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COMMON FIXED POINT RESULTS OF GENERALIZED CONTRACTIVE CONDITION IN METRIC SPACES

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ABSTRACT

In this paper, we prove a unique common fixed point theorem for generalized Contractive conition in metric space, Our result generalize , improves the recent results existing in the literature.

Key words: *Fixed point, common fixed point, reciprocally continuous, weakly compatible.*

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1. INTRODUCTION

from 1922 the study of Existence and uniqueness of coincidence points and common fixed points of mappings satisfying certain contractive conditions has been an interesting field of mathematics .In 1968, Banach has proved fixed point theorem it is said to first fixed point theorem in metric space Later on many Mathematicians were improved, generalized and extended the Banach fixed point theorem in many ways for e.g. [13-11]. Recently A.Djoudi [2] proved some results in metric space. Our result is generelization and improved of A.Djoudi [2].

The following are useful in the main results which are [2].

Definition 1.1: Two self maps A and B of a metric space (X, d) are said to be commute if AB = BA., Two self maps S and T of a metric space (X, d) are said to be compatible mappings if $\lim_{n\to\infty} d(ABx_{n\,,}\, BAx_{n\,,}) = 0$,, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_{n=}$ $\lim_{n\to\infty} Bx_{n=}$ t for some $t \in X$.

Definition 1.2: The maps S and T of a metric space (X, d) are said to be reciprocally continuous if $\lim_{n\to\infty} STx_n = S(t)$ and $\lim_{n\to\infty} TSx_n = T(t)$,whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = t$ and $\lim_{n\to\infty} Tx_n = t$, for some $t \in X$.

Definition 1.3: Let A, B: X → X. Then the pair (A, B) is called weakly compatible, if AB z = BAz for all z ∈ X such that Az = Bz.

2. MAIN RESULTS

In this section we obtained a unique common fixed point result for four self mappings with different contractive condition.

The following result is generalized, and improved the results of [2].

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Theorem 2.1: Let A, B, I and J are four self-mappings in a complete metric space (X, d) and satisfying the following conditions

(i) $A(X) \subseteq J(X)$ and $B(X) \subseteq I(X)$
(ii) $d(Ax, By) \leq \{d(Ix, Jy), [d(Ix, Ax) + d(Jy, By) / 2], [d(Ix, By) + d(Jy, Ax) /2]\}$
(iii) (A, I) is reciprocally continuous and Compatible
(iv) (B, J) is weakly compitable
(v) The sequence $Ax_0, Bx_1, Ax_2, Bx_3, \ldots Ax_{2n}, Bx_{2n+1} \ldots$converges to $z \in X$. Then A, B, I and J have a unique common fixed point in X.

Proof: Let (X, d) be complete metric space, for any $x_0 \in X$ and iterated sequence $\{x_n\}$ and the sequence $Ax_0, Bx_1, Ax_2, Bx_3, \ldots Ax_{2n}, Bx_{2n+1} \ldots$ convergent to a point $z \in X$ from (v)

$$Ax_{2n} \to z \quad \text{and} \quad Bx_{2n+1} \to z \quad \text{as} \quad n \to \infty \tag{1}$$

Since (A, B) is reciprocated continuous $AIx_{2n} \to Az$ and $IAx_{2n} \to Iz$ as $n \to \infty$

By the (A, I) compatibility,

$\lim_{n\to\infty} d(AIx_{2n}, IAx_{2n}) = 0.$

$\Rightarrow d(Az, Iz) = 0$. That is, $Az = Iz$. Since, $A(X) \subseteq J(X)$.

$\Rightarrow$ There exists $p \in X$ such that $Jp = z$., and $B(X) \subseteq I(X)$.

$\Rightarrow$ There exists $q \in X$ such that $Iq = z$. To prove $Az = z$. Put $x = z$ and $y = \ = x_{2n+1}$ (ii) we get

$d(Az, Bx_{2n+1}) \leq \{d(Iz, Jx_{2n+1}), [d(Iz, Az)+ d(J_{2n+1}, Bx_{2n+1}) / 2], [d(Iz, Bx_{2n+1}) + d(Jx_{2n+1}, Az) /2]\}.$

Letting $n \to \infty$

$d(Az, z) \leq \{d(Iz, z), [d(Iz, Az) + d(z, z) / 2], [d(Iz, z) + d(z, Az) /2]\}.$
$\leq \{d(Az, z), [d(Az, Az) + d(z, z) / 2], [d(Az, z) +d(z, Az) /2]\}.$
$\leq \{d(Az, z, \ d(z, Az)]\} < d(Az, z)$, which is a contradiction.

