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A NOVEL APPROACH TO SOLVING FUZZY ASSIGNMENT PROBLEM

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ABSTRACT

There have been many methods provided to solve the fuzzy assignment problem by defuzzification of costs or by solving them without conversion to a crisp assignment problem. Here we have demonstrated a new technique to solve fuzzy assignment problems. We model the given fuzzy assignment problem as a graph and extract the degrees of its vertices and edges and solve the problem so as to obtain the optimal solution.

***Keywords**: ranking of fuzzy triangular number, bipartite graph, assignment problem, vertex, edge degree.*

***Mathematics Subject Classification:** 05C12, 03E72, 05C72.*

1. INTRODUCTION

The assignment problem is an important problem in operation research. There have been many methods given by various authors for solving these problems. The ranking of fuzzy numbers was provided in [5], for using it in decision-making. Bortolan and Degani [11] looked into some ranking methods for these numbers. The ranking based on the max and min set was given by [6]. [6] Proposed a new method of ranking trapezoidal fuzzy numbers based on centroid points. The ranking of triangular fuzzy numbers is given by [3].The ranking of fuzzy numbers by the centroid method was given by [4]. The Hungarian method to solve the fuzzy assignment problem was given by [8], whereas the ranking of numbers is given by Reuben's method. Santhi Maheswari and Sekar dealt with neighbourly irregular graphs [9, 10]. Amit *et.al*, [2] solved the fuzzy assignment problem, without the defuzzification of numbers.

In this, we develop a new method, considering the given fuzzy assignment problem as a graph model, and solve it by finding the degree of the vertices and edges in the graph. The solution thus obtained is more optimized than other proposed methods. We have solved the fuzzy assignment as given in [2] and [8] so that a comparison with the proposed methods can be made.

2. PRELIMINARIES

Definition 2.1: *A Fuzzy graph $G:(\sigma,\mu)$ on the graph $G^*:(V,E)$ is a pair of functions (σ,μ), where $\sigma: V \longrightarrow [0,1]$ is a fuzzy subset of a set V and $\mu: V \times V \longrightarrow [0,1]$ is a symmetric fuzzy relation on σ such that for all u,v in V, the relation $\mu(u,v) = \mu(uv) \le \sigma(u) \wedge \sigma(v)$ is satisfied.*

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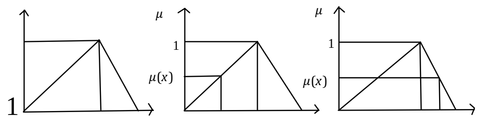
Definition 2.2: *(Triangular membership function)*
The triangle which fuzzifies the input is determined by three parameters a, b and c, where c is the base and b is the height of the triangle. The triangular membership function is given below and its graphical representation is also given. ***µtriangular (x; a; b; c) = max (min (x –a/b –a, c –x/c −b), 0)***

µ x = b$a \le x \le b$$b \le x \le c$



a bc axbc a bx c

$\mu(x) = 1$ $\mu(x) = x - a/b - a$ $\mu(x) = c - x/c - b$

$\mu(x) = 0$, x < a, x > c

Triangular membership function

Definition 2.3: *(Defuzzification by centroid rank method)*
The given triangular FN can be defuzzied by centroid rank method and it is given as $R(x) = (x_1 + x_2 + x_3)/3$

Definition 2.4: *(Assignment problem)*
The assignment problem is nothing but assigning n jobs to n people so that the cost of allocating such jobs to people results in minimum cost or maximum profit. It can be represented as

$$min\ \tilde{z} = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij},$$

subject to the constraints $\sum_{i=1}^{n} m_{ij} = 1,\ \sum_{j=1}^{n} m_{ij} = 1$, *where* $m_{ij} \in [0,1]$.
job allotted to i^{th} *person*

$$m_{ij} = \begin{cases} 1, & if j^{th} \\ 0 & otherwise \end{cases}$$

Definition 2.5: *If G is a fuzzy graph, then the degree of a vertex is given as* $d_G(u) = \sum \mu(uv)$, *where−* $v \in N(u)$*. And the degree of the edge is given as∈* $d_G(xy) = d_G(x) + d_G(y)\ 2\mu(xy)$, *where xy E(G).*

Definition 2.6: *A fuzzy graph G =* (σ,μ) *is said to be bipartite, if the vertex set V is partitioned into two disjoint unions of vertex sets* V_1 *and* V_2*, such that* $x_i, y_i \in V_1 or V_2$.

