

CERTAIN TRANSFORMATION BASIC AND POLY-BASIC HYPERGEOMETRIC SERIES

DR. BRIJESH PRATAP SINGH*¹

Department of Mathematics,
Raja Harpal Singh Mahavidyalay, Singramau, Jaunpur (U.P.), India.

DR. RAJEEV KUMAR SINGH²

Department of Mathematics,
P. B. P. G. College, Pratapgarh City, Pratapgarh (U.P.), India.

(Received On: 17-10-22; Revised & Accepted On: 18-11-22)

ABSTRACT

In this work, new transformations of fundamental and poly-fundamental hypergeometric series were established using the Bailey lemma. Using Bailey's transform and known sums of partial series, specific transformations of poly-basic hypergeometric series have been tried. There are some established transformations of basic multi-base hypergeometric series. Some of them connect two products from the Q -series. The bi-basic and poly-basic q -series are altered in some really fascinating ways as a result of these discoveries.

Keyword: summation formula, transformation formula, basic hypergeometric series, poly - basic hypergeometric series.

INTRODUCTION

The multiplicity-free Wigner and Racah coefficients ($3j$ and $6j$ -symbols) of the group $SU(n+1)$ are multiplicity-free hypergeometric series linked to root systems that first surfaced implicitly in the work of Alisaukas, Jucys and Jucysas well as Chac'on, Cifan, and Biedenharn in the context of unitary group representation theory in 1972. A first summation theorem was produced by Holman, Biedenharn, and Louck after they organized their examination of these coefficients as generalized hypergeometric series. [1] Because they include explicit summands and contain the root system's Weyl denominator, the preceding series can be considered hypergeometric series linked to the root system A .

Using q -analogue generalizations, elliptic hypergeometric series generalize generalized hypergeometric series, which in turn generalizes basic hypergeometric series. They are also known as q -hypergeometric series. The series x_n is referred to as "hypergeometric" when the relationship between the subsequent terms, x_{n+1}/x_n , is a rational function of n . A fundamental hypergeometric series' terms are ordered in ratio according to a rational function of qn . The base's name is Q . [2]

Simple hypergeometric series have gained in importance over the last forty years or so due to its applications in a range of disciplines such as additive number theory, combinatorial analysis, statistical and quantum physics, vector spaces, and so on. They also developed a highly useful method for analysts to aggregate and total distinct independent discoveries in number theory. Basic hypergeometric series, also known as q -hypergeometric series, have been studied in such depth that they are now regarded as a separate field with a strong reputation as opposed to being merely a generalization of more prevalent hypergeometric series. [3]

Define $[q]$ 1 for the a and q complex numbers as usual.

$$\begin{aligned} [a; q]_0 &= 1 \\ [a; q]_n &= (1-a)(1-aq)(1-aq^2) \dots (1-aq^{n-1}), \quad \text{for } n \in N, \\ [a_1, a_2, a_3, \dots, a_r; q]_n &= [a_1; q]_n [a_2; q]_n [a_3; q]_n \dots [a_r; q]_n, \\ [a; q]_\infty &= \prod_{r=0}^{\infty} (1-aq^r). \end{aligned}$$

Corresponding Author: Dr. Brijesh Pratap Singh*¹

Department of Mathematics, Raja Harpal Singh Mahavidyalay, Singramau, Jaunpur (U.P.), India.

What constitutes an $r\Phi_s$ fundamental hypergeometric series?

$${}_r\Phi_s \left[\begin{matrix} a_1, a_2, a_3, \dots, a_r; q; z \\ b_1, b_2, b_3, \dots, b_s \end{matrix} \right] = \sum_{n=0}^{\infty} \frac{[a_1, a_2, a_3, \dots, a_r; q]_n}{[q, b_1, b_2, b_3, \dots, b_s; q]_n} [(-)^n q^{n(n-1)/2}]^{1+s-r} z^n. \quad (1.1)$$

