

WEAKER FORM OF SOFT NANO CONTRA CONTINUOUS FUNCTIONS

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(Received On: 28-08-22; Revised & Accepted On: 14-09-22)

ABSTRACT

In this paper, we have introduced a new class of Continuous function in soft nano Topological spaces; An attempt has been made to introduce contra weakly generalized continuous functions in soft nano topological spaces. Further we have investigated the properties of the above functions in soft nano Topological spaces. Also, we introduced Contra SNwg-irresolute function and few of its properties were investigated.

AMS Subject Classifications: 54A05, 54A10.

Keywords— contra SNwg –continuius function, irresolute function, soft nano topology.

1. INTRODUCTION

Dontchev [5] introduced the concept of a contra-continuous function, which is a modified version of continuity that requires inverse images of open sets to be closed rather than open. In 1971, Gentry and Hoyle [8] defined the class of somewhat continuous functions. These functions, which are a generalized form of continuity that require nonempty inverse images of open sets to have nonempty interiors instead of being open, have proven to be extremely useful in topology. Long and Herrington [12] defined a new type of function known as a strongly q -continuous function. Noiri and Popa [13] proposed and investigated the quasi q -continuous function. Ganster and Reily [7] established and investigated the concept of LC-Continuous functions. Following this, numerous authors proposed further generalisations of contra-continuity like contra- q -continuous [4], perfectly continuous [14], and contra-pre continuous [9].

Generalized contra continuous (contra g -continuous) functions were first developed by Caldas, Jafari, Noiri, and Simoes [4] in 2007. Contra $g\#p$ continuous function by Alli [1] and contra g_s continuous [6] and contra α^* continuous functions, almost contra α^* continuous function [15], and contra π_{gr} continuous, almost contra π_{gr} continuous [10] are new forms of contra generalised continuity.

Nano open sets have been used to describe and determine the characteristics of nano continuous function [11]. Additionally, several weak types of nano open sets, nano semi-open sets, and nano pre-open sets were established. Benchalli *et al* [2] developed the concept of soft nano topological spaces based on Nano topology and soft set theory by utilising a soft set equivalence relation on the universal set. In addition, [3] introduces and studies the concept of soft nano continuity, as well as weaker forms of soft nano open sets and weaker and generalised forms of soft nano continuous functions in soft nano topological spaces.

The purpose of this study is to present certain properties of contra SNwg-continuity using the concepts of SNwg closed set and SNwg continuity. Different conditions for a function to be a Contra SNwg-continuous function are also established. In section four, the Contra SNwg-irresolute function was introduced and few of its properties were investigated.

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2. PRELIMINARIES

We recollect the subsequent definitions which can be useful in the sequel.

Definition 2.1: [2] Let set of objects be denoted by \mathcal{U} , \mathcal{R} is a soft equivalence relation and $\tau_{\mathcal{R}}(\mathcal{X}) = \{\mathcal{U}, \emptyset, (L_{\mathcal{R}}(\mathcal{X}), O), (U_{\mathcal{R}}(\mathcal{X}), O), (B_{\mathcal{R}}(\mathcal{X}), O)\}$ satisfies the following axioms:

- i) U and $\emptyset \in \tau_{\mathcal{R}}(\mathcal{X})$.
- ii) The union of the elements of any finite subcollection $\tau_{\mathcal{R}}(\mathcal{X})$ is in $\tau_{\mathcal{R}}(\mathcal{X})$
- iii) The intersection of the elements of any finite subcollection $\tau_{\mathcal{R}}(\mathcal{X})$ is in $\tau_{\mathcal{R}}(\mathcal{X})$

Then $\tau_{\mathcal{R}}(\mathcal{X})$ is soft nano topology on \mathcal{U} with respect to \mathcal{X} , elements of the soft nano topology are known as the soft nano open sets and $(\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, O)$ is called a soft nano topological space.

Definition 2.2[3] Let $(\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{E})$ and $(\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$ be two soft nano topological spaces. Then, the map $\varphi: (\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{E}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$ is SNWg- continuous on \mathcal{U} if the inverse image of every soft nano closed set in \mathcal{V} is SNWg-closed in \mathcal{U} .

3. CONTRA SNWg-CONTINUOUS FUNCTION

This section introduces the Contra SNWg-continuous function and examines some of its properties.

