

MULTIPLICATIVE NIRMALA AND BANHATTI-NIRMALA INDICES
OF CERTAIN NANOSTAR DENDRIMERS

V. R. KULLI*

Department of Mathematics, Gulbarga University, Gulbarga-585106, India.

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ABSTRACT

In this paper, we introduce the multiplicative Nirmala index and the multiplicative Banhatti-Nirmala index of a graph. Also we determine these newly defined multiplicative Nirmala indices for some standard graphs. Furthermore, we compute the multiplicative Nirmala and multiplicative Banhatti-Nirmala indices of certain important nanostar dendrimers such as $NS_1[n]$, $NS_2[n]$, $NS_3[n]$, $D_1[n]$ and $D_3[n]$ nanostar dendrimers.

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1. INTRODUCTION

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u . The edge connecting the vertices u and v will be denoted by uv . We refer the book [1] for undefined term and notation.

Chemical Graph Theory is a branch of Mathematical Chemistry. It has an important effect on the development of the Chemical Sciences. A chemical graph is a simple graph related to the structure of a chemical compound. Each vertex of this graph represents an atom of the molecule and its edges to the bonds between atoms. Numerous topological indices are useful for establishing correlations between the structure of a molecular compound and its physicochemical properties, see [2].

In [3], Kulli introduced the Nirmala index of a graph G and defined it as

$$N(G) = \sum_{uv \in E(G)} \sqrt{d_G(u) + d_G(v)}.$$

Recently, some Nirmala indices were studied, for example, in [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

The Banhatti-Nirmala index of a graph was introduced by Kulli in [16] and defined it as

$$BN(G) = \sum_{ue} \sqrt{d_G(u) + d_G(e)}.$$

In [17], the multiplicative Nirmala index of a graph G is defined as

$$NII(G) = \prod_{uv \in E(G)} \sqrt{d_G(u) + d_G(v)}.$$

Inspired by work on Nirmala indices, we introduce the multiplicative Banhatti-Nirmala index of a graph as follows:

The multiplicative Banhatti-Nirmala index of a graph G is defined as

$$BNII(G) = \prod_{ue} \sqrt{d_G(u) + d_G(e)}.$$

Corresponding Author: V. R. Kulli*

Department of Mathematics, Gulbarga University, Gulbarga-585106, India.

We can express the multiplicative Banhatti-Nirmala index as

$$BNII(G) = \prod_{uv \in E(G)} \left[\sqrt{d_G(u) + (d_G(u) + d_G(v) - 2)} + \sqrt{d_G(v) + (d_G(u) + d_G(v) - 2)} \right].$$

Recently, some multiplicative indices were studied in [18].

In this paper, we compute the multiplicative Nirmala index and the multiplicative Banhatti-Nirmala index of certain nanostar dendrimers.

2. RESULTS FOR SOME STANDARD GRAPHS

Proposition 1: Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$, and $s \geq 2$ vertices. Then

$$NII(K_{r,s}) = (\sqrt{r+s})^{rs}.$$

Proof: Let $G = K_{r,s}$ be a complete bipartite graph with $r+s$ vertices and rs edges such that $|V_1|=r$, $|V_2|=s$, $V(K_{r,s}) = V_1 \cup V_2$ for $1 \leq r \leq s$; $s \geq 2$. Every vertex of V_1 is incident with s edges and every vertex of V_2 is incident with r edges.

$$NII(K_{r,s}) = \prod_{uv \in E(G)} \sqrt{d_G(u) + d_G(v)} = (\sqrt{r+s})^{rs}.$$

Corollary 1.1: Let $K_{r,r}$ be a complete bipartite graph with $r \geq 2$. Then

$$NII(K_{r,r}) = (\sqrt{2r})^{r^2}.$$

Corollary 1.2: Let $K_{1,r}$ be a star with $r \geq 1$. Then

$$NII(K_{1,r}) = (\sqrt{1+r})^r.$$

Proposition 2: If G is an r -regular graph with n vertices and $r \geq 2$, then

$$NII(G) = (2r)^{\frac{nr}{4}}.$$

Proof: Let G be an r -regular graph with n vertices, $r \geq 2$ and $\frac{nr}{2}$ edges.

$$\begin{aligned} NII(G) &= \prod_{uv \in E(G)} \sqrt{d_G(u) + d_G(v)}. \\ &= (\sqrt{r+r})^{\frac{nr}{2}} = (2r)^{\frac{nr}{4}}. \end{aligned}$$

Corollary 2.1: Let C_n be a cycle with $n \geq 3$ vertices. Then $NII(C_n) = 2^n$.

