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NANO #R^G - CONTINUOUS FUNCTIONS IN NANO TOPOLOGICAL SPACES

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ABSTRACT

In this article, we introduce nano #regular ^ generalized continuous functions (shortly N#r^g-continuous functions) in nano topological spaces. Also we examine some of its properties. Furthermore we study the relationship between these nano #regular ^generalized continuous functions with already existing nano continuous functions in nano topological spaces.

Keywords: *Nano #regular ^generalized continuous function, nano #regular ^ generalized open maps and closed maps.*

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1. INTRODUCTION

Continuous functions are one of the main concepts of Topology. L. Thivagar *et al* [16] introduced the concept of Nano Topological spaces which was defined in terms of approximations and boundary region of a subset of a universe U using an equivalence relation on it and also defined nano closed sets, nano interior and nano closure. He has also defined nano continuous functions, nano open mapping, nano closed mapping and nano homeomorphism. Different type of generalizations of continuous functions was studied by various authors in the recent development of topology. In 1991, Balachandran *et al* [3], introduced and studied the notions of generalized continuous functions. The concept of regular continuous functions was first introduced by Arya. S. P and Gupta [2]. The concept of Nano generalized continuous functions in nano topological space was introduced by K. Bhuvaneswari and K. Mythili Gnanapriya [6]. P. Sulochana Devi and Dr. K. Bhuvaneswari [21] introduced the concept of nano regular generalized continuous functions. The concept of Regular ^Generalized continuous functions was introduced by Savithiri. D and Janaki. C [18]. She also introduced #Regular ^Generalized continuous functions in the topological spaces. The researchers conducted a case study to identify the key factors for the cause of heart failure. Heart failure is a condition in which the heart cannot pump enough blood to meet the body's needs. A "heart attack" occurs when the flow of oxygen-rich blood to a section of heart muscle is blocked suddenly and when the heart does not receive the required oxygen. If blood flow is not restored quickly, the section of muscle begins to die. It is possible to eliminate many of these risk factors in order to reduce the chance of having a first or subsequent heart attack. High blood pressure is caused due to obesity, smoking, high cholesterol or diabetes that increase the risk of heart failure.

As an extension, in this paper we introduce a new class of continuous functions called Nano #Regular ^Generalized continuous functions.

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2. PRELIMINARIES

Definition2.1: Let U be a non-empty finite set of objects called the Universe and R be an equivalence relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$, then

1) The lower approximation of X with respect to R is the set of all objects which can before certain classified as X with respect to R and is denoted by $L_R(x)$. That is
$L_R(X) = \bigcup_{x\in U}\{R(x): R(x)\subseteq X\}$ where R(x) denotes the equivalence class determined by $x\in U$.
2) The upper approximation of X with respect to is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$. That is
$U_R(X) = \bigcup_{x\in U}\{R(x): R(x) \cap X\neq \emptyset\}$.
3) The boundary of the region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X) = U_R(X - L_R(X))$.

Property 2.2: If (U, R) is an approximation space and X, $Y \subseteq U$, then

1) $L_R(X) \subseteq X \subseteq U_R(X)$.
2) $L_R(\phi) = U_R(\phi) = \phi$.
3) $L_R(U) = U_R(U) = U$.
4) $U_R(X\cup Y) = U_R(X) \cup U_R(Y)$.
5) $U_R(X\cap Y) \subseteq U_R(X) \cap U_R(Y)$.
6) $L_R(X\cup Y) \supseteq L_R(X) \cup L_R(Y)$.
7) $L_R(X\cap Y) = L_R(X) \cap L_R(Y)$.
8) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X\subseteq Y$.
9) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$.
10) $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$.
11) $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$.

Definition 2.3: Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{\phi, L_R(X), U_R(X), B_R(X), U\}$ where $X \subseteq U$.

1) U and $\phi\in\tau_R(X)$.
2) The union of elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
3) The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on V called the nano topology U with respect to X. (U, $\tau_R(X)$) is called the nano topological space. Elements of nano topology are known as nano open sets in U. Elements of $[\tau_R(X)]^c$ are called nano closed sets with $[\tau_R(X)]^c$ being called the dual nano topology of $\tau_R(X)$.

Throughout this paper (U,$\tau_R(X)$) is a nano topological space with respect to X where $X \subseteq U$, R isan equivalence relation on U, U/R denotes the family of equivalence classes of U by R.

