_n be a silicate network. Then

$$HEB_2(SL_n) = 6n\left(\frac{4}{15n^2+3n-3}\right)^4 + (18n^2 + 6n)\left(\frac{49}{(15n^2+3n-3)(15n^2+3n-6)}\right)^2$$
$$+(18n^2 = 12n)\left(\frac{10}{15n^2+3n-6}\right)^4.$$

Proof: From definition and by cardinalities of the Banhatti edge partition of SL_n, we obtain

$$HEB_2(SL_n) = \sum_{uv\in E(SL_n)} [B(u)B(v)]^2$$
$$= 6n\left(\frac{4}{(15n^2+3n-3)}\times\frac{4}{(15n^2+3n-3)}\right)^2 + (18n^2 + 6n)\left(\frac{7}{15n^2+3n-3}\times\frac{7}{15n^2+3n-6}\right)^2$$
$$+(18n^2 = 12n)\left(\frac{10}{15n^2+3n-6}\times\frac{10}{15n^2+3n-6}\right)^2$$

gives the desired result after simplification.

By using definitions and by cardinalities of the Banhatti edge partition of SL_n, we obtain the first and second hyper E-Banhatti polynomials of SL_n.

Theorem 11: The first hyper E-Banhatti polynomial of SL_n is given by

$$HEB_1(SL_n, x) = 6nx^{\left(\frac{8}{15n^2+3n-3}\right)^2} + (18n^2 + 6n)x^{\left(\frac{7}{15n^2+3n-3}+\frac{7}{15n^2+3n-6}\right)^2} + (18n^2 - 12n)x^{\left(\frac{20}{15n^2+3n-6}\right)^2}.$$

Theorem 12: The second hyper E-Banhatti polynomial of SL_n is given by

$$HEB_2(SL_n, x) = 6nx^{\left(\frac{4}{15n^2+3n-3}\right)^4} + (18n^2 + 6n)x^{\left(\frac{49}{(15n^2+3n-3)(15n^2+3n-6)}\right)^2} + (18n^2 - 12n)x^{\left(\frac{10}{15n^2+3n-6}\right)^4}.$$

6. RESULTS FOR HONEYCOMB NETWORKS

Honeycomb networks are useful in Computer Graphics and Chemistry. A honeycomb network of dimension n is denoted by HC_n, where n is the number of hexagons between central and boundary hexagon. A 4-dimensional honeycomb network is shown in Figure 4.

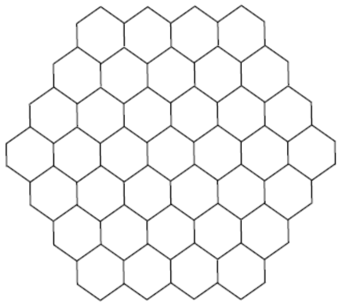

Figure-4: A 4-dimensional honeycomb network

Let G be the graph of a honeycomb network HC_n. By calculation, we obtain that G has $6n^2$ vertices and $9n^2–3n$ edges. In G, by algebraic method, there are three types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv \in E(HC_n) \mid d_G(u) = d_G(v) = 2\}$, $\quad |E_1| = 6$.

$E_2 = \{uv \in E(HC_n) \mid d_G(u) = 2, d_G(v) = 3\}$, $\quad |E_2| = 12n – 12$.

$E_3 = \{uv \in E(HC_n) \mid d_G(u) = d_G(v) = 3\}$, $\quad |E_3| = 9n^2 – 15n + 6$.

Therefore, in HC_n, there are three types of Banhatti edges based on Banhatti degrees of end vertices of each edge as follow:

$$BE_1 = \left\{ uv \in E\left(HC_n\right) \mid \; B(u) = B(v) \;\; \frac{2}{6n^2-2} \right\}, \qquad |BE_1| = 6.$$

$$BE_2 = \left\{ uv \in \mathrm{E}\left(\mathrm{HC_n}\right) B(u) = \frac{2}{6n^2-2}, B(v) = \frac{3}{6n^2-3} \right\}, \qquad |BE_2| = 12n - 12.$$

$$BE_3 = \left\{ uv \in E\left(HC_n\right) \mid \; B(u) = B(v) = \frac{4}{6n^2-3} \right\}, \qquad |BE_2| = 9n^2 - 15n + 6.$$

We now compute the first hyper E-Banhatti index of a honeycomb network HC_n.