Corollary 2.2: Let K_n be a complete graph with $n \geq 3$ vertices. Then

$$NII(K_n) = [2(n-1)]^{\frac{n(n-1)}{4}}.$$

Proposition 3: If P_n is a path with n vertices and $n \geq 3$, then $NII(P_n) = 3 \times 2^{n-3}$.

Proposition 4: Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$, and $s \geq 2$ vertices. Then

$$BNII(K_{r,s}) = [\sqrt{2r+s-2} + \sqrt{r+2s-2}]^{rs}.$$

Proof: Let $G = K_{r,s}$ be a complete bipartite graph with $r+s$ vertices and rs edges such that $|V_1|=r$, $|V_2|=s$, $V(K_{r,s}) = V_1 \cup V_2$ for $1 \leq r \leq s$; $s \geq 2$. Every vertex of V_1 is incident with s edges and every vertex of V_2 is incident with r edges.

$$\begin{aligned} BNII(G) &= \prod_{uv \in E(G)} \left[\sqrt{d_G(u) + (d_G(u) + d_G(v) - 2)} + \sqrt{d_G(v) + (d_G(u) + d_G(v) - 2)} \right] \\ &= [\sqrt{r+(r+s-2)} + \sqrt{s+(r+s-2)}]^{rs} \\ &= [\sqrt{2r+s-2} + \sqrt{r+2s-2}]^{rs}. \end{aligned}$$

Corollary 4.1: Let $K_{r,r}$ be a complete bipartite graph with $r \geq 2$. Then

$$BNII(K_{r,r}) = (2\sqrt{3r-2})^{r^2}.$$

Corollary 4.2: Let $K_{1,r}$ be a star with $r \geq 1$. Then

$$BNII(K_{1,r}) = (\sqrt{r} + \sqrt{2r-1})^r.$$

Proposition 5: If G is an r -regular graph with n vertices and $r \geq 2$, then

$$BNII(G) = (2\sqrt{3r-2})^{\frac{nr}{2}}.$$

Proof: Let G be an r -regular graph with n vertices, $r \geq 2$ and $\frac{nr}{2}$ edges.

$$\begin{aligned} BNII(G) &= \prod_{uv \in E(G)} \left[\sqrt{d_G(u) + (d_G(u) + d_G(v) - 2)} + \sqrt{d_G(v) + (d_G(u) + d_G(v) - 2)} \right] \\ &= \left[\sqrt{r + (r + r - 2)} + \sqrt{r + (r + r - 2)} \right]^{\frac{nr}{2}}. \\ &= (2\sqrt{3r-2})^{\frac{nr}{2}}. \end{aligned}$$

Corollary 5.1: Let C_n be a cycle with $n \geq 3$ vertices. Then $BNII(C_n) = 4^n$.

Corollary 5.2: Let K_n be a complete graph with $n \geq 3$ vertices. Then

$$BNII(K_n) = \left[2\sqrt{3n-5} \right]^{\frac{n(n-1)}{2}}.$$

Proposition 6: If P_n is a path with n vertices and $n \geq 3$, then $BII(P_n) = (\sqrt{2} + \sqrt{3})^2 \times 4^{n-3}$.

3. RESULTS FOR $NS_1[n]$ DENDRIMER NANOSTARS

In this section, we focus on the polypropylenimine octaamine dendrimer, denoted by $NS_1[n]$, where n is the steps of growth in this type of dendrimer. The graph of $NS_1[n]$ nanostar dendrimer is presented in Figure 1.