Example 2.4: U={a, b, c, d}, U/R={{a},{b},{c, d}}, X= {b, c}$\subseteq$U, where $L_R(U)$={b}, $U_R(U)$={b, c, d}, $B_R(U)$={c, d}. Then the nano open sets are $\tau_R(U) = \{\phi,\{b\},\{c, d\},\{b, c, d\},U\}$ and the nano closed sets are $\tau_{R''}(U) = \{\phi,\{a\},\{a, b\},\{a, c, d\},U\}$.

Remark 2.5: If $\tau_R(X)$ is the nano topology on U with respect to X, then the set B = {U, $L_R(X)$, $B_R(X)$} is the basis for $\tau_R(X)$.

Definition 2.6: If (U,$\tau_R(X)$) is a nano topological space with respect to X where $X \subseteq U$ and if $A\subseteq U$, then

1) The Nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by NInt(A). NInt(A) is the largest nano open subset of A.
2) The Nano closure of the set A is defined as the intersection of all nano closed sets containing A and is denoted by NCl(A). NCl(A) is the smallest nano closed set containing A.

Definition 2.7
Let (U, $\tau_R(X)$) be a nano topological space and $A \subseteq U$. Then A is said to be

1) Nano semi-open if$A\subseteq$NCl(NInt(A)).
2) Nano pre-open if$A\subseteq$NInt(NCl(A)).
3) Nano α-open if$A\subseteq$NInt(NCl(NInt(A))).
4) Nano regular open if A=NInt(NCl(A)).

Definition 2.8: Let (U, $\tau_R(X)$) be a nano topological space and A $\subseteq$ U. A is said to be nano semi-closed, nanopre-closed, nano α-closed and nano regular closed if its complements is respectively nano semi- open, nano pre-open, nano α-open and nano regular open.

Definition 2.9: Let (U,$\tau_R(X)$) be a nano topological space. The finite union of nano regular open set in U is known as Nano π-open. The compliment of a Nano π-open is said to be Nano π-closed.

Definition 2.10: Let (U, $\tau_R(X)$) be a nano topological space and A $\subseteq$ U. Then A is said to be Nano regular semi open if there is an open set G in U such that G$\subseteq$ A $\subseteq$ cl(G). The family of all nano regular semi open sets of U is denoted by NRSO(U).

Definition 2.11: A subset A of a nano topological space (U, $\tau_R(X)$) is called

1) Nano generalized closed (briefly Nano g-closed) set if NCl(A) $\subseteq$ V whenever A $\subseteq$ V and V is nano open in U.
2) Nano semi generalized closed (briefly Nano sg –closed) if Nscl(A) $\subseteq$ V whenever A$\subseteq$V and V is nano semi open inU.
3) Nano α generalized closed (briefly Nano αg-closed) if NαCl(A) $\subseteq$ V whenever A $\subseteq$V and
4) V is nano open in U.
5) Nano regular generalized closed (briefly Nano rg-closed) if Nrcl(A) $\subseteq$ V whenever A $\subseteq$V and V is nano regular open inU.
6) Nano generalized regular closed (briefly Nano gr-closed) if Nrcl(A) $\subseteq$ V whenever A $\subseteq$V and V is nano open in U.
7) Nano strongly generalized closed (briefly Nano *g-closed) if Ncl(A) $\subseteq$ V whenever A $\subseteq$V and V is nano w open in U.
8) Nano generalized semiclosed(briefly Nano gs-closed) if Nscl(A)$\subseteq$ V whenever A$\subseteq$V and V is nano open in U.
9) Nano weakly closed (briefly Nano w-closed) if Ncl(A) $\subseteq$ V whenever A $\subseteq$V and V is nano semi open in U.
10) Nano regular weakly closed (briefly Nano rw-closed) if Ncl(A)$\subseteq$ V whenever A$\subseteq$V and V is Nano regular semi open inU.
11) Nano weakly generalized semi closed (briefly Nano wg-closed) if Ncl(Nint(A)) $\subseteq$V
12) whenever A $\subseteq$V and V is nano open in U.
13) Nano weakly generalized closed (briefly Nano wg- closed) if Ncl(Nint(A)) $\subseteq$ V whenever A$\subseteq$V and V is open in U.
14) Nano semi weakly generalized closed (briefly Nano swg-closed) if Ncl(Nint(A)) $\subseteq$ V whenever A$\subseteq$V and V is nano semiopen in U.
15) Nano π generalized closed if Ncl(A) $\subseteq$ V whenever A $\subseteq$V and V is nano π open in U.
16) Nano regular ^ generalized closed (briefly Nano r^g- closed) if Ngcl(A) $\subseteq$ V whenever A$\subseteq$V and V is nano regular open in U.
17) Nano #regular generalized closed (briefly Nano #rg closed) if Ncl(A) $\subseteq$ V whenever A$\subseteq$V and V is nano rw-open in U.
18) Nano #regular ^generalized closed (briefly Nano #r^ g closed) if Ngcl(A)$\subseteq$V whenever A
19) $\subseteq$V and V is nano regular weakly-open. The compliment of N#R^G-closed set is known as N#R^G-open set.
20) Nano mildly #regular ^generalized closed (briefly Nano M#R^G-closed)if Ncl(Nint(A))
21) $\subseteq$V whenever A$\subseteq$ V and V is N#R^G-open in U. The compliment of Nano M#R^G- closed set is Nano M#R^G-open set.