The definition of a poly-basic hypergeometric series is

$$\begin{aligned} \phi \left[\begin{matrix} a_1, a_2, \dots, a_r; c_{1,1}, \dots, c_{1,r_1}; \dots; c_{m,1}, \dots, c_{m,r_m}; q, q_1, \dots, q_m; z \\ b_1, b_2, \dots, b_s; d_{1,1}, \dots, d_{1,s_1}; \dots; d_{m,1}, \dots, d_{m,s_m} \end{matrix} \right] \\ = \sum_{n=0}^{\infty} \frac{[a_1, a_2, a_3, \dots, a_r; q]_n}{[q, b_1, b_2, b_3, \dots, b_s; q]_n} [(-)^n q^{n(n-1)/2}]^{1+s-r} z^n \\ \times \prod_{j=1}^m \frac{[c_{j,1}, c_{j,2}, \dots, c_{j,r_j}; q_j]_n}{[d_{j,1}, d_{j,2}, \dots, d_{j,s_j}; q_j]_n} [(-)^n q^{n(n-1)/2}]^{s_j-r_j} \end{aligned} \quad (1.2)$$

We'll employ the subsequent known outcomes. [4]

$$\sum_{n=0}^{\infty} (-a)^n q^{n(n+1)/2} = \frac{1}{1+1} \frac{aq}{1+1} \frac{a(q^2-q)}{1+1} \frac{aq^3}{1+1} \frac{a(q^4-q^2)}{1+\dots} \quad (1.3)$$

[Andrews & Berndt 1; (6.2.29) p. 152]

$$\sum_{n=0}^{\infty} a^n q^{3n(n+1)/2} = [q; q]_{\infty} \sum_{n=0}^{\infty} \frac{[-aq^{n+1}; q]_n q^n}{[q; q]_n} \quad (1.4)$$

[Andrews & Berndt 1; (9.3.11) p. 230]

If

$$P_n(a) = [q^2; q^2]_{\infty} [-aq; q^2]_n \sum_{j=0}^n \begin{bmatrix} n \\ j \end{bmatrix}_{q^2} \frac{q^{2j}}{[-aq; q^2]_j}$$

Then

$$\lim_{n \rightarrow \infty} P_n(a) = \sum_{n=0}^{\infty} a^j q^{j^2}.$$

So,

$$\sum_{n=0}^{\infty} a^j q^{j^2} = [q^2; q^2]_{\infty} [-aq; q^2]_{\infty} \sum_{j=0}^{\infty} \frac{q^{2j}}{[q^2; q^2]_j [-aq; q^2]_j}. \quad (1.5)$$

[Andrews & Berndt 1; (13.8.2) p. 298]

If we put $m=0$ in [Gaspar & Rahman 2; App.II (II.36)] we get

$$\begin{aligned} \sum_{k=0}^n \frac{(1-adp^k q^k) \left(1 - \frac{bp^k}{dq^k}\right) [a, b; p]_k \left[c, \frac{ad^2}{bc}; q\right]_k q^k}{(1-ad)(1-b/d)[dq, adq/b; q]_k [adp/c, bcp/d; p]_k} \\ \sum_{k=0}^n \frac{(1-adp^k q^k) \left(1 - \frac{bp^k}{dq^k}\right) [a, b; p]_k \left[c, \frac{ad^2}{bc}; q\right]_k q^k}{(1-ad)(1-b/d)[dq, adq/b; q]_k [adp/c, bcp/d; p]_k} \\ = \frac{(1-a)(1-b)(1-c) \left(1 - \frac{ad^2}{bc}\right) [ap, bp; p]_n \left[cq, \frac{ad^2q}{bc}; q\right]_n}{\frac{d(1-ad)(1-b/d)(1-c/d)(1-ad/bc)[dq, adq/b; q]_n [adp/c, bcp/d; p]_n}{a^2 d(1-c/ad)(1-d/bc)(1-1/d)(1-b/ad)}} \\ = \frac{(1-a)(1-b)(1-c)(1-ad/bc)}{(1-ad)(1-b/d)(1-c/d)(1-ad/bc)}. \end{aligned} \quad (1.6)$$

RESEARCH METHODOLOGY

A research technique is a standard method for resolving a study topic through data collection, data analysis, and conclusions based on the study's findings. A research technique is a method for carrying out a research investigation. The deliberate collecting and evaluation of data with the goal of expanding knowledge in any discipline is known as research. The goal of the research is to use methodical ways to address real-world and philosophical issues. The current study is descriptive in nature and is based on secondary data gathered from a variety of sources such as books, journals, scholarly papers, government publications, printed reference materials, and development, education, and education.

