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CERTAIN TRANSFORMATION BASIC AND POLY-BASIC HYPERGEOMETRIC SERIES

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ABSTRACT

In this work, new transformations of fundamental and poly-fundamental hypergeometric series were established using the Bailey lemma. Using Bailey's transform and known sums of partial series, specific transformations of poly-basic hypergeometric series have been tried. There are some established transformations of basic multi-base hypergeometric series. Some of them connect two products from the Q-series. The bi-basic and poly-basic q-series are altered in some really fascinating ways as a result of these discoveries.

Keyword: *summation formula, transformation formula, basic hypergeometric series, poly - basic hypergeometric series.*

INTRODUCTION

The multiplicity-free Wigner and Racah coefficients (3j and 6j-symbols) of the group SU(n+ 1) are multiplicity-free hypergeometric series linked to root systems that first surfaced implicitly in the work of Alisauskas, Jucys and Jucysas well as Chac'on, Ciftan, and Biedenharn in the context of unitary group representation theory in 1972. A first summation theorem was produced by Holman, Biedenharn, and Louck after they organized their examination of these coefficients as generalized hypergeometric series. [1] Because they include explicit summands and contain the root system's Weyl denominator, the preceding series can be considered hypergeometric series linked to the root system A.

Using q-analogue generalizations, elliptic hypergeometric series generalize generalized hypergeometric series, which in turn generalizes basic hypergeometric series. They are also known as q-hypergeometric series. The series x_n is referred to as "hypergeometric" when the relationship between the subsequent terms, xn+1/xn, is a rational function of n. A fundamental hypergeometric series' terms are ordered in ratio according to a rational function of qn. The base's name is Q. [2]

Simple hypergeometric series have gained in importance over the last forty years or so due to its applications in a range of disciplines such as additive number theory, combinatorial analysis, statistical and quantum physics, vector spaces, and so on. They also developed a highly useful method for analysts to aggregate and total distinct independent discoveries in number theory. Basic hypergeometric series, also known as q-hypergeometric series, have been studied in such depth that they are now regarded as a separate field with a strong reputation as opposed to being merely a generalization of more prevalent hypergeometric series. [3]

Define |q| 1 for the a and q complex numbers as usual.

$$[a;q]_0 = 1$$
$$[a;q]_n = (1-a)(1-aq)(1-aq^2)\dots(1-aq^{n-1}), \quad for\ n \in N,$$
$$[a_1,a_2,a_3,\dots,a_r;q]_n = [a_1;q]_n[a_2;q]_n[a_3;q]_n\dots[a_r;q]_n,$$
$$[a;q]_\infty = \prod_{r=0}^{\infty}(1-aq^r).$$

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What constitutes an rΦs fundamental hypergeometric series?

$$ {}_r\Phi_s\left[\begin{matrix} a_1,a_2,a_3,\dots,a_r;q;z \\ b_1,b_2,b_3,\dots,b_s \end{matrix}\right] = \sum_{n=0}^{\infty}\frac{[a_1,a_2,a_3,\dots,a_r;q]_n}{[q,b_1,b_2,b_3,\dots,b_s;q]_n}\left[(-)^n q^{n(n-1)/2}\right]^{1+s-r} z^n. \qquad (1.1)$$

The definition of a poly-basic hypergeometric series is

$$\phi\left[\begin{matrix} a_1,a_2,\dots,a_r : c_{1,1},\dots,c_{1,r_1};\dots;c_{m,1},\dots,c_{m,r_m};q,q_1,\dots,q_m;z \\ b_1,b_2,\dots,b_s : d_{1,1},\dots,d_{1,s_1};\dots;d_{m,1},\dots,d_{m,s_m} \end{matrix}\right]$$
$$= \sum_{n=0}^{\infty}\frac{[a_1,a_2,a_3,\dots,a_r;q]_n}{[q,b_1,b_2,b_3,\dots,b_s;q]_n}\left[(-)^n q^{n(n-1)/2}\right]^{1+s-r} z^n$$
$$\times\prod_{j=1}^{m}\frac{\left[c_{j,1},c_{j,2},\dots,c_{j,r_j};q_j\right]_n}{\left[d_{j,1},d_{j,2},\dots,d_{j,s_j};q\right]_n}\left[(-)^n q^{n(n-1)/2}\right]^{s_j-r_j} \qquad (1.2)$$