Definition 2.12: Let (U, $\tau_R(X)$) and (V, $\tau_{R''}(X)$) be two nano topological spaces. Then a mapping f:(U,$\tau_R(X)$) $\rightarrow$ (V,$\tau_{R''}(X)$) is said to be

1) Nano continuous if $f^{-1}(B)$ is nano open in U for every nano open set B in V.
2) Nano generalized continuous if $f^{-1}(B)$ is nano g-open in U for every nano open set B in V.
3) Nano regular continuous if $f^{-1}(B)$ is nano regular open in U for every nano open set B in V.
4) Nano α-continuous if $f^{-1}(B)$ is nano α-open in U for every nano open set B in V.
5) Nano semi-continuous if $f^{-1}(B)$ is nano semi-open in U for every nano open set B in V.
6) Nano pre-continuous if $f^{-1}(B)$ is nano pre-open in U for every nano open set B in V.
7) Nano regular generalized continuous if $f^{-1}(B)$ is nano regular open in U for every nano open set B in V.

Definition 2.13: Let (U, τ_R (U)) and (V, $\tau_{R''}$ (V)) be two nano topological spaces. Then a mapping f:(U, $\tau_R(U)$)$\rightarrow$ (V,$\tau_{R''}(V)$) is said to be a Nano weakly generalized continuous if every $f^{-1}(B)$ is Nano weakly generalized closed in U for every Nano closed set B in V.

Definition 2.14: Let (U, τ_R(U)) and (V,$\tau_{R''}$(V)) be two nano topological spaces. Then a mapping f:(U,τ_R(U)) →(V,$\tau_{R''}$(V)) is said to be Nano semi weakly generalized continuous if every f^{-1}(B) is Nano semi weakly generalized closed for every Nano closed set B in V.

Definition 2.15: Let (U, τ_R(X)) and (V, $\tau_{R''}$(X)) be two topological spaces. Then a mapping f:(U, τ_R(X))→(V,$\tau_{R''}$(X)) is called regular ^generalized continuous if every f^{-1}(B) is regular^generalized closed in U for every closed set B in V.

Definition 2.16: Let (U, τ_R(x)) and (V,$\tau_{R''}$(X)) be two topological spaces. Then a mapping f:(U, τ_R(X))→(V,$\tau_{R''}$(X)) is said to be #regular ^generalized-continuous if f^{-1}(B) is #regular ^generalized closed in U for every closed set B in V.

2. NANO #R^G - CONTINUOUS AND IRRESOLUTE FUNCTIONS

Definition 3.1: Let (U, τ_R(X)) and (V, $\tau_{R''}$(Y)) be nano topological spaces. A map f: (U, τ_R(X)) → (V, $\tau_{R''}$(Y)) is said to be Nano #regular ^generalized continuous (N#R^G-continuous) if the inverse image of every nano closed subset of V is nano #regular ^generalized closed inU.

Example 3.2: Let U={a, b, c, d}, X={b, c}, U/R={{a},{b},{c, d}},τ_R(U)={ϕ,{b},{c, d},{b, c, d},U}, V={a, b, c, d}, Y={a, b}, V/R={{a},{b, c},{d}}, τ_R(V)={ϕ,{a},{b, c},{a, b, c},U}. Then define f: (U, τ_R(X)) → (V, $\tau_{R''}$(Y)) by f(a)=d, f(b) = a, f(c) =c, f(d)=b. Then f is N#R^G - continuous.

