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NEUTROSOPHIC BETA OMEGA OPEN SETS IN NEUTROSOPHIC TOPOLOGICAL SPACES

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ABSTRACT

Exploring a new type of neutrosophic set in neutrosophic topological spaces is the major aim of our research. In this paper, the concept "Neutrosophic Beta Omega Open Sets" is newly defined and their properties and some interesting theorems are discussed. We have analyzed the relationships between this newly introduced set and the already existing neutrosophic sets.

Keywords: *neutrosophic beta omega open set, neutrosophic beta omega interior.*

AMS Mathematics Subject Classification: *18B30, 03E72.*

1. INTRODUCTION

Fuzzy set theory has played a vital role in the research of mathematics. The research on fuzzy set theory has been witnessing an exponential growth in mathematics. Zadeh [13] introduced the fuzzy set as an extension of a classical notion of crisp set in 1965. K. Atanassov, established the intuitionistic fuzzy set as a extension of fuzzy set. Then Florentin Smarandache [5] extended the concept intuitionistic fuzzy sets as Neutrosophic sets in 1999. Later A. Salama and S. A. Alblowi [9] studied the concept of neutrosophic topological spaces.

2. PRELIMINARIES

Definition 2.1: [4] Let X be a non-empty fixed set. A neutrosophic set (NS) G is an object having the form $G=\{< x, \mu_G(x), \sigma_G(x), \upsilon_G(x) > : x \in X\}$ where $\mu_G(x)$, $\sigma_G(x)$ and $\upsilon_G(x)$ represent the degree of membership, degree of indeterminacy and the degree of nonmembership respectively of each element $x \in X$ to the set G. A neutrosophic set $G = \{< x, \mu_G(x), \sigma_G(x), \upsilon_G(x) >: x \in X\}$ can be identified as an ordered triple $< \mu_G, \sigma_G, \upsilon_G >$ in $]^-0, 1^+[$ on X.

Definition 2.2: [1] For any two sets G and H,

1. $G \subseteq H \Leftrightarrow \mu_G(x) \le \mu_H(x), \sigma_G(x) \le \sigma_H(x)$ and $\upsilon_G(x) \ge \upsilon_H(x), x \in X$
2. $G \cap H = < x, \mu_G(x) \wedge \mu_H(x), \sigma_G(x) \wedge \sigma_H(x), \upsilon_G(x) \vee \upsilon_H(x) >$
3. $G \cup H = < x, \mu_G(x) \vee \mu_H(x), \sigma_G(x) \vee \sigma_H(x), \upsilon_G(x) \wedge \upsilon_H(x) >$
4. $G^C = \{< x, \upsilon_G(x), 1- \sigma_G(x), \mu_G(x) >: x \in X\}$
5. $0_N = \{< x, 0, 0, 1 >: x \in X \}$
6. $1_N = \{< x, 1, 1, 0 >: x \in X\}$.

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Definition 2.3: [9] A neutrosophic topology (NT) on a non-empty set X is a family τ of neutrosophic subsets in X satisfies the following axioms:

1. $0_N, 1_N \subseteq \tau$
2. $G_1 \cap G_2 \subseteq \tau$ for any $G_1, G_2 \subseteq \tau$
3. $\cup G_i \subseteq \tau$ where $\{G_i : i \subseteq J\} \subseteq \tau$

Here the pair (X, τ) is a neutrosophic topological space (NTS) and any neutrosophic set in τ is known as a neutrosophic open set (N-open set) in X. A neutrosophic setG is a neutrosophic closed set (N-closed set) if and only if its complement G^C is a neutrosophic open set in X.

Definition 2.4: [12] A subset G of a neutrosophic topological space (X, τ_N) is called,

(1). a **neutrosophic semi open** set (NSO set) if $G \subseteq Cl_N(Int_N(G))$ and a **neutrosophic semi closed** set(NSC set) if $Int_N(Cl_N(G)) \subseteq G$.
(2). a **neutrosophic pre open** set (NPO set) if $G \subseteq Int_N(Cl_N(G))$ and a **neutrosophic pre closed** set(NPC set) if $Cl_N(Int_N(G)) \subseteq G$.
(3). a **neutrosophic α open** set (NαO set) if $G \subseteq Int_N(Cl_N(Int_N(G)))$ and an **neutrosophic α closed**(NαC set) set if $Cl_N(Int_N(Cl_N(G))) \subseteq G$.
(4). a **neutrosophic semi pre open** set (NSPO set) if $G \subseteq Cl_N(Int_N(Cl_N(G))))$ and a **neutrosophic semi pre closed** set (NSPC set) if $Int_N(Cl_N(Int_N(G))) \subseteq G$.
(5). a **neutrosophic regular open** (NRO) set if $G = Int_N(Cl_N(G))$ and a **neutrosophic regular closed** (NRC) set if $G = Cl_N(Int_N(G))$.