Definition 3.1: The map $\varphi: (\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{E}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$ is Contra SNWg-continuous on \mathcal{U} if the inverse image of every soft nano open set in \mathcal{V} is soft nano weakly generalized closed set in \mathcal{U} .

Example 3.2: Let $\mathcal{U} = \{a, b, c, d, e\}, \mathcal{E} = \{h_1, h_2, h_3\}$ and $\mathcal{X} = \{a, b\} \subseteq \mathcal{U}$ with $\mathcal{U}/\mathcal{R} = \{F(h_1), F(h_2), F(h_3)\} = \{\{a\}, \{b\}, \{c\}, \{d\}, \{e\}\}$. Then soft nano topology is $(\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{E}) = \{\mathcal{U}, \emptyset, \{(h_1, \{a, b, c\}), (h_2, \{a, b, c\}), (h_3, \{a, b, c\})\}\}$. Let $\mathcal{V} = \{p, q, r, s, t\}, \mathcal{K} = \{k_1, k_2, k_3\}$ and $\mathcal{Y} = \{p, q, r\} \subseteq \mathcal{V}$. Let $\mathcal{V}/\mathcal{R}' = \{F(h_1), F(h_2), F(h_3)\} = \{\{q\}\{p, r\}, \{s, t\}\}$. Then, soft nano topology is $(\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K}) = \{\mathcal{V}, \emptyset, \{(k_1, \{p, q\}), (k_2, \{p, q\}), (k_3, \{p, q\})\}\}$. Define $\varphi: (\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{E}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$, let us consider $\varphi: \mathcal{U} \rightarrow \mathcal{V}$ and $p: \mathcal{E} \rightarrow \mathcal{K}$ by $\varphi(a) = s, \varphi(b) = t, \varphi(c) = p, \varphi(d) = q, \varphi(e) = r$. Then φ is contra SNWg-continuous function.

Theorem 3.3: Let $\varphi: (\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{E}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$ be a Contra SNWg-continuous function, if and only if inverse image of every soft nano closed set in \mathcal{V} is soft nano weakly generalized open in \mathcal{U} .

Proof: Let $\varphi: (\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{E}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$ and (G, \mathcal{E}) be a soft nano closed set in \mathcal{V} . Since φ is Contra SNWg-continuous function $\varphi^{-1}(\mathcal{V} - (G, \mathcal{E})) = \mathcal{U} - \varphi^{-1}((G, \mathcal{E}))$ is soft nano closed set in \mathcal{U} . Hence $\varphi^{-1}((G, \mathcal{E}))$ is SNWg-open set in \mathcal{V} .

Conversely, let (G, \mathcal{E}) be a soft nano open set in \mathcal{V} . By assumption is $\varphi^{-1}(\mathcal{V} - (G, \mathcal{E}))$ SNWg-open set. $\varphi^{-1}(\mathcal{V} - (G, \mathcal{E})) = \mathcal{U} - \varphi^{-1}((G, \mathcal{E}))$, $\varphi^{-1}((G, \mathcal{E}))$ is SNWg-closed set in \mathcal{U} . Hence φ is Contra SNWg-continuous function.

Theorem 3.4: Let $\varphi: (\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{E}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$ be a function from $(\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{E})$ to $(\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$ then the following conditions are equivalent.

- i) Inverse image of every soft nano closed set in \mathcal{V} is soft nano weakly generalized open in \mathcal{U} .
- ii) For $x \in \mathcal{U}$ and each soft nano closed set (G, \mathcal{E}) in \mathcal{V} with $\varphi(x) \in (G, \mathcal{E})$ there exist an SNWg-open set in \mathcal{U} such that $\varphi(\mathcal{A}, \mathcal{E}) \subseteq (G, \mathcal{E})$.

Proof: (i) \rightarrow (ii)

Let (G, \mathcal{K}) be soft nano closed set in \mathcal{V} , such that $\varphi(x) \in (G, \mathcal{K}), x \in \mathcal{U}$. Let $(\mathcal{A}, \mathcal{E}) = \varphi^{-1}((G, \mathcal{K})). x \in (\mathcal{A}, \mathcal{E})$, and $\varphi((\mathcal{A}, \mathcal{E})) \subseteq (G, \mathcal{K})$.