3. ASSIGNMENT PROBLEM SOLVED BY A GRAPH MODEL

The assignment problem is a square matrix with equal resources and people. It can be applied in manufacturing units, delivery apps, and for easy scheduling of tasks in industries. It is used to designate the resources to the people so that the cost of designating them is minimized or maximized. That is, if we have n resources and n people, we find an optimal way to allocate the resources to the people so that the total cost is minimized or maximized. Each resource is given to exactly one and not more than one resource is allocated to the person.

The major objective of this, as said before is to min or max the total profit subject to certain constraints. If we consider i resources and j people, then c_{ij} denotes the cost to allot i^{th} resource to j^{th} person. As we use the fuzzy cost here, we represent it as $\tilde{c}_{ij}$. We defuzzied the triangular fuzzy numbers based on the centroid ranking method.

After defuzzification, we consider the set of resources and set of people as distinct sets of vertices of a bipartite graph, let us denote them as $V_1 = resources$, $V_2 = jobs$, and $V_1 \cap V_2 = 0$. We take the defuzzied cost c_{ij} as the membership value of edge ij. If the given problem is not balanced, i.e., the number of resources ≠ number of people, then we balance them out by adding a dummy resource or people. This dummy resource or people can be considered as the vertex of degree 0 in our graph model.

The proposed method provided a quick solution just by finding the degree of vertices and edges in our graph model. As we have n resources and n people, the total number of edges joining these two vertex sets is n^2. We then find the minimal optimal solution by allocating resources to a person based on the max degree of the edge connecting them. Similarly, for a maximal optimal solution, we take up the minimum degree of edges connecting the resource and person. We present here a numerical example and compare it with the results obtained from other methods.

3.1. Finding the Optimal Solution by the Proposed Method

Example 3.1: *Given 3 people and 3 jobs, and the cost of allocating them in terms of fuzzy cost. Find the optimal allocation of jobs to people so that the total cost is minimized also find another allocation which results in maximum cost. We consider the below problem from [2].*

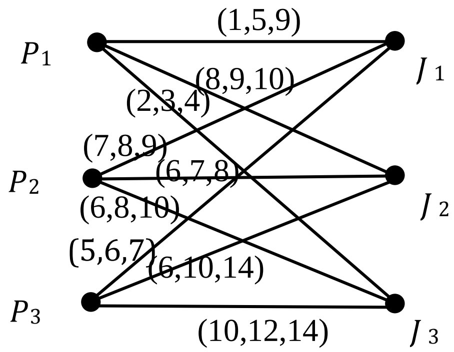
Person/job	J_1	J_2	J_3
P_1	(1,5,9)	(8,9,10)	(2,3,4)
P_2	(7,8,9)	(6,7,8)	(6,8,10)
P_3	(5,6,7)	(6,10,14)	(10,12,14)

We solve this by the proposed method.

Step-1: *Defuzzy the given problem using the centroid ranking method. Then we obtain the following*

Person/job	J_1	J_2	J_3
P_1	5	9	3
P_2	8	7	8
P_3	6	10	12

Step-2: *Consider P_1, P_2, P_3 and J_1, J_2, and J_3 as 2 distinct vertex sets of a bipartite graph in a fuzzy environment and the defuzzied values as membership values of the edges connecting these vertices.*



Step-3: *Find the degree of the vertices and the degree of all the edges. The degree of vertices is given as*
$d(P_1) = 5 + 9 + 3 = 17$, $d(P_2) = 23$, $d(P_3) = 28$, $d(J_1) = 5 + 8 + 6 = 19$, $d(J_2) = 26$, $d(J_3) = 23$.
The degree of the edges is $d(P_1J_1) = 17 + 19 - 2(5) = 26$, $d(P_1J_2) = 25$, $d(P_1J_3) = 34$, $d(P_2J_1) = 26$, $d(P_2J_2) = 35$, $d(P_2J_3) = 30$, $d(P_3J_1) = 35$, $d(P_3J_2) = 34$, $d(P_3J_3) = 27$.