RESULT AND DISCUSSION

Theorem 1: If

$$\beta_n = \sum_{r=0}^n \alpha_r, \tag{1.7}$$

After that, multiply both sides of (1.7) by $\Omega_n z^n$ and sum over n from 0 to ∞

We have,

$$\sum_{n=0}^{\infty} \Omega_n \beta_n z^n = \sum_{n=0}^{\infty} \Omega_n z^n \sum_{r=0}^n \alpha_r$$

Which by an appeal of the identity,

$$\sum_{n=0}^{\infty} \sum_{r=0}^n A(n, r) = \sum_{n=0}^{\infty} \sum_{r=0}^{\infty} A(n + r, r),$$

Gives,

$$\sum_{n=0}^{\infty} \Omega_n \beta_n z^n = \sum_{n=0}^{\infty} \sum_{r=0}^{\infty} \Omega_{n+r} z^{n+r} \alpha_r. \tag{1.8}$$

The series utilizing telescope is the source of the majority of summation equations for poly-basic hypergeometric series. Therefore, by using the terms of such a series for β_n in our equation, we may obtain α_n . (1.7). Again, we may obtain transformation equations for poly-basic hypergeometric series by inserting these α_n and β_n values into (1.8). The following theorem will be illustrated using this technique. [5-7]

Theorem 2:

choosing $\alpha_r = \frac{(1-adp^r q^r) \left(1 - \frac{bp^r}{dq^r}\right) [a, b; p]_r \left[c, \frac{ad^2}{bc}; q\right]_r q^r}{(1-ad)(1-b/d)[dq, adq/b; q]_r [adp/c, bcp/d; p]_r}$ in (1.7) and

Using (1.8), we get

$$\beta_n = \frac{(1-a)(1-b)(1-c) \left(1 - \frac{ad^2}{bc}\right) [ap, bp; p]_n \left[cq, \frac{ad^2 q}{bc}; q\right]_n}{d(1-ad)(1-b/d)(1-c/d)(1-ad/bc)[dq, adq/b; q]_n [adp/c, bcp/d; p]_n} - \frac{a^2 d(1-c/ad)(1-d/bc)(1-1/d)(1-b/ad)}{(1-ad)(1-b/d)(1-c/d)(1-ad/bc)}$$

Putting these values of α_n and β_n in (1.8) we have,

$$\begin{aligned} & \frac{(1-a)(1-b)(1-c) \left(1 - \frac{ad^2}{bc}\right)}{d(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \times \sum_{n=0}^{\infty} \Omega_n z^n \frac{[ap, bp; p]_n \left[cq, \frac{ad^2 q}{bc}; q\right]_n}{[dq, adq/b; q]_n [adp/c, bcp/d; p]_n} \\ & - \frac{a^2 d(1-c/ad)(1-d/bc)(1-1/d)(1-b/ad)}{(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \sum_{n=0}^{\infty} \Omega_n z^n \\ & \sum_{n=0}^{\infty} \sum_{r=0}^{\infty} \Omega_{n+r} z^{n+r} \frac{(1-adp^r q^r) \left(1 - \frac{bp^r}{dq^r}\right) [a, b; p]_r \left[c, \frac{ad^2}{bc}; q\right]_r q^r}{(1-ad)(1-b/d)[dq, adq/b; q]_r [adp/c, bcp/d; p]_r} \end{aligned} \tag{1.9}$$

[8-10]

MAIN TRANSFORMATION FORMULAE

The previous part's theorems will be used to establish the transformation equations in this section. [11-13]

(i) Taking $\Omega_n = q_1^{n(n+1)/2}$ in (1.9) we get,

$$\begin{aligned} & \frac{(1-a)(1-b)(1-c) \left(1 - \frac{ad^2}{bc}\right)}{d(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \times \sum_{n=0}^{\infty} \frac{[ap, bp; p]_n \left[cq, \frac{ad^2 q}{bc}; q\right]_n}{[dq, adq/b; q]_n [adp/c, bcp/d; p]_n} z^n q_1^{r(r+1)/2} \\ & - \frac{a^2 d(1-c/ad)(1-d/bc)(1-1/d)(1-b/ad)}{(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \sum_{n=0}^{\infty} q_1^{r(r+1)/2} z^n \end{aligned}$$

$$\begin{aligned}
 &= \sum_{r=0}^{\infty} \frac{(1 - adp^r q^r) \left(1 - \frac{bp^r}{dq^r}\right) [a, b; p]_r \left[c, \frac{ad^2}{bc}; q\right]_r (zq)^r q_1^{r(r+1)/2}}{(1 - ad)(1 - b/d)[dq, adq/b; q]_r [adp/c, bcp/d; p]_r} \times \\
 &\times \sum_{r=0}^{\infty} (zq_1^r)^n q_1^{n(n+1)/2}.
 \end{aligned} \tag{2.1}$$