Definition 2.5: A subset G_N of a neutrosophic topological space (X, τ_N) is called

(1). a **neutrosophic generalized closed** set (NG-closed set) [4] if $cl_N(G_N) \subseteq U_N$ whenever $G_N \subseteq U_N$ and U_N is N-open in (X, τ_N).
(2). a **neutrosophic generalized semi closed** set (briefly NGS-closed**)** [11] if $Scl_N(G_N) \subseteq U_N$ whenever $G_N \subseteq U_N$ and U_N is N- open in (X, τ_N).
(3). a **neutrosophic ω closed** set (Nω-closed set**)** [9] if $cl_N(G_N) \subseteq U_N$ whenever $G_N \subseteq U_N$ and U_N is NS- open in (X, τ_N).
(4). a **neutrosophic α generalized closed** set (briefly NαG-closed**)** [7] if $\alpha cl_N(G_N) \subseteq U_N$ whenever $G_N \subseteq U_N$ and G_N is N- open in (X, τ_N).
(5). a **neutrosophic generalized regular closed** set (briefly NGR-closed**)** [2] if $Rcl_N(G_N) \subseteq U_N$ whenever $G_N \subseteq U_N$ and U_N is N- open in (X, τ_N).

Definition 2.6: [8] A neutrosophic set G of a neutrosophic topological space (X, τ_N) is called **neutrosophic beta omega closed** (**N$\beta\omega$-closed**) if $\beta cl_N(G) \subseteq U$ whenever $G \subseteq U$ and U is Nω- open in (X, τ_N).

3. NEUTROSOPHIC BETA OMEGA OPEN SET

Definition 3.1: A neutrosophic set G of a neutrosophic topological space (X, τ_N) is called neutrosophic beta omega open (N$\beta\omega$-open) if the complement of G is N$\beta\omega$-closed set.

Example 3.1: Let X = {a, b, c}, $\tau_N = \{0_N, G, 1_N\}$ where G = $< x, \left(\frac{a}{0.2}, \frac{b}{0.2}, \frac{c}{0.2}\right), \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3}\right), \left(\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.7}\right) >$. Then τ_N is a NT and consider W= $x, \left(\frac{a}{0.7}, \frac{b}{0.9}, \frac{c}{0.8}\right), \left(\frac{a}{0.8}, \frac{b}{0.9}, \frac{c}{0.8}\right), \left(\frac{a}{0.2}, \frac{b}{0.2}, \frac{c}{0.1}\right) >$. Whenever $W \subseteq U$ and U is Nω-open, we get $\beta cl_N(W) \subseteq U$. Then W^C is N$\beta\omega$-closed. Hence W is N$\beta\omega$-open.

Theorem 3.1: Every N-open set in (X, τ_N) is N$\beta\omega$-open in (X, τ_N).

Proof: Let G be N-open in (X, τ_N). Then G^C is N-closed set. Let U be any Nω-open such that $G^C \subseteq U$. Since G^C is N-closed, we get $cl_N(G^C) = G^C$. Therefore, $G^C \subseteq U$ implies $\beta cl_N(G^C) \subseteq cl_N(G^C) \subseteq U$. Therefore $\beta cl_N(G^C) \subseteq U$. Therefore G^C is N$\beta\omega$-closed set. Hence G is N$\beta\omega$-open set.

Remark 3.1: The converse of the above theorem need not be true.

Example 3.2: Let X = {a, b, c}, $\tau_N = \{0_N, G, H, 1_N\}$ where G = $<x, \left(\frac{a}{0.4}, \frac{b}{0.3}, \frac{c}{0.4}\right), \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.2}\right), \left(\frac{a}{0.7}, \frac{b}{0.6}, \frac{c}{0.6}\right)$, H $=< x, \left(\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.3}\right), \left(\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.5}\right) >$. Then τ_N is a NT and consider W = $< x, \left(\frac{a}{0.8}, \frac{b}{0.8}, \frac{c}{0.7}\right), \left(\frac{a}{0.8}, \frac{b}{0.7}, \frac{c}{0.8}\right), \left(\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.2}\right) >$. Then W is N$\beta\omega$-open. But W is not N-open.

Theorem 3.2: Every Nβ-open set in (X, τ_N) is Nβω-open in (X, τ_N).

Proof: Let G be Nβ-open in (X, τ_N). Then G^C is Nβ-closed in (X, τ_N). Let U be Nω-open such that $G^C \subseteq U$. Since G^C is Nβ-closed, we have $\beta cl_N(G^C) = G^C \subseteq U$. Therefore G^C is Nβω-closed set. Hence G is Nβω-open set.

Remark 3.2: The converse of the above theorem need not be true.