(ii) \rightarrow (i): Let (G, \mathcal{K}) be any Soft nano closed in \mathcal{V} , $x \in \mathcal{U}, \varphi^{-1}(x) \in (G, \mathcal{K})$. There exists an SNWg open set \mathcal{U}_x such that $\varphi(\mathcal{U}_x) \subseteq (G, \mathcal{K}), \varphi^{-1}((G, \mathcal{K})) = \cup \{\mathcal{U}_x, x \in \varphi^{-1}((G, \mathcal{K})) \in SNO(x)\}, \varphi^{-1}((G, \mathcal{K}))$ is SNWg - open set in \mathcal{U} .

Remark 3.5: Composition of two Contra SNWg-continuous function need not be Contra SNWg continuous function as shown in the following example

Example 3.6: Let $\mathcal{U} = \{a, b, c, d, e\}, \mathcal{H} = \{h_1, h_2, h_3\}$ and $\mathcal{X} = \{a, b\} \subseteq \mathcal{U}$ with $\mathcal{U} / \mathcal{R} = \{F(h_1), F(h_2), F(h_3)\} = \{\{a\}, \{b\}, \{c\}, \{d\}, \{e\}\}$. Then soft nano topology is $(\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{H}) = \{\mathcal{U}, \emptyset, \{(h_1, \{b\}), (h_2, \{b\}), (h_3, \{b\})\}, \{(h_1, \{a, c\}), (h_2, \{a, c\}), (h_3, \{a, c\})\}, \{(h_1, \{a, b, c\}), (h_2, \{a, b, c\}), (h_3, \{a, b, c\})\}\}$

Let $\mathcal{V} = \{p, q, r, s, t\}, \mathcal{K} = \{h_1, h_2, h_3\}$ and $\mathcal{Y} = \{p, q\} \subseteq \mathcal{V}$.

Let $\mathcal{V} / \mathcal{R}' = \{F(h_1), F(h_2), F(h_3)\} = \{\{p, q\}, \{s\}, \{r, t\}\}$. Then, soft nano topology is $(\tau_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K}) = \{\mathcal{V}, \emptyset, \{(h_1, \{p, q\}), (h_2, \{p, q\}), (h_3, \{p, q\})\}\}$. Let $\mathcal{W} = \{i, j, k, l, m\}, \mathcal{E} = \{e_1, e_2, e_3\}$ and $\mathcal{Z} = \{k, m\}$ with $\mathcal{W} / \tau_{\mathcal{R}''} = \{F(e_1), F(e_2), F(e_3)\} = \{\{i\}, \{j\}, \{k\}, \{l\}, \{m\}\}$. Then $(\tau_{\mathcal{R}''}(\mathcal{Z}), \mathcal{W}, \mathcal{E}) = \{\mathcal{W}, \emptyset, \{(e_1, \{l, m\}), (e_2, \{l, m\}), (e_3, \{l, m\})\}\}$.

Define $\varphi: (\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{H}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$, let us consider $\varphi: \mathcal{U} \rightarrow \mathcal{V}$ and $p: \mathcal{H} \rightarrow \mathcal{K}$ by $\varphi(a) = r, \varphi(b) = s, \varphi(c) = t, \varphi(d) = p, \varphi(e) = q, \psi: (\tau_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K}) \rightarrow (\tau_{\mathcal{R}''}(\mathcal{Z}), \mathcal{W}, \mathcal{E})$ by $\psi(p) = l, \varphi(q) = m, \varphi(r) = k, \psi(s) = m, \psi(t) = i$. Function φ and ψ are contra SNwg- continuous function but their composition is not contra SNwg-continuous function. Since $\varphi^{-1}(\psi^{-1}(\{k, m\})) = \varphi^{-1}(\{r, s\}) = \{a, b\}$ is not SNwg-closed.

Remark 3.7: Contra SNwg-continuous function and SNwg-continuous function are independent.