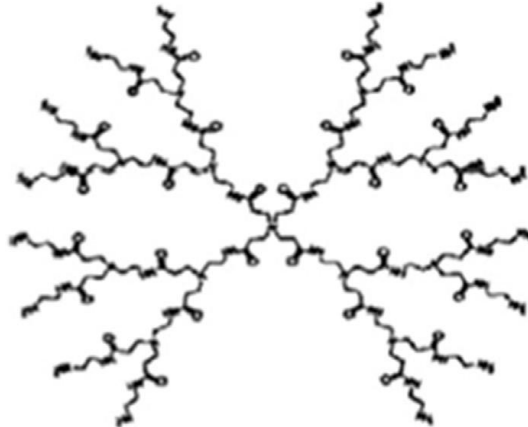


Figure-1: The molecular graph of $NS_1[n]$

Let G be the graph of polypropylenimine octaamine dendrimer $NS_1[n]$. By calculation, we obtain that G has $32 \times 2^n - 29$ edges. Also by calculation, we obtain that the edge set $E(NS_1[n])$ can be divided into four partitions based on the degree of end vertices of each edge as follows:

$$\begin{aligned} E_{12} &= \{uv \in E(G) \mid d_G(u) = 1, d_G(v) = 2\}, & |E_{12}| &= 2 \times 2^n. \\ E_{13} &= \{uv \in E(G) \mid d_G(u) = 1, d_G(v) = 3\}, & |E_{13}| &= 4 \times 2^n - 4. \\ E_{22} &= \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}, & |E_{22}| &= 12 \times 2^n - 11. \\ E_{23} &= \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}, & |E_{23}| &= 14 \times 2^n - 14. \end{aligned}$$

In the following theorem, we compute the multiplicative Nirmala index of $NS_1[n]$.

Theorem 1: Let G be the graph of a dendrimer $NS_1[n]$. Then

$$NII(G) = (3)^{2^n} \times (2)^{16 \times 2^n - 15} \times (5)^{7 \times 2^n - 7}.$$

Proof: From the definition and by cardinalities of the partition of $NS_1[n]$, we obtain

$$\begin{aligned} NII(G) &= \prod_{uv \in E(G)} \sqrt{d_G(u) + d_G(v)} \\ &= (\sqrt{1+2})^{2 \times 2^n} \times (\sqrt{1+3})^{4 \times 2^n - 4} \times (\sqrt{2+2})^{12 \times 2^n - 11} \times (\sqrt{2+3})^{14 \times 2^n - 14}. \end{aligned}$$

After simplification, we obtain the desired result.

In the following theorem, we compute the multiplicative Banhatti-Nirmala index of $NS_1[n]$.

Theorem 2: Let G be the graph of a dendrimer $NS_1[n]$. Then

$$BNII(G) = (\sqrt{2} + \sqrt{3})^{2 \times 2^n} \times (\sqrt{3} + \sqrt{5})^{4 \times 2^n - 4} \times (4)^{12 \times 2^n - 11} \times (\sqrt{5} + \sqrt{6})^{14 \times 2^n - 14}.$$

Proof: From the definition and by cardinalities of the partition of $NS_1[n]$, we obtain

$$\begin{aligned} BNII(G) &= \prod_{uv \in E(G)} \left[\sqrt{d_G(u) + (d_G(u) + d_G(v) - 2)} + \sqrt{d_G(v) + (d_G(u) + d_G(v) - 2)} \right] \\ &= \left[\sqrt{1 + (1 + 2 - 2)} + \sqrt{2 + (1 + 2 - 2)} \right]^{2 \times 2^n} \times \left[\sqrt{1 + (1 + 3 - 2)} + \sqrt{3 + (1 + 3 - 2)} \right]^{4 \times 2^n - 4} \\ &\quad \times \left[\sqrt{2 + (2 + 2 - 2)} + \sqrt{2 + (2 + 2 - 2)} \right]^{12 \times 2^n - 11} \times \left[\sqrt{2 + (2 + 3 - 2)} + \sqrt{3 + (2 + 3 - 2)} \right]^{14 \times 2^n - 14} \end{aligned}$$

gives the desired result after simplification.

4. RESULTS FOR $NS_2[n]$ DENDRIMER NANOSTARS

In this section, we focus on the polypropylenimine octaamine dendrimer, denoted by $NS_2[n]$, where n is the steps of growth in this type of dendrimer. The graph of $NS_2[n]$ dendrimer nanostar is presented in Figure 2.