We'll employ the subsequent known outcomes. [4]

$$\sum_{n=0}^{\infty}(-a)^n q^{n(n+1)/2} = \frac{1}{1+}\frac{aq}{1+}\frac{a(q^2-q)}{1+}\frac{aq^3}{1+}\frac{a(q^4-q^2)}{1+\cdots} \qquad (1.3)$$

[Andrews & Berndt 1; (6.2.29) p. 152]

$$\sum_{n=0}^{\infty}a^n q^{3n(n+1)/2} = [q;q]_\infty\sum_{n=0}^{\infty}\frac{[-aq^{n+1};q]_n q^n}{[q;q]_n} \qquad (1.4)$$

[Andrews & Berndt 1; (9.3.11) p. 230]

If

$$P_n(a) = [q^2;q^2]_\infty[-aq;q^2]_n\sum_{j=0}^{n}\begin{bmatrix} n \\ j \end{bmatrix}_{q^2}\frac{q^{2j}}{[-aq;q^2]_j}$$

Then

$$\lim_{n\to\infty}P_n(a) = \sum_{n=0}^{\infty}a^j q^{j^2}.$$

So,

$$\sum_{n=0}^{\infty}a^j q^{j^2} = [q^2;q^2]_\infty[-aq;q^2]_\infty\sum_{j=0}^{\infty}\frac{q^{2j}}{[q^2;q^2]_j[-aq;q^2]_j}. \qquad (1.5)$$

[Andrews & Berndt 1;(13.8.2) p. 298]

If we put m=0 in [Gasper & Rahman 2; App.II (II.36)we get

$$\sum_{k=0}^{n}\frac{(1-adp^k q^k)\left(1-\frac{bp^k}{dq^k}\right)[a,b;p]_k\left[c,\frac{ad^2}{bc};q\right]_k q^k}{(1-ad)(1-b/d)[dq,adq/b;q]_k[adp/c,bcp/d;p]_k}$$
$$\sum_{k=0}^{n}\frac{(1-adp^k q^k)\left(1-\frac{bp^k}{dq^k}\right)[a,b;p]_k\left[c,\frac{ad^2}{bc};q\right]_k q^k}{(1-ad)(1-b/d)[dq,adq/b;q]_k[adp/c,bcp/d;p]_k}$$
$$=\frac{(1-a)(1-b)(1-c)\left(1-\frac{ad^2}{bc}\right)[ap,bp;p]_n\left[cq,\frac{ad^2q}{bc};q\right]_n}{d(1-ad)(1-b/d)(1-c/d)(1-ad/bc)[dq,adq/b;q]_n[adp/c,bcp/d;p]_n}$$
$$-\frac{a^2d(1-c/ad)(1-d/bc)(1-1/d)(1-b/ad)}{(1-ad)(1-b/d)(1-c/d)(1-ad/bc)}. \qquad (1.6)$$

RESEARCH METHODOLOGY

A research technique is a standard method for resolving a study topic through data collection, data analysis, and conclusions based on the study's findings. A research technique is a method for carrying out a research investigation. The deliberate collecting and evaluation of data with the goal of expanding knowledge in any discipline is known as research. The goal of the research is to use methodical ways to address real-world and philosophical issues. The current study is descriptive in nature and is based on secondary data gathered from a variety of sources such as books, journals, scholarly papers, government publications, printed reference materials, and development, education, and education.

RESULT AND DISCUSSION

Theorem 1: If

$$\beta_n = \sum_{r=0}^{n} \alpha_r , \tag{1.7}$$

After that, multiply both sides of (1.7) by $\Omega_n z^n$ and sum over n from 0 to ∞

We have,

$$\sum_{n=0}^{\infty} \Omega_n \beta_n z^n = \sum_{n=0}^{\infty} \Omega_n z^n \sum_{r=0}^{n} \alpha_r$$

Which by an appeal of the identity,

$$\sum_{n=0}^{\infty}\sum_{r=0}^{n} A(n,r) = \sum_{n=0}^{\infty}\sum_{r=0}^{\infty} A(n+r,r) ,$$

Gives,

$$\sum_{n=0}^{\infty} \Omega_n \beta_n z^n = \sum_{n=0}^{\infty}\sum_{r=0}^{\infty} \Omega_{n+r} z^{n+r} \alpha_r . \tag{1.8}$$

The series utilizing telescope is the source of the majority of summation equations for poly-basic hypergeometric series. Therefore, by using the terms of such a series for β_n in our equation, we may obtain α_n. (1.7). Again, we may obtain transformation equations for poly-basic hypergeometric series by inserting these α_n and β_n values into (1.8). The following theorem will be illustrated using this technique. [5-7]