Definition 3.3: Let (U, τ_R(X)) and (V,$\tau_{R''}$(Y)) be nano topological spaces. A map f:(U,τ_R(X))→(V,$\tau_{R''}$(Y)) is said to be Nano #regular ^generalized irresolute if every f^{-1}(W) is nano #regular ^generalized closed in U for every nano #regular ^generalized closed set W in V.

Example 3.4: Let U= {a, b, c, d}, X={a, c},U/R={{a},{b},{c, d}},τ_R(U) ={ϕ,{a},{c, d},{a, c, d}, U}, V={a, b, c, d} Y={b, c}, V/R={{a},{b},{c, d}},$\tau_{R''}$(V)={ϕ,{b},{c, d},{b, c, d},U}. Then define f:(U,τ_R(X)) → (V,$\tau_{R''}$(Y)) by f(a)=b, f(b)=a, f(c)=c, f(d)=d. Then f is N#R^G-irresolute.

Remark 3.5: Every Nano #regular ^generalized irresolute function is Nano #regular ^generalized-continuous but the converse is not true.

Example 3.6: Let U={a, b, c, d},X={b, c},U/R={{a},{b},{c, d}},τ_R (U)={ϕ,{b},{c, d},{b, c, d},U},V={a, b, c, d}, Y={a, b},V/R{{a},{b, c},{d}},$\tau_{R''}$(V)={ϕ,{a},{b, c},{a, b, c},U}. Then define f: (U, τ_R(X)) → (V,$\tau_{R''}$(Y)) by f(a)=d, f(b)=a, f(c)=b, f(d)=c. Then f is Nano #regular ^generalized irresolute function, which is Nano #regular ^generalized continuous function.

Example 3.7: Let U={a, b, c, d}, X={b, c}, U/R={{a},{b},{c, d}},τ_R(U)={ϕ,{b},{c, d},{b, c, d},U}, V={a, b, c, d}, Y={c, d}, V/R={{a},{b},{c, d}},$\tau_{R''}$(V)={ϕ,{c, d},V}. Then define f: (U, τ_R(X)) → (V, $\tau_{R''}$(Y)) byf(a) = a, f(b) = b, f(c) = c, f(d)=d. Then f is Nano #regular ^generalized continuous function but not Nano #regular ^generalized irresolute function.

Remark 3.8: Every Nano continuous function is Nano #regular ^generalized continuous but the converse is not true.

Example 3.9: Let U={a, b, c, d}, X={a, b}, U/R={{a},{b, c},{d}}, τ_R(U)={ϕ,{a},{b, c},{a, b, c},U}, V={a, b, c, d}, Y={c, d}, V/R={{a},{b, d},{c}}, $\tau_{R''}$(V)={ϕ,{d},{b, c},{b, c, d},V}. Then define f: (U, τ_R(X)) → (V, $\tau_{R''}$(Y)) by f(a) = d, f(b) = c, f(c) = b, f(d)=a. Then f is nano-continuous, which is also a Nano #regular ^generalized continuous function.

Example 3.10: Let U={a, b, c, d}, X={b, c}, U/R={{a},{b},{c, d}}, τ_R(U)={ϕ ,{b},{c, d},{b, c, d},U}, V={a, b, c, d}, Y={a, b}, V/R={{a},{b, c},{d}}, $\tau_{R''}$(V)={ϕ,{a},{b, c},{a, b, c},V}. Then define f:(U, τ_R(X)) → (V,$\tau_{R''}$(Y)) by f(a)=d, f(b)=b, f(c)=c, f(d)=a. Then f is Nano #regular ^generalized continuous function but it is not a Nano continuous function.

Remark 3.11: Nano-αcontinuous and Nano #regular ^ generalized continuous are independent to each other.

Example 3.12: Let U={a, b, c, d}, X={b, c}, U/R={{a},{b},{c, d}}, τ_R(U)={ϕ ,{b},{c, d},{b, c, d},U}, V={a, b, c, d}, Y={a, b}, V/R={{a},{b, c},{d}}, $\tau_{R''}$(V)={ϕ,{a},{b, c},{a, b, c},V}. Then define f: (U,τ_R(X))→(V,$\tau_{R''}$(Y)) by f(a)=d, f(b)=b, f(c)=c, f(d)=a. Then f is Nano #regular ^generalized continuous but not Nano α-continuous function.