Example 3.3: Let X = {a, b, c}, $\tau_N = \{0_N, G, 1_N\}$ where G = $< x, \left(\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.1}\right), \left(\frac{a}{0.7}, \frac{b}{0.8}, \frac{c}{0.8}\right), \left(\frac{a}{0.6}, \frac{b}{0.8}, \frac{c}{0.7}\right) >$. Then τ_N is a NT and consider W $= < x, \left(\frac{a}{0.2}, \frac{b}{0.2}, \frac{c}{0.1}\right), \left(\frac{a}{0.2}, \frac{b}{0.1}, \frac{c}{0.2}\right), \left(\frac{a}{0.7}, \frac{b}{0.8}, \frac{c}{0.8}\right) >$. Then W is Nβω-open. But W is not Nβ-open.

Theorem 3.3: Every NG*-open set in (X, τ_N) is Nβω-open in (X, τ_N).

Proof: Let G be NG*-open in (X, τ_N). Then G^C is a NG*-closed set in (X, τ_N). Let U be Nω-open such that $G^C \subseteq U$. Since U is Nω-open, U is NG-open. Therefore, we have $\beta cl_N(G^C) \subseteq cl_N(G^C) \subseteq G^C$. Therefore G^C is Nβω-closed set. Hence G is Nβω-open set.

Remark 3.3: The converse of the above theorem need not be true.

Example 3.4: Let X = {a, b, c}, $\tau_N = \{0_N, G, H, I, 1_N\}$ where G = $< x, \left(\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.3}, \frac{c}{0.4}\right), \left(\frac{a}{0.7}, \frac{b}{0.8}, \frac{c}{0.7}\right) >$, H $= < x, \left(\frac{a}{0.43}, \frac{b}{0.34}, \frac{c}{0.44}\right), \left(\frac{a}{0.45}, \frac{b}{0.43}, \frac{c}{0.42}\right), \left(\frac{a}{0.61}, \frac{b}{0.73}, \frac{c}{0.71}\right)$, I $= < x, \left(\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.4}\right), \left(\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.4}\right), \left(\frac{a}{0.8}, \frac{b}{0.85}, \frac{c}{0.8}\right) >$. Then τ_N is a NT and consider W$= < x, \left(\frac{a}{0.8}, \frac{b}{0.88}, \frac{c}{0.87}\right), \left(\frac{a}{0.21}, \frac{b}{0.13}, \frac{c}{0.37}\right) >$. Then W is Nβω-open. But W is not NG*-open.

Theorem 3.4: Every Nψ-open set in (X, τ_N) is Nβω-open in (X, τ_N).

Proof: Let G be Nψ-open in (X, τ_N). Then G^C is N ψ-closed in (X, τ_N). Let U be any Nω-open set such that $G^C \subseteq U$. Since every Nω-open is NSG-open, U is NSG-open, we have $Scl_N(G^C) \subseteq U$. But $\beta cl_N(G^C) \subseteq Scl_N(G^C)$. Then we get $\beta cl_N(G^C) \subseteq U$. Therefore G^C is Nβω-closed set. Hence G is Nβω-open set.

Remark 3.4: The converse of the above theorem need not be true.

Example 3.5: Let X = {a, b, c}, $\tau_N = \{0_N, G, H, 1_N\}$ where G = $< x, \left(\frac{a}{0.27}, \frac{b}{0.38}, \frac{c}{0.41}\right), \left(\frac{a}{0.41}, \frac{b}{0.38}, \frac{c}{0.27}\right), \left(\frac{a}{0.63}, \frac{b}{0.65}, \frac{c}{0.66}\right)$, H $= < x, \left(\frac{a}{0.33}, \frac{b}{0.49}, \frac{c}{0.49}\right), \left(\frac{a}{0.45}, \frac{b}{0.49}, \frac{c}{0.4}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}\right) >$. Then τ_N is a NT and consider
W $= < x, \left(\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.7}\right), \left(\frac{a}{0.7}, \frac{b}{0.8}, \frac{c}{0.9}\right), \left(\frac{a}{0.1}, \frac{b}{0.2}, \frac{c}{0.3}\right) >$. Then W is Nβω-open. But W is not Nψ-open.

Theorem 3.5: Every NWG*-open set in (X, τ_N) is Nβω-open in (X, τ_N).

Proof: Let G be NWG*-open in (X, τ_N). Then G^C is NWG*-closed in (X, τ_N). Let U be any Nω-open set such that $G \subseteq U$. Since every Nω-open is NG-open, U is NG-open, we have $cl_N(int_N(G)) \subseteq U$. But $int_N(cl_N(int_N(G))) \subseteq cl_N(int_N(G))$ which implies $\beta cl_N(G) \subseteq U$. Therefore G^C is Nβω-closed set. Hence G is Nβω-open set.

Remark 3.5: The converse of the above theorem need not be true.