Theorem 2:

$$choosing\ \alpha_r = \frac{(1-adp^r q^r)\left(1-\frac{bp^r}{dq^r}\right)[a,b;p]_r \left[c,\frac{ad^2}{bc};q\right]_r q^r}{(1-ad)(1-b/d)[dq,adq/b;q]_r [adp/c,bcp/d;p]_r}\ in\ (1.7) \text{ and}$$

Using (1.8), we get

$$\beta_n = \frac{(1-a)(1-b)(1-c)\left(1-\frac{ad^2}{bc}\right)[ap,bp;p]_n \left[cq,\frac{ad^2q}{bc};q\right]_n}{d(1-ad)(1-b/d)(1-c/d)(1-ad/bc)[dq,adq/b;q]_n [adp/c,bcp/d;p]_n} - \frac{a^2d(1-c/ad)(1-d/bc)(1-1/d)(1-b/ad)}{(1-ad)(1-b/d)(1-c/d)(1-ad/bc)}$$

Putting these values of α_n and β_n in (1.8) we have,

$$\frac{(1-a)(1-b)(1-c)\left(1-\frac{ad^2}{bc}\right)}{d(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \times \sum_{n=0}^{\infty} \Omega_n z^n \frac{[ap,bp;p]_n \left[cq,\frac{ad^2q}{bc};q\right]_n}{[dq,adq/b;q]_n [adp/c,bcp/d;p]_n}$$
$$- \frac{a^2d(1-c/ad)(1-d/bc)(1-1/d)(1-b/ad)}{(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \sum_{n=0}^{\infty} \Omega_n z^n$$
$$\sum_{n=0}^{\infty}\sum_{r=0}^{\infty} \Omega_{n+r} z^{n+r} \frac{(1-adp^r q^r)\left(1-\frac{bp^r}{dq^r}\right)[a,b;p]_r \left[c,\frac{ad^2}{bc};q\right]_r q^r}{(1-ad)(1-b/d)[dq,adq/b;q]_r [adp/c,bcp/d;p]_r} \tag{1.9}$$

[8-10]

MAIN TRANSFORMATION FORMULAE

The previous part's theorems will be used to establish the transformation equations in this section. [11-13]

(i)Taking $\Omega_n = q1^{n(n+1)/2}$ in (1.9) we get,

$$\frac{(1-a)(1-b)(1-c)\left(1-\frac{ad^2}{bc}\right)}{d(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \times \sum_{n=0}^{\infty} \frac{[ap,bp;p]_n \left[cq,\frac{ad^2q}{bc};q\right]_n z^n q_1^{r(r+1)/2}}{[dq,adq/b;q]_n [adp/c,bcp/d;p]_n}$$
$$- \frac{a^2d(1-c/ad)(1-d/bc)(1-1/d)(1-b/ad)}{(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \sum_{n=0}^{\infty} q_1^{r(r+1)/2} z^n$$

$$= \sum_{r=0}^{\infty} \frac{(1-adp^r q^r)\left(1-\frac{bp^r}{dq^r}\right)[a,b;p]_r \left[c, \frac{ad^2}{bc};q\right]_r (zq)^r q_1^{r(r+1)/2}}{(1-ad)(1-b/d)[dq, adq/b\,;q]_r [adp/c\,, bcp/d\,;p]_r} \times$$
$$\times \sum_{r=0}^{\infty} (zq_1^r)^n q_1^{n(n+1)/2}. \tag{2.1}$$