Remark 3.13: The concepts of nano swg continuous and nano #regular ^generalized-continuous are independent to each other.

Example 3.14: Let U={a, b, c, d},X={c, d},U/R={{a},{b, d},{c}}, τR(U)={ϕ ,{d},{b, c},{b, c, d},U},V={a, b, c, d}, Y={a, c, d},V/R={{a},{b},{c, d}}, τR''(V)={ϕ,{a, c, d},V}.Then define f:(U,τR(X))→(V,τR''(Y)) by f(a)=a, f(b)=b, f(c)=c, f(d)=d. Then f is Nano semi weakly generalized continuous function but not Nano #regular ^generalized continuous function.

Example 3.15: Let U={a, b, c, d}, X={a, c}, U/R={{a},{b},{c, d}}, τR(U)={ϕ,{a},{c, d},{a, c, d},U}, V={a, b, c, d}, Y={c, d},V/R={{a},{b},{c, d}},τR''(V)={ϕ,{c ,d},V}. Then define f:(U,τR(X))→ (V,τR''(Y)) by f(a)=a, f(b)=b, f(c) = c, f(d) = d. Then f is Nano #regular ^generalized continuous function but not Nano semi weakly generalized continuous function.

Remark 3.16: Every nano generalized continuous function is Nano #regular ^generalized-continuous but the converse is not true.

Example 3.17: Let U={a, b, c, d}, X={c, d}, U/R={{a},{b, d},{c}}, τR(U)={ϕ,{d},{b, c},{b, c, d},U}, V={a, b, c, d}, Y={b. c}, V/R={{a, d},{b}{c}}, τR''(V)={ϕ,{b},{a, d},{a, b, d},V}. Then define f: (U, τR(X)) →(V,τR''(Y)) by f(a) = c, f(b) = b, f(c) = a, f(d) = d. Then f is Nano generalized continuous function which is Nano #regular ^generalized continuous function.

Remark 3.18: Every Nano #regular ^generalized continuous function is Nano regular ^generalized continuous but the converse is not true.

Example 3.19: Let U={a, b, c, d}, X={a, d}, U/R={{a},{b},{c, d}},τR(U)={ϕ,{b},{c, d},{b, c, d},U}, V={a, b, c, d}, Y={c, d},V/R={{a},{b, c},{d}}, τR''(V)={ϕ,{d},{b, c},{b, c, d},V}. Then define f: (U, τR(X)) → (V,τR''(Y)) by f(a)=a, f(b)=d, f(c)=b, f(d)=c. Then f is Nano #regular ^generalized continuous function, also which is Nano regular ^generalized continuous function.

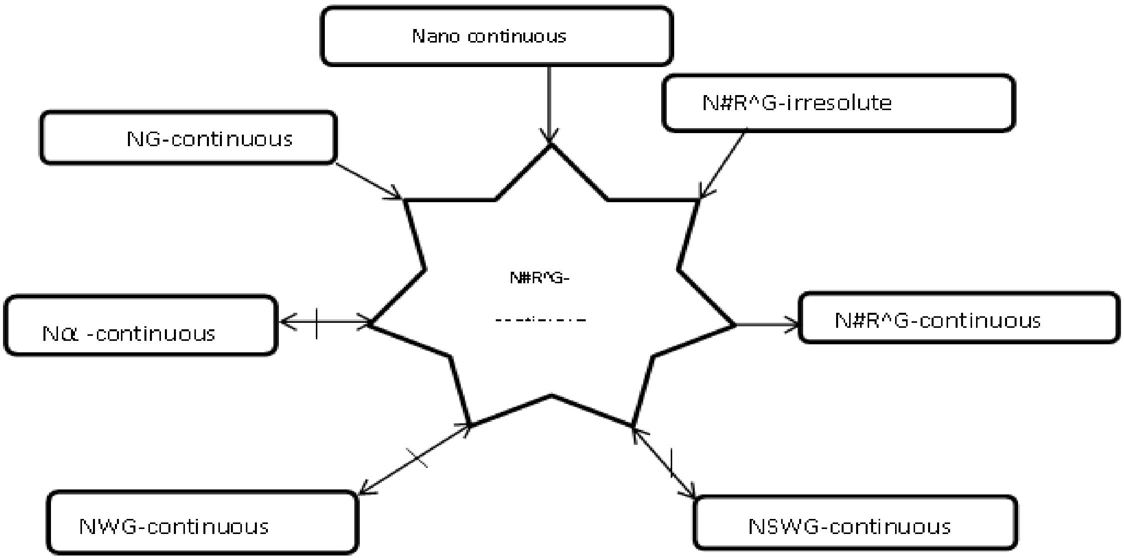
Example 3.20: Let U={a, b, c, d}, X={b, c}, U/R={{a},{b},{c, d}},τR(U)={ϕ ,{b},{c, d},{b, c, d},{U},V={a, b, c, d}, Y={b, c, d}, V/R={{a,b},{c},{d}},τR''(V)={ϕ,{a, b},{c, d},V}. Then define f:(U,τR(X))→ (V,τR''(Y)) by f(a)=a, f(b)=d, f(c)=c, f(d)=b. Then f is Nano regular ^generalized continuous but not Nano #regular ^generalized continuous function.