Theorem 13: Let HC_n be a honeycomb network. Then

$$HEB_1\left(HC_n\right) = 6\left(\frac{4}{6n^2-2}\right)^2 + (12n-12)\left(\frac{2}{6n^2-2} + \frac{3}{6n^2-3}\right)^2 + \left(9n^2-15n+6\right)\left(\frac{8}{6n^2-3}\right)^2.$$

Proof: From definition and by cardinalities of the Banhatti edge partition of HC_n, we obtain

$$HEB_1\left(HC_n\right) = \sum_{uv \in E\left(HC_n\right)} \left[B(u) + B(v)\right]^2$$

$$= 6\left(\frac{2}{6n^2-2} + \frac{2}{6n^2-2}\right)^2 + (12n-12)\left(\frac{2}{6n^2-2} + \frac{3}{6n^2-3}\right)^2$$

$$+ \left(9n^2-15n+6\right)\left(\frac{4}{6n^2-3} + \frac{4}{6n^2-3}\right)^2.$$

After simplification, we obtain the desired result.

We determine the second hyper E-Banhatti index of a honeycomb network HC_n.

Theorem 14: Let HC_n be a honeycomb network. Then

$$HEB_2\left(HC_n\right) = 6\left(\frac{2}{6n^2-2}\right)^4 + (12n-12)\left(\frac{6}{\left(6n^2-2\right)\left(6n^2-3\right)}\right)^2 + \left(9n^2-15n+6\right)\left(\frac{4}{6n^2-3}\right)^4.$$

Proof: From definition and by cardinalities of the Banhatti edge partition of HC_n, we obtain

$$HEB_2\left(HC_n\right) = \sum_{uv \in E\left(HC_n\right)} \left[B(u) B(v)\right]^2$$

$$= 6\left(\frac{2}{6n^2-2} \times \frac{2}{6n^2-2}\right)^2 + (12n-12)\left(\frac{2}{6n^2-2} \times \frac{3}{6n^2-3}\right)^2$$

$$+ \left(9n^2-15n+6\right)\left(\frac{4}{6n^2-3} \times \frac{4}{6n^2-3}\right)^2$$

gives the desired result after simplification.

By using definitions and by cardinalities of the Banhatti edge partition of HC_n, we obtain the first and second hyper E-Banhatti polynomials of HC_n.

Theorem 15: The first hyper E-Banhatti polynomial of HC_n is given by

$$HEB_1(HC_n, x) = 6x^{\left(\frac{4}{6n^2-2}\right)^2} + (12n-12)\,x^{\left(\frac{2}{6n^2-2}+\frac{3}{6n^2-3}\right)^2} + (9n^2-15n+6)\,x^{\left(\frac{8}{6n^2-3}\right)^2}.$$

Theorem 16: The second hyper E-Banhatti polynomial of HC_n is given by

$$HEB_2(HC_n, x) = 6x^{\left(\frac{2}{6n^2-2}\right)^4} + (12n=12)\,x^{\left(\frac{6}{(6n^2-2)(6n^2-3)}\right)^2} + (9n^2-15n+6)\,x^{\left(\frac{4}{6n^2-3}\right)^4}.$$

7. CONCLUSION

In this study, we have introduced the first and second hyper E-Banhatti indices of a graph. Furthermore, we have determined these newly defined indices for some standard graphs, wheel graphs, friendship graphs and certain networks. This study is a new direction in The Theory of Graph Index in Graphs.

Many questions are suggested by this research, among them are the following:

1. Obtain properties of the first and second hyper E-Banhatti indices.
2. Compute these two indices for other chemical nanostructures.

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