(ii) Using (1.3) in (2.1) we get

$$\begin{aligned}
 &\frac{(1 - a)(1 - b)(1 - c) \left(1 - \frac{ad^2}{bc}\right)}{d(1 - ad)(1 - b/d)(1 - c/d)(1 - ad/bc)} \times \sum_{n=0}^{\infty} \frac{[ap, bp; p]_n \left[cq, \frac{ad^2 q}{bc}; q\right]_n z^n q_1^{n(n+1)/2}}{[aq, adq/b; q]_n [adp/c, bcp/d; p]_n} \\
 &= \sum_{r=0}^{\infty} \frac{(1 - adp^r q^r) \left(1 - \frac{bp^r}{dq^r}\right) [a, b; p]_r \left[c, \frac{ad^2}{bc}; q\right]_r (zq)^r q_1^{r(r+1)/2}}{(1 - ad)(1 - b/d)[dq, adq/b; q]_r [adp/c, bcp/d; p]_r} \times \\
 &\times \left\{ \frac{1}{1 - 1 +} \frac{zq_1^{r+1} zq_1^{r+1} (1 - q_1)}{1 - 1 +} \frac{zq_1^{r+3} zq_1^{r+2} (1 - q_1^2)}{1 - \dots} \right\} \\
 &+ \frac{a^2 d(1 - c/ad)(1 - d/bc)(1 - 1/d)(1 - b/ad)}{(1 - ad)(1 - b/d)(1 - c/d)(1 - ad/bc)} \times \\
 &\times \left\{ \frac{1}{1 - 1 +} \frac{zq_1 zq_1 (1 - q_1)}{1 - 1 +} \frac{zq_1^3 zq_1^2 (1 - q_1^2)}{1 - \dots} \right\}
 \end{aligned} \tag{2.2}$$

(iii) Taking $\Omega_n = q_1^{n^2}$ in (1.9) we get,

$$\begin{aligned}
 &\frac{(1 - a)(1 - b)(1 - c) \left(1 - \frac{ad^2}{bc}\right)}{d(1 - ad)(1 - b/d)(1 - c/d)(1 - ad/bc)} \times \sum_{n=0}^{\infty} \frac{[ap, bp; p]_n \left[cq, \frac{ad^2 q}{bc}; q\right]_n z^n q_1^{n^2}}{[dq, adq/b; q]_n [adp/c, bcp/d; p]_n} \\
 &= \sum_{r=0}^{\infty} \frac{(1 - adp^r q^r) \left(1 - \frac{bp^r}{dq^r}\right) [a, b; p]_r \left[c, \frac{ad^2}{bc}; q\right]_r (zq)^r q_1^r}{(1 - ad)(1 - b/d)[dq, adq/b; q]_r [adp/c, bcp/d; p]_r} \sum_{n=0}^{\infty} (zq_1^{2r})^n q_1^{n^2} \\
 &+ \frac{a^2 d(1 - c/ad)(1 - d/bc)(1 - 1/d)(1 - b/ad)}{(1 - ad)(1 - b/d)(1 - c/d)(1 - ad/bc)} \sum_{n=0}^{\infty} z^n q_1^{n^2}
 \end{aligned} \tag{2.3}$$

(iv) Using (1.5), we discover using the right side of (2.3),

$$\begin{aligned}
 &\frac{(1 - a)(1 - b)(1 - c) \left(1 - \frac{ad^2}{bc}\right)}{d(1 - ad)(1 - b/d)(1 - c/d)(1 - ad/bc)} \times \sum_{n=0}^{\infty} \frac{[ap, bp; p]_n \left[cq, \frac{ad^2 q}{bc}; q\right]_n z^n q_1^{n^2}}{[dq, adq/b; q]_n [adp/c, bcp/d; p]_n} \\
 &= [q_1^2; q_1^2]_{\infty} [-zq_1 q_1^2]_{\infty} \times \\
 &\times \sum_{n=0}^{\infty} \frac{(1 - adp^r q^r) \left(1 - \frac{bp^r}{dq^r}\right) [a, b; p]_r \left[c, \frac{ad^2}{bc}; q\right]_r (zq)^r q_1^{r^2}}{(1 - ad)(1 - b/d)[dq, adq/b; q]_r [adp/c, bcp/d; p]_r} {}_2\phi_1 \left[\begin{matrix} 0, 0; q_1^2; q_1^2 \\ -zq_1^{2r+1} \end{matrix} \right] \\
 &+ [q_1^2; q_1^2]_{\infty} [-zq_1; q_1^2]_{\infty} \frac{a^2 d(1 - c/ad)(1 - d/bc)(1 - 1/d)(1 - b/ad)}{(1 - ad)(1 - b/d)(1 - c/d)(1 - ad/bc)} \\
 &\times {}_2\phi_1 \left[\begin{matrix} 0, 0; q_1^2; q_1^2 \\ -zq_1 \end{matrix} \right]
 \end{aligned} \tag{2.4}$$