(ii) Using (1.3) in (2.1) we get

$$\frac{(1-a)(1-b)(1-c)\left(1-\frac{ad^2}{bc}\right)}{d(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \times \sum_{n=0}^{\infty} \frac{[ap,bp;p]_n \left[cq, \frac{ad^2q}{bc};q\right]_n z^n q_1^{n(n+1)/2}}{[aq, adq\;/\;b;q]_n [adp\;/\;c, bcp\;/\;d;p]_n}$$
$$= \sum_{r=0}^{\infty} \frac{(1-adp^r q^r)\left(1-\frac{bp^r}{dq^r}\right)[a,b;p]_r \left[c, \frac{ad^2}{bc};q\right]_r (zq)^r q_1^{r(r+1)/2}}{(1-ad)(1-b\;/\;d)[dq, adq\;/\;b;q]_r [adp\;/\;c, bcp\;/\;d;p]_r} \times$$
$$\times \left\{\frac{1}{1-}\frac{zq_1^{r+1}}{1+}\frac{zq_1^{r+1}(1-q_1)}{1-}\frac{zq_1^{r+3}}{1+}\frac{zq_1^{r+2}(1-q_1^2)}{1-\cdots}\right\}$$
$$+\frac{a^2d(1-c\;/\;ad)(1-d\;/\;bc)(1-1\;/\;d)(1-b\;/\;ad)}{(1-ad)(1-b\;/\;d)(1-c\;/\;d)(1-ad\;/\;bc)} \times$$
$$\times \left\{\frac{1}{1-}\frac{zq_1}{1+}\frac{zq_1(1-q_1)}{1-}\frac{zq_1^3}{1+}\frac{zq_1^2(1-q_1^2)}{1-\cdots}\right\} \tag{2.2}$$

(iii) Taking $\Omega_n = q_1^{n^2}$ in (1.9) we get,

$$\frac{(1-a)(1-b)(1-c)\left(1-\frac{ad^2}{bc}\right)}{d(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \times \sum_{n=0}^{\infty} \frac{[ap,bp;p]_n \left[cq, \frac{ad^2q}{bc};q\right]_n z^n q_1^{n^2}}{[dq, adq/b\,;q]_n [adp/c\,, bcp/d\,;p]_n}$$
$$= \sum_{r=0}^{\infty} \frac{(1-adp^r q^r)\left(1-\frac{bp^r}{dq^r}\right)[a,b;p]_r \left[c, \frac{ad^2}{bc};q\right]_r (zq)^r q_1^r}{(1-ad)(1-b/d)[dq, adq/b\,;q]_r [adp/c\,, bcp/d\,;p]_r} \sum_{n=0}^{\infty} (zq_1^{2r})^n q_1^{n^2}$$
$$+\frac{a^2d(1-c/ad)(1-d/bc)(1-1/d)(1-b/ad)}{(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \sum_{n=0}^{\infty} z^n q_1^{n^2} \tag{2.3}$$

(iv) Using (1.5), we discover using the right side of (2.3),

$$\frac{(1-a)(1-b)(1-c)\left(1-\frac{ad^2}{bc}\right)}{d(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \times \sum_{n=0}^{\infty} \frac{[ap,bp;p]_n \left[cq, \frac{ad^2q}{bc};q\right]_n z^n q_1^{n^2}}{[dq, adq/b\,;q]_n [adp/c\,, bcp/d\,;p]_n}$$
$$= [q_1^2;q_1^2]_\infty [-zq_1 q_1^2]_\infty \times$$
$$\times \sum_{n=0}^{\infty} \frac{(1-adp^r q^r)\left(1-\frac{bp^r}{dq^r}\right)[a,b;p]_r \left[c, \frac{ad^2}{bc};q\right]_r (zq)^r q_1^{r^2}}{(1-ad)(1-b/d)[dq, adq/b\,;q]_r [adp/c\,, bcp/d\,;p]_r} \;{}_2\phi_1 \left[\begin{matrix}0,0;q_1^2;q_1^2\\ -zq_1^{2r+1}\end{matrix}\right]$$
$$+[q_1^2;q_1^2]_\infty [-zq_1;q_1^2]_\infty \frac{a^2d(1-c/ad)(1-d/bc)(1-1/d)(1-b/ad)}{(1-ad)(1-b/d)(1-c/d)(1-ad/bc)}$$
$$\times \;{}_2\phi_1 \left[\begin{matrix}0,0;q_1^2;q_1^2\\ -zq_1\end{matrix}\right] \tag{2.4}$$

(v) Now, using (1.4) in (2.5) we get, [14-15]