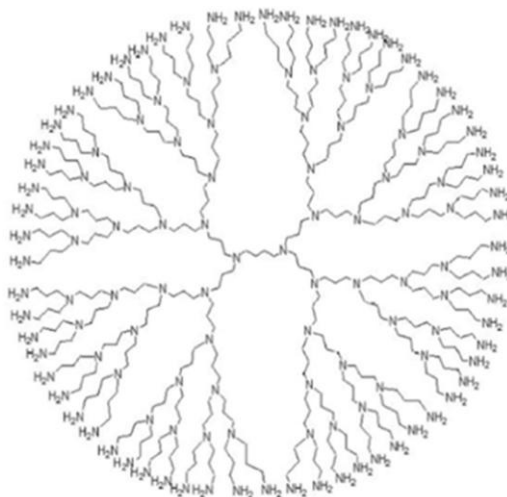


Figure-2: The structure of $NS_2[n]$

Let G be the graph of polypropylenimine octaamine dendrimer $NS_2[n]$. By calculation, we obtain that G has $16 \times 2^n - 11$ edges. Also by calculation, we obtain that the edge set $E(NS_2[n])$ can be divided into three partitions based on the degree of end vertices of each edge as follows:

$$\begin{aligned} E_{12} &= \{uv \in E(G) \mid d_G(u) = 1, d_G(v) = 2\}, & |E_{12}| &= 2 \times 2^n. \\ E_{22} &= \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}, & |E_{22}| &= 8 \times 2^n - 5. \\ E_{23} &= \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}, & |E_{23}| &= 6 \times 2^n - 6. \end{aligned}$$

In the following theorem, we compute the multiplicative Nirmala index of $NS_2[n]$.

Theorem 3: Let G be the graph of a dendrimer $NS_2[n]$. Then

$$NII(G) = (3)^{2^n} \times (2)^{8 \times 2^n - 5} \times (5)^{3 \times 2^n - 3}.$$

Proof: From the definition and by cardinalities of the partition of $NS_2[n]$, we get

$$\begin{aligned} NII(G) &= \prod_{uv \in E(G)} \sqrt{d_G(u) + d_G(v)} \\ &= (\sqrt{1+2})^{2 \times 2^n} \times (\sqrt{2+2})^{8 \times 2^n - 5} \times (\sqrt{2+3})^{6 \times 2^n - 6} \end{aligned}$$

gives the desired result after simplification.

In the following theorem, we compute the multiplicative Banhatti-Nirmala index of $NS_2[n]$.

Theorem 4: Let G be the graph of a dendrimer $NS_2[n]$. Then

$$BNII(G) = (\sqrt{2} + \sqrt{3})^{2 \times 2^n} \times (4)^{8 \times 2^n - 5} \times (\sqrt{5} + \sqrt{6})^{6 \times 2^n - 6}.$$

Proof: From the definition and by cardinalities of the partition of $NS_2[n]$, we obtain

$$\begin{aligned} BNII(G) &= \prod_{uv \in E(G)} \left[\sqrt{d_G(u) + (d_G(u) + d_G(v) - 2)} + \sqrt{d_G(v) + (d_G(u) + d_G(v) - 2)} \right] \\ &= \left[\sqrt{1 + (1 + 2 - 2)} + \sqrt{2 + (1 + 2 - 2)} \right]^{2 \times 2^n} \times \left[\sqrt{2 + (2 + 2 - 2)} + \sqrt{2 + (2 + 2 - 2)} \right]^{8 \times 2^n - 5} \\ &\quad \times \left[\sqrt{2 + (2 + 3 - 2)} + \sqrt{3 + (2 + 3 - 2)} \right]^{6 \times 2^n - 6}. \end{aligned}$$

After simplification, we obtain the desired result.

5. RESULTS FOR $NS_3[n]$ DENDRIMER NANOSTARS

In this section, we focus on the molecular graph structure of the first class of dendrimer nanostars. This family of dendrimer nanostars is denoted by $NS_3[n]$, where n is the steps of growth in this type of dendrimer nanostars. The molecular graph structure of $NS_3[3]$ dendrimer nanostar is presented in Figure 3.