Remark 3.21: The concepts of Nano weakly generalized continuous and Nano #regular ^generalized continuous are independent to each other.

Example 3.22: Let U={a, b, c, d}, X={b, c, d}, U/R={{a, b},{c},{d}}, τR(U)={ϕ ,{a, b},{c, d},U}, V={a, b, c, d}, Y={b,d}, V/R={{a, c},{b},{d}}, τR''(V)={ϕ ,{b, d}, V}.Then define f: (U, τR(X)) →(V,τR''(Y)) by f(a) = a, f(b) = c, f(c)=b, f(d)=d. Then f is Nano #regular ^generalized continuous but not Nano weakly generalized continuous function.

Example 3.23: Let U={a, b, c, d}, X={a, b}, U/R={{a},{b, c},{d}}, τR(U)={ϕ,{a},{b, c},{a, b, c},U}, V={a, b, c, d}, Y={a, b, d},V/R={{a},{b, c},{d}},τR''(V)={ϕ,{a, d},{b, c},V}. Then define f: (U,τR(X)) → (V,τR''(Y)) by f(a)=b, f(b)=a, f(c)=d, f(d)=c. Then f is Nano weakly generalized continuous but not nano #regular ^generalized continuous function.

The above discussions are implemented in the following diagram.



4. NANO#R^G-OPEN MAPS and NANO#R^G-CLOSEDMAPS

Definition 4.1: A mapping f: (U, $\tau_R(X)$) → (V, $\tau_{R''}(Y)$) is nano #regular ^generalized-open map (N#R^G-open map) if the image of every nano open set of X is N#R^G open set in Y.

Example 4.2: Let U={a,b,c,d}, X={b,c}⊆U, U/R={{a},{b},{c, d}}, τR(U) ={ϕ,{b},{c, d},{b, c, d},U}, V={a, b, c, d}, Y={b, c, d}⊆V,V/R={{a, b},{c},{d}},τR''(V)={ϕ,{a, b},{c, d},U}.Then define f: (U,τR(X)) → (V,τR''(Y)) by f(a)=a, f(b) = b, f(c) = c, f(d)=d. Then f is N#R^G-open map.

Theorem 4.3: A mapping f: (U, τR(X)) →(V,τR''(Y)) is N#R^G open map if and only if every nano open set U of X, f(Nint(U))⊆ N#R^Gint(f(U)).

Proof: Necessity: Let f be an N#R^G-open mapping and U be an open set in X. Nint(U) ⊆ U which implies that f(Nint(U)) ⊆ f(U). Since f is N#R^G-open map, f(Nint(U)) is N#R^G open set in Y such that f(Nint(U)) ⊆ f(U). Therefore f(Nint(U)) ⊆ N#R^Gint(U).

Sufficiency: Suppose that U is nano open set of X. Then f(U) = f(Nint(U)) ⊆ N#R^Gint(f(U)). But N#R^Gint(f(U)) ⊆ f(U). Consequently, f(U) = N#R^Gint(U) which implies that f(U) is an N#R^G- open set of Y. Thus f is N#R^G open map.

Theorem 4.4: A mapping f: (U, τR(X)) →(V,τR''(Y)) is N#R^G-open if and only if for every nano closed set S of Y and for each nano closed set U of X containing $f^{-1}(S)$ there is an N#R^G-closed set V of Y such that S⊆ V and $f^{-1}(V)$⊆ U.