Step-4: *We consider the edge degrees $d(P_iJ_1)$, $i=1,2,3$ and find the edge with the maximum degree. Here we have $d(P_1J_3)$ as the maximum value and no other edge degree connecting J_3 with another person is maximum, so we allot P_1 the job J_3.*
Next, consider $d(P_iJ_2)$, $i=2,3$ and $d(P_2J_2)$ are found maximum, so allot P_2 with J_2.
The remaining job J_1 is allocated to P_3.

Step-5: *The optimal assignment thus obtained is $P_1 \rightarrow J_3$, $P_2 \rightarrow J_2$, $P_3 \rightarrow J_1$.*

Step-6: *The minimal cost can be found by adding the defuzzied cost, here* (13,16,19). *Thus, the minimal cost is 16.*

Step-7: *We need to find the maximal cost, with another optimal assignment. Consider $d(P_iJ_1)$, $i=1,2,3$, and find the edge with a minimum degree, here we have $d(P_1J_2)$ as minimal and no other edge with minimal value. So, allot P_1 the job J_2.*
Next, find the minimum degree among the edges $d(P_iJ_2)$, $i=2,3$ and $d(P_2J_1)$ is minimal so allot P_2 with J_1. The remaining P_3 is allowed job J_3.

Step-8: *The other allotment for maximization is obtained as $P_1 \rightarrow J_2$, $P_2 \rightarrow J_1$, $P_3 \rightarrow J_3$. The maximal cost is* (25, 29, 33) *i.e., maximal cost=29.*

3.2. Comparison of the Proposed Model to Existing Models

Example 3.2: *We consider the problem, whose objective is to find the optimal solution so the total cost of such assignment is minimum [8]*

The optimal solution as obtained in [8] is given as $P_1 \to J_2$, $P_2 \to J_5$, $P_3 \to J_7$, $P_4 \to J_1$, $P_5 \to J_6$, $P_6 \to J_4$, $P_7 \to J_3$. *The minimal cost is* (31, 40, 87) *i.e., the minimal cost is 59.................(*)*

We solve it by the proposed method. The defuzzied values are tabulated below.

P/J	J_1	J_2	J_3	J_4	J_5	J_6	J_7
P_1	(11,12,13)	(5,6,13)	(7, 9, 11)	(3,10,11)	(3, 8, 13)	(12,14,16)	(7,9,11)
P_2	(7,9,11)	(9,10,11)	(5,8,11)	(8,10,12)	(5,6,7)	(9,11,13)	(8,10,11)
P_3	(3,9,15)	(7,8,13)	(6,7,9)	(4,5,12)	(4,6,12)	(8,15,18)	(3,4,17)
P_4	(5,6,13)	(13,17,21)	(5,7,13)	(8,10,16)	(8,15,18)	(7,11,15)	(3,9,13)
P_5	(6,8,10)	(6,9,12)	(9,11,13)	(11,13,15)	(8,9,10)	(6,8,12)	(4,8,12)
P_6	(10,14,18)	(9,13,17)	(4,6,10)	(2,4,18)	(3,8,17)	(6,12,14)	(7,8,13)
P_7	(4,7, 10)	(7,8,13)	(5,6,7)	(1,8,13)	(8,9,14)	(5,12,19)	(7,8,17)