Example 3.8: Let $\mathcal{U} = \{a, b, c, d, e\}, \mathcal{H} = \{h_1, h_2, h_3\}$ and $\mathcal{X} = \{a, b\} \subseteq \mathcal{U}$ with $\mathcal{U} / \mathcal{R} = \{F(h_1), F(h_2), F(h_3)\} = \{\{a, c\}, \{b\}, \{d\}, \{e\}\}$. Then soft nano topology is $(\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{H}) = \{\mathcal{U}, \emptyset, \{(h_1, \{b\}), (h_2, \{b\}), (h_3, \{b\})\}, \{(h_1, \{a, c\}), (h_2, \{a, c\}), (h_3, \{a, c\})\}, \{(h_1, \{a, b, c\}), (h_2, \{a, b, c\}), (h_3, \{a, b, c\})\}\}$

Let $\mathcal{V} = \{p, q, r, s, t\}, \mathcal{K} = \{h_1, h_2, h_3\}$ and $\mathcal{Y} = \{r, t\} \subseteq \mathcal{V}$.

Let $\mathcal{V} / \mathcal{R}' = \{F(h_1), F(h_2), F(h_3)\} = \{\{p\}, \{q, s\}, \{r, t\}\}$. Then, soft nano topology is $(\tau_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K}) = \{\mathcal{V}, \emptyset, \{(h_1, \{r, t\}), (h_2, \{r, t\}), (h_3, \{r, t\})\}\}$. Define $\varphi: (\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{H}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$, let us consider $\varphi: \mathcal{U} \rightarrow \mathcal{V}$ and $p: \mathcal{H} \rightarrow \mathcal{K}$ by $\varphi(a) = r, \varphi(b) = t, \varphi(c) = p, \varphi(d) = s, \varphi(e) = q$. Then φ is SNwg-continuous function but not contra SNwg- continuous function. Since $\varphi^{-1}(\{r, t\}) = \{a, b\}$ is not SNwg-closed in \mathcal{U} .

Example 3.9: Let $\mathcal{U} = \{a, b, c, d, e\}, \mathcal{H} = \{h_1, h_2, h_3\}$ and $\mathcal{X} = \{a, b\} \subseteq \mathcal{U}$ with $\mathcal{U} / \mathcal{R} = \{F(h_1), F(h_2), F(h_3)\} = \{\{a\}, \{b\}, \{c\}, \{d\}, \{e\}\}$. Then soft nano topology is $(\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{H}) = \{\mathcal{U}, \emptyset, \{(h_1, \{b\}), (h_2, \{b\}), (h_3, \{b\})\}, \{(h_1, \{a, c\}), (h_2, \{a, c\}), (h_3, \{a, c\})\}, \{(h_1, \{a, b, c\}), (h_2, \{a, b, c\}), (h_3, \{a, b, c\})\}\}$

Let $\mathcal{V} = \{p, q, r, s, t\}, \mathcal{K} = \{h_1, h_2, h_3\}$ and $\mathcal{Y} = \{p, q\} \subseteq \mathcal{V}$. Let $\mathcal{V} / \mathcal{R}' = \{F(h_1), F(h_2), F(h_3)\} = \{\{p, q\}, \{s\}, \{r, t\}\}$. Then, soft nano topology is $(\tau_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K}) = \{\mathcal{V}, \emptyset, \{(h_1, \{p, q\}), (h_2, \{p, q\}), (h_3, \{p, q\})\}\}$. Define $\varphi: (\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{H}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$, let us consider $\varphi: \mathcal{U} \rightarrow \mathcal{V}$ and $p: \mathcal{H} \rightarrow \mathcal{K}$ by $\varphi(a) = r, \varphi(b) = s, \varphi(c) = t, \varphi(d) = p, \varphi(e) = q$. Then φ is contra SNwg- continuous function but not SNwg- continuous function. Since $\varphi^{-1}(\{p, q\}) = \{d, e\}$ is not SNwg-closed in \mathcal{U} .

Theorem 3.10: Every soft Soft nanoContra continuous function is Contra SNwg-continuous function.

Proof: $\varphi: (\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{E}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$ be a soft nano Contra continuous function. (G, \mathcal{E}) be soft Nano open set in \mathcal{V} . Since φ is soft nano Contra function $\varphi^{-1}((G, \mathcal{E}))$ is closed set in \mathcal{U} . Therefore $\varphi^{-1}((G, \mathcal{E}))$ is SNwg-closed in \mathcal{U} . Hence φ is Contra SNwg-continuous function.