Proof: Necessity: Suppose that f is an N#R^G-open map. Let S be a nano closed set of Y and U be a nano closed set of X such that $f^{-1}(S) \subseteq U$. Then $V=f^{-1}(U^c)^c$ is a N#R^G closed set of Y such that $f^{-1}(V) \subseteq U$.
Sufficiency: Let F be an open setofX.Then$f^{-1}(f(F))^c \subseteq F^c$ and F^c is a nano closed set in X. By the hypothesis there is a N#R^G-closed set V of Y such that$(f(F))^c \subseteq V$and$f^{-1}(V) \subseteq F^c$. Therefore $F \subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f(F) \subseteq f((f^{-1}(V))^c) \subseteq V^c$ that is, $f(F)=V^c$ which is N#R^G-open in Y and thus f is N#R^G-open map.

Theorem 4.5: If a mapping f: (U, τR(X)) → (V, τR''(Y)) is N#R^G-open then $Nint(f^{-1}(G)) \subseteq f^{-1}(N\#R\hat{}int(G))$ for every nano set G of Y.

Proof: Let G be nano set of Y. Then $Nint(f^{-1}(G))$ is a nano open set in X. Since f is N#R^G-open $f(Nint(f^{-1}(G))) \subseteq$ N#R^Gint($f(f^{-1}(G))$ ⊆ N#R^Gint(G)). Thus $Nint(f^{-1}(G)) \subseteq f^{-1}$(N#R^Gint(G)).

Definition 4.6: A mapping f: (U, τR(X)) → (V,τR''(Y)) is Nano #regular ^generalized closed map(N#R^G-closed map) if the image of every nano closed set of X is N#R^G-closed set in Y.

Definition 4.7: A mapping f: (U, τR(X)) → (V,τR''(Y)) is nano M^1#R^G-open if f(U) is N#R^G-open set in Y for every N#R^G-open set U of X.
A mapping f: (U, τR(X)) → (V,τR''(Y)) is nano M^1#R^G-closed if f(U) is N#R^G-closed set in Y for every N#R^G-closed set U of X.

Theorem 4.8: A mapping f: (U, τR(X)) →(V,τR''(Y)) is N#R^G-closed if and only if for every nano set S of Y and for each nano open set U of X containing $f^{-1}(S)$ there is a N#R^G-open set V of Y such that S⊆V and $f^{-1}(V) \subseteq U$.

Proof:
Necessity: Suppose that f is an N#R^G-closed map. Let S be a nano closed set of Y and U be a nano open set of X such that $f^{-1}(S) \subseteq U$. Then $V=f^{-1}(U^c)^c$ is a N#R^G-closed set Y such that $f^{-1}(V) \subseteq U$. Then $V= f^{-1}(U^c)$ is an N#R^G-open set of Y such that $f^{-1}(V) \subseteq U$.
Sufficiency: Let F be a nano closed set of X. Then$f^{-1}(f(F))^c \subseteq F^c$ and F^c is an nano open set in X. By hypothesis there is a N#R^G-open set V of Y such that $(f(F))^c \subseteq V$ and $f^{-1}(V) \subseteq F^c$. Therefore $F \subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f(F) \subseteq f(f^{-1}(V))^c \subseteq V^c$ that is $f(F)=V^c$ which is N#R^G-closed in Y and thus f is N#R^G-closed map.

Theorem 4.9: For any bijection f: (U, τR(X)) → (V,τR''(Y)) , the following are equivalent.
(I) f is a N#R^G-open map.
(II) f is a N#R^G-closed map.
(III) f^{-1}: V →U is N#R^G-continuous.

Proof:
(I) ⇒(II): Let F be a nano closed set in X. Then X-U is nano open set in X. By hypothesis,
f(X- U) is N#R^G-open in Y. Then Y-f(X-U) = f(U) is N#R^G-closed in Y and hence f is an N#R^G- closed map.
(II) ⇒(III): Let U be a nano closed set in X. From (II), f(U) is N#R^G-closed in Y and f(U)= $(f^{-1})^{-1}(U)$ which implies that f^{-1} is N#R^G-continuous.
(III) ⇒(I): Let V be a nano open set in X. By (III), $(f^{-1})^{-1}(V)=f(V)$ is N#R^G-open in Y. Hence f is an N#R^G-open map.

Theorem 4.10: For any bijection f: (U, τR(X)) → (V,τR''(Y)), the following statements are equivalent.
(i) f^{-1}:V →U is N#R^G-irresolute.
(ii) f is an NM^1#R^G-openmap.
(iii) f is an NM^1#R^G-closedmap.