Therefore, $Az = z$.

To prove, $Bp= z$. Put $x = x_{2n}$ and $y = p$ in (ii) we get

$d(Ax_{2n}, Bp) \leq \{d(Ix_{2n}, Jp), [d(Ix_{2n}, Ax_{2n}) + d(Jp, Bp) / 2], [d(z, Bp) + d(Jp, z) /2]\}.$

Letting $n \to \infty$

$d(z, Bp) \leq \{d(z, Jp), [d(z, z) + d(z, Bp) / 2], [d(z, Bp) + d(Jp, z)] /2]\}.$
$\leq \{d(z, z), [d(z, z) + d(z, Bp) / 2], [d(z, Bp) +d(z, z) /2] \}.$
$\leq \{d(z, Bp)/2, \ d(z, Bz)/2]\}< d(z, Bp)$, which is a contradiction.

Therefore, $Bp = z$.

Hence $Bp = Jp = z$.

Since (I, J) is weakly compatible

$\Rightarrow BJp = JBp \Rightarrow Bz = Jz$.

To prove $Bz = z$. Put $x = x_{2n}$, $y = z$ in (ii) we get

$d(Ax_{2n}, Bz) \leq \{d(Ix_{2n}, Jz), [d(Ix_{2n}, Ax_{2n})+ d(Jz, Bz)] / 2, [d(Ix_{2n}, Bz) + d(Jz, Ax_{2n})] /2]\}.$

Letting $n \to \infty$

$d(z,Bz) \leq \{d(z, Jz), [d(z, z) + d(z, Bz)] / 2], [d(z, Bz) + d(Jz, z)] /2]\}.$
$\leq \{d(z, z), [d(z, z) + d(z, Bz)] / 2], [d(z, Bz) + d(z, z)] /2]\}.$
$< d(z, Bz)$, which is a contradiction.

Therefore, $Bz = z$.

Hence $Az = Bz = z$.

To prove, $Iz = z$.

Put, $x = Iz$ and $y = x_{2n+1}$ in (ii) we get

$d(AIx_{2n}, Bx_{2n+1}) \leq \{d(IIz, Jx_{2n+1}), [d(IIz, AIz) + d(Jx_{2n+1} Bx_{2n+1}) / 2], [d(IIz, Bx_{2n+1}) + d(Jx_{2n+1}, AIz) /2]\}.$

Letting $n \to\infty$

$d(Iz, z) \leq \{d(Iz, z), [d(Iz, Iz) + d(z, z) / 2], [d(Iz, z) + d(z, Jz) /2]\}.$

$\leq \{d(Iz, z), [d(Iz, z) / 2], d(z, Iz) /2]\}.$

$\leq d(Iz, z)$, which is a contradiction.

Therefore, Iz = z.

To prove Jz = z.

put x = z, and y = Jz in (ii) we get

$d(Az, BJz) \leq \{d(Iz, JJz), [d(Iz, Az) + d(JJz, BJz) / 2], [d(Iz, BJz) + d(JJz, Az) /2]\}.$

$d(z, Jz) \leq\{d(z, Jz), [d(z, z) + d(Jz, Jz) / 2], [d(z,J z) + d(Jz, z) /2]\}.$

$\leq \{d(z, Jz), d(z, Jz) / 2], d(z, Jz) /2]\}.$

$< d(z, Jz)$, which is a contradiction.

Therefore, Jz = z.

Therefore, Jz = Iz = z.

Hence, Bz = Az = = Jz = Iz = z.

Therefore, z is a common fixed point of A, B, I and J.

It is easily prove that A, B, I and J have a unique common fixed point in X.

Remark: Our theorem generalized and improved the results of [2], and which is the more general the results of [2].

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