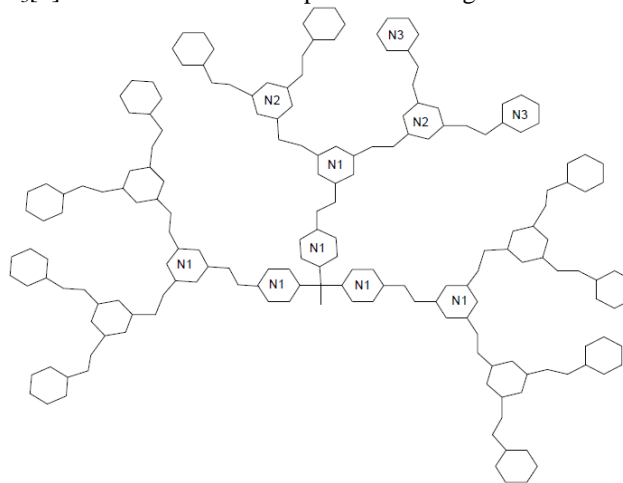


Figure-3: The structure of $NS_3[3]$

Let G be the molecular graph of a dendrimer nanostar $NS_3[n]$. By calculation, we obtain that G has $27 \times 2^n - 5$ edges. Also by calculation, we obtain that the edge set $E(NS_3[n])$ can be divided into four partitions based on the degree of end vertices of each edge as follows:

$$\begin{aligned} E_{14} &= \{uv \in E(G) \mid d_G(u) = 1, d_G(v) = 4\}, & |E_{14}| &= 1 \\ E_{22} &= \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}, & |E_{22}| &= 9 \times 2^n + 3. \\ E_{23} &= \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}, & |E_{23}| &= 18 \times 2^n - 12. \\ E_{34} &= \{uv \in E(G) \mid d_G(u) = 3, d_G(v) = 4\}, & |E_{34}| &= 3. \end{aligned}$$

In the following theorem, we compute the multiplicative Nirmala index of $NS_3[n]$.

Theorem 5: Let G be the graph of a dendrimer $NS_3[n]$. Then

$$NII(G) = (\sqrt{5})^1 \times (2)^{9 \times 2^n + 3} \times (5)^{9 \times 2^n - 6} \times (\sqrt{7})^3.$$

Proof: From the definition and by cardinalities of the partition of $NS_3[n]$, we have

$$\begin{aligned} NII(G) &= \prod_{uv \in E(G)} \sqrt{d_G(u) + d_G(v)} \\ &= (\sqrt{1+4})^1 \times (\sqrt{2+2})^{9 \times 2^n + 3} \times (\sqrt{2+3})^{18 \times 2^n - 12} \times (\sqrt{3+4})^3. \end{aligned}$$

After simplification, we obtain the desired result.

In the following theorem, we compute the multiplicative Banhatti-Nirmala index of $NS_3[n]$.

Theorem 6: Let G be the graph of a dendrimer $NS_3[n]$. Then

$$BNII(G) = (2 + \sqrt{7})^1 \times (4)^{9 \times 2^n + 3} \times (\sqrt{5} + \sqrt{6})^{18 \times 2^n - 12} \times (2\sqrt{2} + 3)^3.$$

Proof: From the definition and by cardinalities of the partition of $NS_3[n]$, we obtain

$$\begin{aligned} BNII(G) &= \prod_{uv \in E(G)} \left[\sqrt{d_G(u) + (d_G(u) + d_G(v) - 2)} + \sqrt{d_G(v) + (d_G(u) + d_G(v) - 2)} \right] \\ &= \left[\sqrt{1 + (1 + 4 - 2)} + \sqrt{4 + (1 + 4 - 2)} \right]^1 \times \left[\sqrt{2 + (2 + 2 - 2)} + \sqrt{2 + (2 + 2 - 2)} \right]^{9 \times 2^n + 3} \\ &\quad \times \left[\sqrt{2 + (2 + 3 - 2)} + \sqrt{3 + (2 + 3 - 2)} \right]^{18 \times 2^n - 12} \times \left[\sqrt{3 + (3 + 4 - 2)} + \sqrt{4 + (3 + 4 - 2)} \right]^3 \end{aligned}$$

gives the desired result after simplification.

6. RESULTS FOR $D_1[n]$ DENDRIMER NANOSTARS

In this section, we consider a family of dendrimer nanostars with n growth stages, denoted by $D_1[n]$. The graph of $D_1[n]$ with 4 growth stages is depicted in Figure 4.