Remark 3.11: If $\varphi: (\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{E}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$ is Contra SNwg-continuous function then φ need not be soft nano Contra continuous function as shown in the following example.

Example 3.12: Let $\mathcal{U} = \{a, b, c, d, e\}, \mathcal{E} = \{h_1, h_2, h_3\}$ and $\mathcal{X} = \{a, b\} \subseteq \mathcal{U}$ with $\mathcal{U} / \mathcal{R} = \{F(h_1), F(h_2), F(h_3)\} = \{\{a\}, \{b\}, \{c\}, \{d\}, \{e\}\}$. Then soft nano topology is $(\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{E}) = \{\mathcal{U}, \emptyset, \{(h_1, \{a, b, c\}), (h_2, \{a, b, c\}), (h_3, \{a, b, c\})\}\}$. Let $\mathcal{V} = \{p, q, r, s, t\}, \mathcal{K} = \{h_1, h_2, h_3\}$ and $\mathcal{Y} = \{p, q, r\} \subseteq \mathcal{V}$. Let $\mathcal{V} / \mathcal{R}' = \{F(h_1), F(h_2), F(h_3)\} = \{\{q\}, \{p, r\}, \{s, t\}\}$. Then, soft nano topology is $(\tau_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K}) = \{\mathcal{V}, \emptyset, \{(h_1, \{p, q\}), (h_2, \{p, q\}), (h_3, \{p, q\})\}\}$. Define $\varphi: (\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{E}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$, let us consider $\varphi: \mathcal{U} \rightarrow \mathcal{V}$ and $p: \mathcal{E} \rightarrow \mathcal{K}$ by $\varphi(a) = s, \varphi(b) = t, \varphi(c) = p, \varphi(d) = q, \varphi(e) = r$. Then φ is contra SNwg-continuous function but not soft nano Contra continuous function. Since $\varphi^{-1}(\{p, q\}) = \{c, d\}$ is SNwg-closed but not soft nano closed in \mathcal{U} .

Theorem 3.13: A function φ from soft nano topological space \mathcal{U} to soft nano topological space \mathcal{V} is Contra SNwg-continuous function if the only soft nano open set containing the inverse image of every soft nano open (G, \mathcal{E}) of \mathcal{V} is \mathcal{U} .

Proof: Let (G, \mathcal{E}) be a Soft nano open set \mathcal{V} and \mathcal{U} is the only Soft nano open set such that $\varphi^{-1}((G, \mathcal{E})) \subseteq \mathcal{U}$. Then $Ncl(Nint(\varphi^{-1}((G, \mathcal{E}))) \subseteq \mathcal{U}$. (i.e.) $\varphi^{-1}((G, \mathcal{E}))$ is SNwg-closed in \mathcal{U} . φ is Contra SNwg-continuous function.

Corollary 3.14: Let $\varphi: (\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{E}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$ be a function and $Nint(\varphi^{-1}((G, \mathcal{E}))) = \phi$ for every soft nano open set (G, \mathcal{E}) of \mathcal{V} then φ is Contra SNwg-continuous function.

Theorem 3.15: Let $\varphi: (\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{E}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$ be a Contra SNwg-continuous function then $\varphi(Nint_{wg}(G, \mathcal{E})) \subseteq Ncl(\varphi((G, \mathcal{E}))$ for every subset $(G, \mathcal{E}) \subseteq \mathcal{V}$.

Proof: Let $(G, \mathcal{E}) \subseteq \mathcal{U}$ then $Ncl(\varphi(G, \mathcal{E}))$ is a soft nano closed set in \mathcal{V} . Since φ is Contra SNwg-continuous $\varphi^{-1}(Ncl(\varphi(G, \mathcal{E})))$ is SNwg-open set in \mathcal{U} and $Nint_{wg}(\varphi^{-1}(Ncl(\varphi((G, \mathcal{E})))) = \varphi^{-1}(Ncl(\varphi(G, \mathcal{E})))$. $\varphi((G, \mathcal{E})) \subseteq Ncl(\varphi((G, \mathcal{E})))$, $Nint_{wg}((G, \mathcal{E})) \subseteq Nint_{wg}(\varphi^{-1}(Ncl(\varphi((G, \mathcal{E}))))$, $Nint_{wg}((G, \mathcal{E})) \subseteq \varphi^{-1}(Ncl(\varphi(G, \mathcal{E})))$, $f(Nint_{wg}((G, \mathcal{E}))) \subseteq Ncl(\varphi((G, \mathcal{E})))$.