P/J	J_1	J_2	J_3	J_4	J_5	J_6	J_7
P_1	12	8	9	8	8	14	9
P_2	9	10	8	10	6	11	9.6
P_3	9	9.3	7.3	7	7.3	13.6	8
P_4	8	17	8.3	11.3	13.6	11	8.3
P_5	8	9	11	13	9	8.6	8
P_6	14	13	6.6	8	9.3	10.6	9.3
P_7	7	9.3	6	7.3	10.3	12	10.6

The degree of the vertices are given as $d(P_1) = 12 + 8 + 9 + 8 + 8 + 14 + 9 = 68$, $d(P_2) = 63.6$, $d(P_3) = 61.5$, $d(P_4) = 77.5$, $d(P_5) = 66.6$, $d(P_6) = 70.8$, $d(P_7) = 62.5$, $d(J_1) = 67$, $d(J_2) = 75.6$, $d(J_3) = 56.2$, $d(J_4) = 64.6$, $d(J_5) = 63.5$, $d(J_6) = 80.8$, $d(J_7) = 62.8$.

The degree of the edges is given as
$d(P1J1) = 68 + 67 - 2(12) = 111$, $d(P1J2) = 127.6$, $d(P1J3) = 106.2$, $d(P1J4) = 116.6$, $d(P1J5) = 115.5$, $d(P1J6) = 120.8$, $d(P1J7) = 112.8$,

$d(P_2J_1) = 112.6$, $d(P_2J_2) = 119.2$, $d(P_2J_3) = 103.8$, $d(P_2J_4) = 108.2$, $d(P_2J_5) = 115.1$, $d(P_2J_6) = 122.4$, $d(P_2J_7) = 107.2$,

$d(P_3J_1) = 110.5$, $d(P_3J_2) = 118.5$, $d(P_3J_3) = 103.1$, $d(P_3J_4) = 112.1$, $d(P_3J_5) = 110.4$, $d(P_3J_6) = 115.1$, $d(P_3J_7) = 108.3$,

$d(P_4J_1) = 128.5$, $d(P_4J_2) = 119.1$, $d(P_4J_3) = 117.1$, $d(P_4J_4) = 119.5$, $d(P_4J_5) = 113.8$, $d(P_4J_6) = 136.3$, $d(P_4J_7) = 123.7$,

$d(P_5J_1) = 117.6$, $d(P_5J_2) = 124.2$, $d(P_5J_3) = 100.8$, $d(P_5J_4) = 105.2$, $d(P_5J_5) = 112.1$, $d(P_5J_6) = 130.2$, $d(P_5J_7) = 113.4$,

$d(P_6J_1) = 109.8$, $d(P_6J_2) = 120.4$, $d(P_6J_3) = 113.8$, $d(P_6J_4) = 119.4$, $d(P_6J_5) = 115.7$, $d(P_6J_6) = 130.4$, $d(P_6J_7) = 115$,
$d(P_7J_1) = 115.5$, $d(P_7J_2) = 119.5$, $d(P_7J_3) = 106.7$, $d(P_7J_4) = 112.5$, $d(P_7J_5) = 105.4$, $d(P_7J_6) = 119.3$,
$d(P_7J_7) = 104.1$.

1. Consider $d(P_iJ_1)$, i = 1 to 7, and find the edge with the maximum degree, here we have $d(P_4J_1)$ greater than others, but $d(P_4J_6) > d(P_4J_1)$, and no other edge containing J_6 has max value, so allot P_4 the job J_6. Let us consider J_1 later.
2. Next consider $d(P_iJ_2)$, i=1,2,3,5,6,7, here $d(P_1J_2)$ has a maximum value and no other edge containing J_2 has max value, so allot P_1 to J_2.
3. Now find the maximum degree of the edges $d(P_iJ_3)$, i=2,3,5,6,7, here $d(P_6J_3)$ is max, but $d(P_6J_4) > d(P_6J_3)$, and we allot P_6 to J_4.
4. Considering $d(P_iJ_5)$, i=2,3,5,7, the max edge degree is of P_2J_5 and so allocate P_2 the job J_5.
5. Find the max degree in $d(P_iJ_1)$, i=3,5,7 the edge degree of P_5J_1 is max, so allotted is obtained.
6. We are left with only 2 edges, and $d(P_3J_7) > d(P_7J_7)$, so allot P_3 with J_7, and the remaining P_7 with J_3.