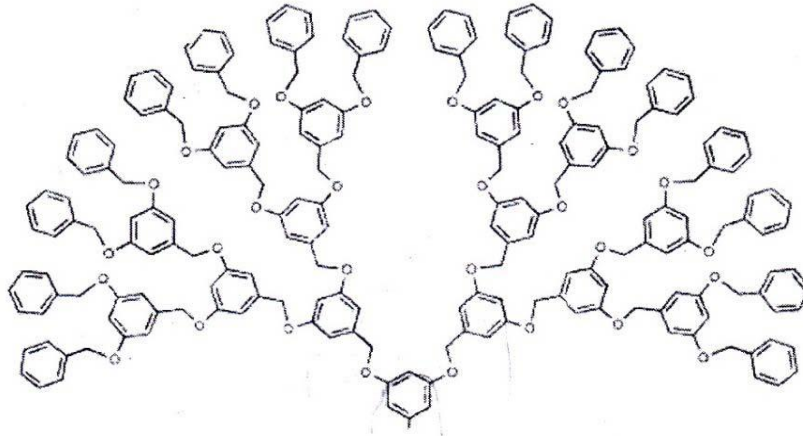


Figure-4: The molecular graph of $D_1[4]$

Let G be the graph of a dendrimer nanostar $D_1[n]$. By calculation, we obtain that G has $18 \times 2^n - 11$ edges. Also by calculation, we obtain that the edge set $E(D_1[n])$ can be divided into three partitions based on the degree of end vertices of each edge as follows:

$$\begin{aligned} E_{13} &= \{uv \in E(G) \mid d_G(u) = 1, d_G(v) = 3\}, & |E_{13}| &= 1. \\ E_{22} &= \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}, & |E_{22}| &= 6 \times 2^n - 2. \\ E_{23} &= \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}, & |E_{23}| &= 12 \times 2^n - 10. \end{aligned}$$

In the following theorem, we compute the multiplicative Nirmala index of $D_1[n]$.

Theorem 7: Let G be the graph of a dendrimer $D_1[n]$. Then

$$NII(G) = (2)^{6 \times 2^n - 1} \times (5)^{6 \times 2^n - 5}.$$

Proof: From the definition and by cardinalities of the partition of $D_1[n]$, we obtain

$$\begin{aligned} NII(G) &= \prod_{uv \in E(G)} \sqrt{d_G(u) + d_G(v)} \\ &= (\sqrt{1+3})^1 \times (\sqrt{2+2})^{6 \times 2^n - 2} \times (\sqrt{2+3})^{12 \times 2^n - 10} \end{aligned}$$

gives the desired result after simplification.

In the following theorem, we compute the multiplicative Banhatti-Nirmala index of $D_1[n]$.

Theorem 8: Let G be the graph of a dendrimer $D_1[n]$. Then

$$BNII(G) = (\sqrt{3} + \sqrt{5})^1 \times (4)^{6 \times 2^n - 2} \times (\sqrt{5} + \sqrt{6})^{12 \times 2^n - 10}.$$

Proof: From the definition and by cardinalities of the partition of $D_1[n]$, we obtain

$$\begin{aligned} BNII(G) &= \prod_{uv \in E(G)} \left[\sqrt{d_G(u) + (d_G(u) + d_G(v) - 2)} + \sqrt{d_G(v) + (d_G(u) + d_G(v) - 2)} \right] \\ &= \left[\sqrt{1 + (1 + 3 - 2)} + \sqrt{3 + (1 + 3 - 2)} \right]^1 \times \left[\sqrt{2 + (2 + 2 - 2)} + \sqrt{2 + (2 + 2 - 2)} \right]^{6 \times 2^n - 2} \\ &\quad \times \left[\sqrt{2 + (2 + 3 - 2)} + \sqrt{3 + (2 + 3 - 2)} \right]^{12 \times 2^n - 10}. \end{aligned}$$

After simplification, we obtain the desired result.

7. RESULTS FOR $D_3[n]$ DENDRIMER NANOSTARS

In this section, we consider the dendrimer nanostar with n growth stages, where $n \geq 0$, denoted by $D_3[n]$. The graph of $D_3[n]$ with 4 growth stages is shown in Figure 5.