Remark 3.16: The converse of the above theorem need not be true as shown in the following example.

Example 3.17: In example 3.8 $\varphi(Nint_{wg}(G, \mathcal{E})) \subseteq Ncl(\varphi((G, \mathcal{E}))$ but φ is not contra SNwg-continuous function.

Remark 3.18: In theorem 3.15, if $\varphi((G, \mathcal{E}))$ is soft nano closed set then $\varphi(Nint_{wg}((G, \mathcal{E}))) = Ncl(\varphi((G, \mathcal{E}))$

Corollary 3.19: Let $\varphi: (\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{E}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$ be a SNwg-continuous function then $\varphi(Nint_{wg}(\varphi^{-1}((G, \mathcal{E})))) \subseteq Ncl((G, \mathcal{E}))$ for every subset $(G, \mathcal{E}) \subseteq \mathcal{V}$.

Theorem 3.20: Let $\varphi: (\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{E}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$ be a SNwg-continuous function then $\varphi^{-1}(Nint(G, \mathcal{E})) \subseteq Nint_{wg}(\varphi^{-1}((G, \mathcal{E})))$ for every subset $(G, \mathcal{E}) \subseteq \mathcal{V}$.

Proof: Let $(G, \mathcal{E}) \subseteq \mathcal{V}$ the $Nint((G, \mathcal{E}))$ is a soft nano open set in \mathcal{V} . Since φ is Contra SNwg-continuous $\varphi^{-1}(Nint(G, \mathcal{E}))$ is SNwg-closed set in \mathcal{U} and $Ncl_{wg}(\varphi^{-1}(Nint(G, \mathcal{E}))) = \varphi^{-1}(Nint(G, \mathcal{E}))$. $Nint((G, \mathcal{E})) \subseteq (G, \mathcal{E})$, $\varphi^{-1}(Nint((G, \mathcal{E}))) \subseteq \varphi^{-1}((G, \mathcal{E}))$, $Ncl_{wg}(\varphi^{-1}(Nint(G, \mathcal{E}))) \subseteq Ncl_{wg}(\varphi^{-1}((G, \mathcal{E})))$, $\varphi^{-1}(Nint(G, \mathcal{E})) \subseteq Ncl_{wg}(\varphi^{-1}((G, \mathcal{E})))$

Remark 3.21: The converse of the above theorem need not be true as shown in the following example

Example 3.22: In example 3.9, φ is not Contra SNwg-continuous function, but $\varphi^{-1}(Nint(G, \mathcal{E})) \subseteq Nint_{wg}(\varphi^{-1}((G, \mathcal{E})))$ for every subset $(G, \mathcal{E}) \subseteq \mathcal{V}$.

Remark 3.23: In theorem 3.20, $\varphi^{-1}(Nint A)) \subseteq Nint_{wg}(\varphi^{-1}((G, \mathcal{E})))$ if (G, \mathcal{E}) is soft nano open.

4. CONTRA SNwg IRRESOLUTE FUNCTION

In this section we define SNwg-irresolute function and examine some of its properties.

Definition 4.1: The map $\varphi: (\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{E}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$ is Contra SNwg-irresolute function on \mathcal{U} if the inverse image of every soft nano weakly generalized open(closed) set in \mathcal{V} is soft nano weakly generalized closed (open) in \mathcal{U} .