The optimal allotment thus obtained is $P_1 \rightarrow J_2$, $P_2 \rightarrow J_5$, $P_3 \rightarrow J_7$, $P_4 \rightarrow J_6$, $P_5 \rightarrow J_1$, $P_6 \rightarrow J_4$, $P_7 \rightarrow J_3$. The minimal cost thus obtained is (33, 45, 87) i.e., minimal cost=55.

3.3 CONCLUSION

The minimal cost obtained from (*) is 59, whereas by the proposed method we get a more optimized value. The proposed method is very easy to solve as it involves only the degree of edges connecting the vertices of both sets. The optimal allotment for maximal cost can be obtained by considering the edges with a minimum degree, while choosing for allotment, the same is illustrated in Example 3.1. Here we considered examples from previous articles, to compare the optimality of the solutions. Any fresh example of an assignment problem with fuzzy costs can be solved using this proposed method easily to obtain a favourable solution.

REFERENCES

1. Amei Yu, Mei Lu, and Feng Tian, (2004),On the Spectral radius of graphs, Linear Algebra and its Applications, 387, 41-49.
2. Amit kumar, Anil Gupta and amarpreet Kaur,(2009), Method For solving Fully fuzzy assignment problems using triangular fuzzy numbers, International journal of Computer and Information Engineering, vol.3, no.7, 18891892.
3. S.Vimala, S.Krishna Prabha,(2016), Assignment Problems with fuzzy costs using Ones Assignment Method, IOSR Journal of Mathematics, Vol.12, Issue 5,85-89
4. Fateen Najwa Azman and Lazim Abdullah, (2012), Ranking Fuzzy Numbers by Centroid Method, Malaysian Journal of Fundamental and Applied Sciences, vol.8, no.3, 121-125.
5. Jain. R.,(1978), A procedure for multi-aspect decision making using fuzzy sets, International Journal of Systems Science, vol. 8, no.1,17.
6. Chen, S.J., Chen,S.M., (2007), Fuzzy Risk Analysis based on the Ranking of Generalized Trapezoidal Fuzzy Numbers, Applied Intelligence 26, 111
7. Chang W., (1981), Ranking of fuzzy utilities with triangular membership functions, Proceedings of International Conference on Policy Analysis and Systems, 263272.
8. Mohamed Muamer, (2020), Fuzzy Assignment problems, Journal of Science, vol.10, 40-47.
9. N.R.Santhi Maheswari and Sekar,(2015), On m-Neighbourly irregular Fuzzy graphs, International Journal of Mathematics and soft computing,vol.5, 145153
10. N.R.Santhi Maheswari and Sekar, (2014), Neighbourly irregular graphs and semi neighbourly irregular graphs, Acta Cienia Indica, vol.XLM, no.1, 71-77.
11. Bortolan .G. and Degani, (1985), A review of some methods for ranking fuzzy subsets, Fuzzy Sets and Systems, vol. 15, no. 1,119.
12. L.Subha Lakshmi and Dr N.R.Santhi Maheswari, On Pseudo Highly and Pseudo Strongly Edge irregular Graphs, Proceedings of International Conference-ICRRMCSA'22, (2022), 61-69
13. L.Subha Lakshmi and Dr N.R.Santhi Maheshwari, On support strongly irregular Fuzzy Soft Graphs, Third International Conference on Applied Mathematics and Intellectual Property Rights, (2022)
14. L.Subha Lakshmi and Dr N.R.Santhi Maheshwari, Support Edge Irregularity of FSG, 3rd International Conference on Mathematical Modelling, Analysis and computing, (2022).
15. S.P.Nandhini and E.Nandhini, Strongly irregular fuzzy graphs, International Journal of Mathematical archive, (2016), vol.5, no.5, pp.110-114

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