(v) Now, using (1.4) in (2.5) we get, [14-15]

$$\begin{aligned}
 &\frac{(1 - a)(1 - b)(1 - c) \left(1 - \frac{ad^2}{bc}\right)}{d(1 - ad)(1 - b/d)(1 - c/d)(1 - ad/bc)} \times \sum_{n=0}^{\infty} \frac{[ap, bp; p]_n \left[cq, \frac{ad^2 q}{bc}; q\right]_n z^n q_1^{3n(n+1)/2}}{[dq, adq/b; q]_n [adp/c, bcp/d; p]_n} \\
 &= [q_1; q_1]_{\infty} \sum_{r=0}^{\infty} \frac{(1 - adp^r q^r) \left(1 - \frac{bp^r}{dq^r}\right) [a, b; p]_r \left[c, \frac{ad^2}{bc}; q\right]_r (zq)^r q_1^{3n(n+1)/2}}{(1 - ad)(1 - b/d)[dq, adq/b; q]_r [adp/c, bcp/d; p]_r} \\
 &\times {}_4\Phi_3 \left[\begin{matrix} i\sqrt{zq_1^{3r+1}}, -i\sqrt{zq_1^{3r+1}}, iq_1\sqrt{zq_1^{3r}}, -iq_1\sqrt{zq_1^{3r}}; q; q \\ 0, 0, -zq_1^{3r} \end{matrix} \right] \\
 &+ [q_1; q_1]_{\infty} \frac{a^2 d(1 - c/ad)(1 - d/bc)(1 - 1/d)(1 - b/ad)}{(1 - ad)(1 - b/d)(1 - c/d)(1 - ad/bc)} \times \\
 &\times {}_4\Phi_3 \left[\begin{matrix} i\sqrt{zq_1}, -i\sqrt{zq_1}, iq_1\sqrt{z}, -iq_1\sqrt{z}; q; q \\ 0, 0, -zq \end{matrix} \right]
 \end{aligned} \tag{2.6}$$

Taking $c = q$ in (3.2) we have,

$$\begin{aligned} & \sum_{n=0}^{\infty} \frac{[ap, bp; p]_n \left[\frac{a}{b}; q\right]_n z^n q_1^{n(n+1)/2}}{[q, aq/b; q]_n [ap/q, bqp; p]_n} \left(\frac{1 - q^{n+1}}{1 - q}\right) \\ &= \sum_{r=0}^{\infty} \frac{(1 - ap^r q^r) \left(1 - \frac{bp^r}{q^r}\right) [a, b; p]_r \left[\frac{a}{bq}; q\right]_r q_1^{r(r+1)/2} (zq)^r}{(1 - a)(1 - b)[aq/b; q]_r [ap/q, bqp; p]_r} \\ & \times \left\{ \frac{1 - zq_1^{r+1}}{1 - 1 +} \frac{zq_1^{r+1}}{1 -} \frac{zq_1^{r+3}}{1 +} \frac{zq_1^{r+2}}{1 -} \frac{(1 - q_1^2)}{1 - \dots} \right\} \end{aligned}$$

(vi) Taking $d=1$ in (2.4) we get,

$$\begin{aligned} & \times \sum_{n=0}^{\infty} \frac{[ap, bp; p]_n \left[cq, \frac{aq}{bc}; q\right]_n z^n q_1^{n^2}}{[q, aq/b; q]_n [ap/c, bcp; p]_n} = [q_1^2; q_1^2]_{\infty} [-zq_1 q_1^2]_{\infty} \times \\ & \times \sum_{r=0}^{\infty} \frac{(1 - ap^r q^r) \left(1 - \frac{bp^r}{q^r}\right) [a, b; p]_r \left[c, \frac{a}{bc}; q\right]_r (zq)^r q_1^{r^2}}{(1 - a)(1 - b)[q, aq/b; q]_r [ap/c, bcp; p]_r [-zq_1 q_1^2]_r} {}_2\phi_1 \left[\begin{matrix} 0, 0; q_1^2; q_1^2 \\ -zq_1^{2r+1} \end{matrix} \right] \end{aligned}$$