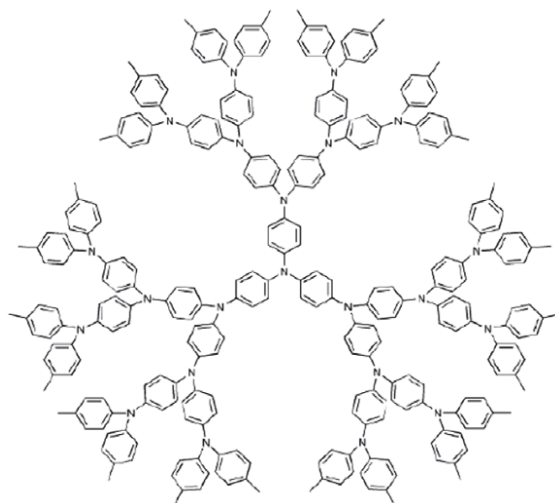


Figure-5: The molecular graph of $D_3[3]$

Let G be the graph of a dendrimer nanostar $D_3[n]$. By calculation, we obtain that G has $24 \times 2^n - 20$ vertices and $24(2^{n+1} - 1)$ edges. Also by calculation, we obtain that the edge set $E(D_3[n])$ can be divided into four partitions based on the degree of end vertices of each edge as follows:

$$\begin{aligned} E_{13} &= \{uv \in E(G) \mid d_G(u) = 1, d_G(v) = 3\}, & |E_{13}| &= 3 \times 2^n. \\ E_{22} &= \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}, & |E_{22}| &= 12 \times 2^n - 6. \\ E_{23} &= \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}, & |E_{23}| &= 24 \times 2^n - 12. \\ E_{33} &= \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}, & |E_{33}| &= 9 \times 2^n - 6. \end{aligned}$$

In the following theorem, we compute the multiplicative Nirmala index of $D_3[n]$.

Theorem 9: Let G be the graph of a dendrimer $D_3[n]$. Then

$$NII(G) = (2)^{15 \times 2^n - 6} \times (5)^{12 \times 2^n - 6} \times (\sqrt{6})^{9 \times 2^n - 6}.$$

Proof: From the definition and by cardinalities of the partition of $D_3[n]$, we obtain

$$\begin{aligned} NII(G) &= \prod_{uv \in E(G)} \sqrt{d_G(u) + d_G(v)} \\ &= (\sqrt{1+3})^{3 \times 2^n} \times (\sqrt{2+2})^{12 \times 2^n - 6} \times (\sqrt{2+3})^{24 \times 2^n - 12} \times (\sqrt{3+3})^{9 \times 2^n - 6} \end{aligned}$$

gives the desired result after simplification.

In the following theorem, we compute the multiplicative Banhatti-Nirmala index of $D_3[n]$.

Theorem 10: Let G be the graph of a dendrimer $D_3[n]$. Then

$$BNII(G) = (\sqrt{3} + \sqrt{5})^{3 \times 2^n} \times (4)^{12 \times 2^n - 6} \times (\sqrt{5} + \sqrt{6})^{24 \times 2^n - 12} \times (2\sqrt{7})^{9 \times 2^n - 6}.$$

Proof: From the definition and by cardinalities of the partition of $D_3[n]$, we obtain

$$\begin{aligned} BNII(G) &= \prod_{uv \in E(G)} \left[\sqrt{d_G(u) + (d_G(u) + d_G(v) - 2)} + \sqrt{d_G(v) + (d_G(u) + d_G(v) - 2)} \right] \\ &= \left[\sqrt{1 + (1 + 3 - 2)} + \sqrt{3 + (1 + 3 - 2)} \right]^{3 \times 2^n} \times \left[\sqrt{2 + (2 + 2 - 2)} + \sqrt{2 + (2 + 2 - 2)} \right]^{12 \times 2^n - 6} \\ &\quad \times \left[\sqrt{2 + (2 + 3 - 2)} + \sqrt{3 + (2 + 3 - 2)} \right]^{24 \times 2^n - 12} \times \left[\sqrt{3 + (3 + 3 - 2)} + \sqrt{3 + (3 + 3 - 2)} \right]^{9 \times 2^n - 6} \end{aligned}$$

After simplification, we obtain the desired result.

8. CONCLUSION

In this study, we have introduced the multiplicative Banhatti-Nirmala index of a graph. We have computed the multiplicative Nirmala index and the multiplicative Banhatti-Nirmala index for some standard graphs. Also we have determined these multiplicative Nirmala indices for some important chemical structures such as nanostar dendrimers.

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