$$\frac{(1-a)(1-b)(1-c)\left(1-\frac{ad^2}{bc}\right)}{d(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \times \sum_{n=0}^{\infty} \frac{[ap,bp;p]_n \left[cq, \frac{ad^2q}{bc};q\right]_n z^n q_1^{3n(n+1)/2}}{[dq, adq/b\,;q]_n [adp/c\,, bcp/d\,;p]_n}$$
$$= [q_1;q_1]_\infty \sum_{r=0}^{\infty} \frac{(1-adp^r q^r)\left(1-\frac{bp^r}{dq^r}\right)[a,b;p]_r \left[c, \frac{ad^2}{bc};q\right]_r (zq)^r q_1^{3n(n+1)/2}}{(1-ad)(1-b/d)[dq, adq/b\,;q]_r [adp/c\,, bcp/d\,;p]_r}$$
$$\times \;{}_4\Phi_3 \left[\begin{matrix}i\sqrt{zq_1^{3r+1}}, -i\sqrt{zq_1^{3r+1}}, iq_1\sqrt{zq_1^{3r}}, -iq_1\sqrt{zq_1^{3r}};q;q\\ 0,0;-zq_1^{3r}\end{matrix}\right]$$
$$+[q_1;q_1]_\infty \frac{a^2d(1-c/ad)(1-d/bc)(1-1/d)(1-b/ad)}{(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \times$$
$$\times \;{}_4\Phi_3 \left[\begin{matrix}i\sqrt{zq_1}, -i\sqrt{zq_1}, iq_1\sqrt{z}, -iq_1\sqrt{z};q;q\\ 0,0;-zq\end{matrix}\right] \tag{2.6}$$

Taking c = q in (3.2) we have,

$$\sum_{n=0}^{\infty}\frac{[ap,bp;p]_n\left[\frac{a}{b};q\right]_n z^n q_1^{n(n+1)/2}}{[q,aq/b\,;q]_n[ap/q\,,bqp;p]_n}\left(\frac{1-q^{n+1}}{1-q}\right)$$
$$=\sum_{r=0}^{\infty}\frac{(1-ap^rq^r)\left(1-\frac{bp^r}{q^r}\right)[a,b;p]_r\left[\frac{a}{bq};q\right]_r q_1^{r(r+1)/2}(zq)^r}{(1-a)(1-b)[aq/b\,;q]_r[ap/q\,,bqp;p]_r}$$
$$\times\left\{\frac{1}{1-}\frac{zq_1^{r+!}}{1+}\frac{zq_1^{r+1}(1-q_1)}{1-}\frac{zq_1^{r+3}}{1+}\frac{zq_1^{r+2}(1-q_1^2)}{1-\cdots}\right\}$$

(vi) Taking d=1 in (2.4) we get,

$$\times\sum_{n=0}^{\infty}\frac{[ap,bp;p]_n\left[cq,\frac{aq}{bc};q\right]_n z^n q_1^{n^2}}{[q,aq/b\,;q]_n[ap/c\,,bcp;p]_n}=[q_1^2;q_1^2]_\infty[-zq_1q_1^2]_\infty\times$$
$$\times\sum_{r=0}^{\infty}\frac{(1-ap^rq^r)\left(1-\frac{bp^r}{q^r}\right)[a,b;p]_r\left[c,\frac{a}{bc};q\right]_r(zq)^r q_1^{r^2}}{(1-a)(1-b)[q,aq/b\,;q]_r[ap/c\,,bcp;p]_r[-zq_1q_1^2]_r}\;{}_2\phi_1\begin{bmatrix}0,0;q_1^2;q_1^2\\-zq_1^{2r+1}\end{bmatrix}$$

(vii) Taking d=1 in (2.6) we have,

$$\sum_{n=0}^{\infty}\frac{[ap,bp;p]_n\left[cq,\frac{aq}{bc};q\right]_n z^n q_1^{3n(n+1)/2}}{[q,aq/b\,;q]_n[ap/c\,,bcp;p]_n}$$
$$=[q_1;q_1]_\infty\sum_{r=0}^{\infty}\frac{(1-ap^rq^r)\left(1-\frac{bp^r}{q^r}\right)[a,b;p]_r\left[c,\frac{a}{bc};q\right]_r(zq)^r q_1^{3r(r+1)/2}}{(1-a)(1-b)[q,aq/b\,;q]_r[ap/c\,,bcp;p]_r}$$
$$\times\;{}_4\Phi_3\begin{bmatrix}i\sqrt{zq_1^{3r+1}},-i\sqrt{zq_1^{3r+1}},iq_1\sqrt{zq_1^{3r}},-iq_1\sqrt{zq_1^{3r}};q;q\\0,0;-zq_1^{3r}\end{bmatrix}$$

CONCLUSION

Using known summations and transformations, we demonstrated in the previous section how the Bailey lemma may be utilized to find new transformations of fundamental hypergeometric series. Some of the transformations in the previous section generalize well-known transformation equations. Using Bailey's transform and various different summations formulae, the section's findings are extremely insightful and fascinating in the context of fundamental hypergeometric functions.

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