Example 4.2: Let $\mathcal{U} = \{a, b, c, d, e\}$, $\mathcal{E} = \{\mathcal{h}_1, \mathcal{h}_2, \mathcal{h}_3\}$ and $\mathcal{X} = \{c, d, e\} \subseteq \mathcal{U}$ with $\mathcal{U} / \mathcal{R} = \{F(\mathcal{h}_1), F(\mathcal{h}_2), F(\mathcal{h}_3)\} = \{\{a\}, \{b\}, \{c, d\}, \{e\}\}$. Then soft nano topology is $(\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{E}) = \{\mathcal{U}, \emptyset, \{(\mathcal{h}_1, \{c, d, e\}), (\mathcal{h}_2, \{c, d, e\}), (\mathcal{h}_3, \{c, d, e\})\}\}$. Let $\mathcal{V} = \{p, q, r, s, t\}$, $\mathcal{K} = \{\mathcal{k}_1, \mathcal{k}_2, \mathcal{k}_3\}$ and $\mathcal{Y} = \{s, t\} \subseteq \mathcal{V}$. Let $\mathcal{V} / \mathcal{R}' = \{F(\mathcal{k}_1), F(\mathcal{k}_2), F(\mathcal{k}_3)\} = \{\{p\}\{q, r\}, \{s, t\}\}$. Then, soft nano topology is $(\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K}) = \{\mathcal{V}, \emptyset, \{(\mathcal{k}_1, \{s, t\}), (\mathcal{k}_2, \{s, t\}), (\mathcal{k}_3, \{s, t\})\}\}$. Define $\varphi: (\tau_{\mathcal{R}}(\mathcal{X}), \mathcal{U}, \mathcal{E}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$, let us consider $\varphi: \mathcal{U} \rightarrow \mathcal{V}$ and $p: \mathcal{E} \rightarrow \mathcal{K}$ by $\varphi(a) = s, \varphi(b) = t, \varphi(c) = q, \varphi(d) = r, \varphi(e) = p$. Then φ is Contra SNwg irresolute function.

Remark 4.3: Composition of two Contra SNwg-irresolute function on U need not be Contra SNwg-irresolute function as given in the following example.

Example 4.4: Let $U = \{a, b, c, d, e\}, \mathcal{E} = \{\mathcal{h}_1, \mathcal{h}_2, \mathcal{h}_3\}$ and $X = \{c, d, e\} \subseteq U$ with $U / \mathcal{R} = \{F(\mathcal{h}_1), F(\mathcal{h}_2), F(\mathcal{h}_3)\} = \{\{a\}, \{b\}, \{c, d\}, \{e\}\}$. Then soft nano topology is $(\tau_{\mathcal{R}}(X), U, \mathcal{E}) = \{U, \emptyset, \{(\mathcal{h}_1, \{c, d, e\}), (\mathcal{h}_2, \{c, d, e\}), (\mathcal{h}_3, \{c, d, e\})\}\}$. Let $\mathcal{V} = \{p, q, r, s, t\}, \mathcal{K} = \{\mathcal{k}_1, \mathcal{k}_2, \mathcal{k}_3\}$ and $\mathcal{Y} = \{s, t\} \subseteq \mathcal{V}$. Let $\mathcal{V} / \mathcal{R}' = \{F(\mathcal{k}_1), F(\mathcal{k}_2), F(\mathcal{k}_3)\} = \{\{p\}\{q, r\}, \{s, t\}\}$. Then, soft nano topology is $(\tau_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K}) = \{\mathcal{V}, \emptyset, \{(\mathcal{k}_1, \{s, t\}), (\mathcal{k}_2, \{s, t\}), (\mathcal{k}_3, \{s, t\})\}\}$. Let $\mathcal{W} = \{i, j, k, l, m\}, \mathcal{E} = \{e_1, e_2, e_3\}$ and $\mathcal{Z} = \{i, j, l\}$ with

$\mathcal{W} / \tau_{\mathcal{R}''} = \{F(e_1), F(e_2), F(e_3)\} = \{\{i\}, \{j\}, \{k\}, \{l\}, \{m\}\}$. Then $(\tau_{\mathcal{R}''}(\mathcal{Z}), \mathcal{W}, \mathcal{E}) = \{\mathcal{W}, \emptyset, \{(e_1, \{i, j, l\}), (e_2, \{i, j, l\}), (e_3, \{i, j, l\})\}\}$. Define $\varphi: (\tau_{\mathcal{R}}(X), U, \mathcal{E}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$, let us consider $\varphi: U \rightarrow \mathcal{V}$ and $p: \mathcal{H} \rightarrow \mathcal{K}$ by $\varphi(a) = s, \varphi(b) = t, \varphi(c) = q, \varphi(d) = r, \varphi(e) = p$. $\psi: (\tau_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K}) \rightarrow (\tau_{\mathcal{R}''}(\mathcal{Z}), \mathcal{W}, \mathcal{E})$ by $\psi(p) = i, \varphi(q) = j, \varphi(r) = l, \psi(s) = k, \psi(t) = m$. Here φ and ψ are Contra SNwg-irresolute function but their composition is not Contra SNwg-irresolute function. Since $\varphi^{-1}(\psi^{-1}(\{k, m\})) = \varphi^{-1}(\{s, t\}) = \{a, b\}$ is not SNwg-open set in U .