(vii) Taking $d=1$ in (2.6) we have,

$$\begin{aligned} & \sum_{n=0}^{\infty} \frac{[ap, bp; p]_n \left[cq, \frac{aq}{bc}; q\right]_n z^n q_1^{3n(n+1)/2}}{[q, aq/b; q]_n [ap/c, bcp; p]_n} \\ &= [q_1; q_1]_{\infty} \sum_{r=0}^{\infty} \frac{(1 - ap^r q^r) \left(1 - \frac{bp^r}{q^r}\right) [a, b; p]_r \left[c, \frac{a}{bc}; q\right]_r (zq)^r q_1^{3r(r+1)/2}}{(1 - a)(1 - b)[q, aq/b; q]_r [ap/c, bcp; p]_r} \\ & \times {}_4\Phi_3 \left[\begin{matrix} i\sqrt{zq_1^{3r+1}}, -i\sqrt{zq_1^{3r+1}}, iq_1\sqrt{zq_1^{3r}}, -iq_1\sqrt{zq_1^{3r}}; q; q \\ 0, 0; -zq_1^{3r} \end{matrix} \right] \end{aligned}$$

CONCLUSION

Using known summations and transformations, we demonstrated in the previous section how the Bailey lemma may be utilized to find new transformations of fundamental hypergeometric series. Some of the transformations in the previous section generalize well-known transformation equations. Using Bailey's transform and various different summations formulae, the section's findings are extremely insightful and fascinating in the context of fundamental hypergeometric functions.

REFERENCES

1. Andrews, G. E. and Berndt, B. C., Ramanujan's Lost Notebook Part I, Springer 2005.
2. Gasper, G. and Rahman, M., Basic Hypergeometric Series, Cambridge University Press 1991.
3. Srivastava, H. M. and Manocha, H. L., A treatise on generating functions, Ellis Horwood limited, Halsted Press; a division of John Wiley and Sons, New York 1984
4. Srivastava HM, Karlsson PW (1985) Multiple Gaussian hypergeometric series, vol 347. Ellis Horwood Limited, Halsted Press, New York
5. Slater LJ (1966) Generalized hypergeometric functions. Cambridge University Press, Cambridge, p 90
6. Verma A, Jain VK (1980) Some summation formulae of basic hypergeometric series. Indian J Pure Appl Math 11 (8):1028, 1033, 1036, 1037
7. Andrews GE, Berndt BC (2005) Ramanujan's lost notebook, part I. Springer, New York, p 146, 150
8. Bailey, W.N., Identities of Rogers-Ramanujan type, Proc. London Math.Soc., 50 (2) (1949), 1-10.
9. Denis, R.Y., On certain summation of q-series and identities of Rogers-Ramanujan type, J. Math. Phys. Sci., 22 (1) (1988), 87-99.
10. Denis, R.Y., Singh, S.N., Singh, S.P., On certain Transformation and Summation formulae for q-series, Italian Journal of Pure and Applied Mathematics, 27 (2010), (79-190).
11. Ernst, T., A Method for q-calculus, J. of Nonlinear Math. Physics, (10) (2003), 487-525.
12. Exton, H., Handbook of Hypergeometric Integrals, Ellis Horwood Limited, Halsted press, John Wiley and Sons, Chichester, New York, 1978
13. Tariboon, J., Ntouyas, S.K., Agarwal, P., New concepts of fractional quantum calculus and applications to impulsive fractional q-difference equations, Advances in Difference Equations, (2015), 18.
14. G. E. Andrews, "A general theory of identities of the Rogers-Ramanujan type," Bulletin of the American Mathematical Society, vol. 80, pp. 1033-1052, 1974.

15. G. E. Andrews, "An analytic generalization of the Rogers-Ramanujan identities for odd moduli," Proceedings of the National Academy of Sciences of the United States of America, vol. 71, pp. 4082–4085, 1974.
16. G. E. Andrews, "Bailey's transform, lemma, chains and tree," in Special Functions 2000: Current Perspective and Future Directions, Tempe, AZ, vol. 30 of NATO Science Series. Series II, Mathematics, Physics and Chemistry, pp. 1–22, Kluwer Academic Publishers, Dordrecht, The Netherlands, 2001.

Source of support: Nil, Conflict of interest: None Declared.

[Copy right © 2022. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]