Proof:
(i) ⇒ (ii): Let U be an N#R^G-open set in X. By (i), $(f^{-1})^{-1}(U)=f(U)$ is N#R^G-open set in Y and thus f is an NM^1-#R^G-open map.

(ii)⇒ (iii): Let V be an N#R^G-closed set in Y. By (ii), f(X-V) is N#R^G-open set in Y. Then Y-f(V) is N#R^G-open in Y and hence f(V) is N#R^G-closed in Y. Therefore, f is an NM^1#R^Gclosedmap.

(iii)⇒ (i): Let V be an N#R^G-closed set in X. By (iii), f(V) is a N#R^G-closed set in Y. Since f^{-1}: V→U is a bijection, $(f^{-1})^{-1}(V)$ is N#R^G-closed in Y. Hence f^{-1} isNM^1-#R^G- irresolute.

Definition 4.11: Let (U, τR (X)) and (V,τR'' (Y)) be two nano topological spaces. Then U is said to be $T_{1/2}$ space if every generalized closed set in U is closed.

Theorem 4.12: If f: (U,τR(X))→(V,τR''(Y)) is a nano closed map and g:(V,τR''(Y)) → (W,τR''''(Z)) is N#R^G-closed map then their composition $g \circ f$: (U,τR(X))→(W,τR''''(Z))is N#R^G-closed map.

Proof: Let F be a nano closed in X. Then by hypothesis, f(F) is nano closed set in Y. Since g is N#R^G- closed map, $g \circ f$ (F)=g(f(F)) is N#R^G-closed set in W. Thus $g \circ f$:(U,τR(X))→(W,τR''''(Z))is N#R^G-closed map.

Theorem 4.13: Let f:(U,τR(X))→ (V,$\tau_{R'}$(Y)) and g:(V,$\tau_{R''}$(Y))→(W,τR''''(Z)) be two mappings such that their composition $g \circ f$:(U,τR(X))→(W,τR''''(Z)) be a N#R^G-closed map. Then if
(i) f is nano continuous and surjective, then g is N#R^G-closed.
(ii) g is N#R^G-irresolute and injective, then f is N#R^G-closed.
(iii) f is N#R^G-continuous, surjective and X is an N#R^G$T_{1/2}$-space, then g is N#R^G-closed.

Proof:
(i) **:** Let V be a nano closed set of Y. By assumption, $f^{-1}(V)$ is a nano closed set in X. By hypothesis, ($g \circ f$ ($f^{-1}(V)$) = g(V) is N#R^G-closed set in Z. Hence g is an N#R^G-closed map.
(ii) **:** Let U be a closed set of X. By hypothesis, g^{-1}($g \circ f$ (U)) is N#R^G-closed set in Y. Therefore, f(U) is an N#R^G-closed set in Y and hence f is an N#R^G-closed map.

(iii) **:** Let V be a nano closed set in Y. By assumption, $f^{-1}(V)$ is N#R^G-closed set in X. Since X is an N#R^GT1/2space, $f^{-1}(V)$ is a nano closed set in X. By hypothesis, $g \circ f$ ($f^{-1}(V)$ =g(V) is an N#R^G-closed and thus g is an N#R^G-closed map.

Theorem 4.14:

(i) Let f:(U,τ_R(X))→ (V,$\tau_{R''}$(Y)) and g:(V,$\tau_{R''}$(Y))→(W,$\tau_{R'''}$(Z))be two N#R^G closed mappings and Y is N#R^GT1/2-space, then their composition $g \circ f$: (U, τ_R(X)) → (W, $\tau_{R'''}$(Z))is an N#R^G-closed map.

(ii) Let f:(U,τ_R(X))→ (V,$\tau_{R''}$(Y))a nano closed map and g:(V,$\tau_{R''}$(Y))→(W,$\tau_{R'''}$(Z)) be an N#R^G-closed map ,then their composition $g \circ f$:(U,τ_R(X))→(W,$\tau_{R'''}$(Z)) is a N#R^G closed map.

(iii) Let f:(U,τ_R(X))→ (V,$\tau_{R''}$(Y)) an N#R^G closed map and g: (V,$\tau_{R''}$(Y))→ (W, $\tau_{R'''}$(Z)) be a nano closed map. Y is an N#R^GT1/2- space, then their composition $g \circ f$: (U,τ_R(X))→(W,$\tau_{R'''}$(Z)) is a nano- closed map.

Proof: Obvious.

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