Theorem 4.5: $\varphi: (\tau_{\mathcal{R}}(X), U, \mathcal{E}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{H}), \psi: (\tau_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{H}) \rightarrow (\tau_{\mathcal{R}''}(\mathcal{Z}), \mathcal{W}, \mathcal{K})$ be two Contra SNwg-irresolute functions, then their composition is SNwg-irresolute function.

Proof: Let (G, \mathcal{E}) be a SNwg-open set in \mathcal{W} . $\varphi^{-1}((G, \mathcal{E}))$ is SNwg-closed set in \mathcal{V} , since ψ is Contra SNwg-irresolute function. $\varphi^{-1}(\psi^{-1}((G, \mathcal{E})))$ is SNwg-open in U because φ is Contra SNwg-irresolute. Hence the composition of φ and ψ is SNwg-continuous function.

Theorem 4.6: Every Contra SNwg-irresolute function is Contra SNwg-continuous function.

Proof: Let $\varphi: (\tau_{\mathcal{R}}(X), U, \mathcal{E}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$ be a Contra SNwg-irresolute function and (G, \mathcal{E}) be a soft nano closed set in $(\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$. (G, \mathcal{E}) is SNwg-closed set since every soft nano closed set is SNwg-closed set. Then $\varphi^{-1}((G, \mathcal{E}))$ is SNwg-open set. Hence φ is Contra SNwg-continuous function.

Remark 4.7: Every Contra SNwg-continuous function need not be Contra SNwg-irresolute function as shown in the example.

Example 4.8: Let $U = \{a, b, c, d, e\}, \mathcal{E} = \{\mathcal{h}_1, \mathcal{h}_2, \mathcal{h}_3\}$ and $X = \{c, d, e\} \subseteq U$ with $U / \mathcal{R} = \{F(\mathcal{h}_1), F(\mathcal{h}_2), F(\mathcal{h}_3)\} = \{\{a\}, \{b\}, \{c\}, \{d\}, \{e\}\}$. Then soft nano topology is $(\tau_{\mathcal{R}}(X), U, \mathcal{E}) = \{U, \emptyset, \{(\mathcal{h}_1, \{c, d, e\}), (\mathcal{h}_2, \{c, d, e\}), (\mathcal{h}_3, \{c, d, e\})\}\}$. Let $\mathcal{V} = \{p, q, r, s, t\}, \mathcal{K} = \{\mathcal{k}_1, \mathcal{k}_2, \mathcal{k}_3\}$ and $\mathcal{Y} = \{s, t\} \subseteq \mathcal{V}$. Let $\mathcal{V} / \mathcal{R}' = \{F(\mathcal{k}_1), F(\mathcal{k}_2), F(\mathcal{k}_3)\} = \{\{p\}\{q, r\}, \{s, t\}\}$. Then, soft nano topology is $(\tau_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K}) = \{\mathcal{V}, \emptyset, \{(\mathcal{k}_1, \{q, r\}), (\mathcal{k}_2, \{q, r\}), (\mathcal{k}_3, \{q, r\})\}\}$. Define $\varphi: (\tau_{\mathcal{R}}(X), U, \mathcal{E}) \rightarrow (\sigma_{\mathcal{R}'}(\mathcal{Y}), \mathcal{V}, \mathcal{K})$, let us consider $\varphi: U \rightarrow \mathcal{V}$ and $p: \mathcal{E} \rightarrow \mathcal{K}$ by $\varphi(a) = s, \varphi(b) = r, \varphi(c) = t, \varphi(d) = p, \varphi(e) = q$. Then φ is Contra SNwgcontinuous function but not Contra SNwg-irresolute function. Since $\varphi^{-1}(\{q, r, s\}) = \{a, b, e\}$ is not SNwg-closed in U .

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Source of support: Nil, Conflict of interest: None Declared.

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