Example 3.6: Let X = {a, b, c}, $\tau_N = \{0_N, G, H, 1_N\}$ where G = $<x, \left(\frac{a}{0.22}, \frac{b}{0.33}, \frac{c}{0.22}\right), \left(\frac{a}{0.33}, \frac{b}{0.22}, \frac{c}{0.27}\right), \left(\frac{a}{0.77}, \frac{b}{0.88}, \frac{c}{0.77}\right) >$, H $= < x, \left(\frac{a}{0.33}, \frac{b}{0.44}, \frac{c}{0.33}\right), \left(\frac{a}{0.44}, \frac{b}{0.33}, \frac{c}{0.33}\right), \left(\frac{a}{0.66}, \frac{b}{0.77}, \frac{c}{0.66}\right) >$. Then τ_N is a NT and consider W $= < x, \left(\frac{a}{0.4}, \frac{b}{0.6}, \frac{c}{0.6}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}\right), \left(\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.5}\right) >$. Then W is Nβω-open, but not NWG*-open.

Theorem 3.6: Every NP-open set in (X, τ_N) is Nβω-open in (X, τ_N).

Proof: Let G be NP-open in (X, τ_N). Then G^C is NP-closed in (X, τ_N). Let U be any Nω-open such that $G \subseteq U$. We have $Pcl_N(G) = G$. But $\beta cl_N(G) \subseteq Pcl_N(G)$. Then we get $\beta cl_N(G) \subseteq U$. Therefore G^C is Nβω-closed set. Hence G is Nβω-open set.

Remark 3.6: The converse of the above theorem need not be true.

Example 3.7: Let X = {a, b, c}, $\tau_N = \{0_N, G, 1_N\}$ where G = $< x, \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.1}\right), \left(\frac{a}{0.4}, \frac{b}{0.2}, \frac{c}{0.2}\right), \left(\frac{a}{0.6}, \frac{b}{0.8}, \frac{c}{0.7}\right) >$. Then τ_N is a NT and consider W = $< x, \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}\right) >$. Then W is Nβω-open but not NP-open.

Remark 3.7: The following examples show that Nβω-open set and NGS-open are independent in (X, τ_N).

Example 3.8: Let X = {a, b, c}, $\tau_N = \{0_N, G, H_N, I, 1_N\}$ where G = $< x, \left(\frac{a}{0.23}, \frac{b}{0.34}, \frac{c}{0.43}\right), \left(\frac{a}{0.28}, \frac{b}{0.38}, \frac{c}{0.43}\right), \left(\frac{a}{0.77}, \frac{b}{0.85}, \frac{c}{0.67}\right) >$ H = $< x, \left(\frac{a}{0.33}, \frac{b}{0.44}, \frac{c}{0.44}\right), \left(\frac{a}{0.33}, \frac{b}{0.48}, \frac{c}{0.44}\right), \left(\frac{a}{0.69}, \frac{b}{0.79}, \frac{c}{0.57}\right) >$, I = x, $\left(\frac{a}{0.22}, \frac{b}{0.24}, \frac{c}{0.4}\right), \left(\frac{a}{0.2}, \frac{b}{0.3}, \frac{c}{0.4}\right), \left(\frac{a}{0.8}, \frac{b}{0.87}, \frac{c}{0.69}\right) >$. Then τ_N is a NT and consider W = $< x, \left(\frac{a}{0.8}, \frac{b}{0.9}, \frac{c}{0.7}\right), \left(\frac{a}{0.82}, \frac{b}{0.72}, \frac{c}{0.68}\right), \left(\frac{a}{0.12}, \frac{b}{0.23}, \frac{c}{0.34}\right) >$. Then W is Nβω-open. But W is not NGS-open.

Example 3.9: Let X = {a, b, c}, $\tau_N = \{0_N, G, 1_N\}$ where G = $< x, \left(\frac{a}{0.7}, \frac{b}{0.8}, \frac{c}{0.7}\right), \left(\frac{a}{0.67}, \frac{b}{0.67}, \frac{c}{0.67}\right), \left(\frac{a}{0.38}, \frac{b}{0.23}, \frac{c}{0.33}\right) >$. Then τ_N is a NT and consider W = $< x, \left(\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.3}\right), \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3}\right), \left(\frac{a}{0.8}, \frac{b}{0.8}, \frac{c}{0.8}\right) >$. Then W is NGS-open. But W is not Nβω-open.

Remark 3.8: The following examples show that Nβω-open set and NαG-open are independent in (X, τ_N).

Example 3.10: Let X = {a, b, c}, $\tau_N = \{0_N, G, H, 1_N\}$ where G = $< x, \left(\frac{a}{0.42}, \frac{b}{0.41}, \frac{c}{0.32}\right), \left(\frac{a}{0.43}, \frac{b}{0.22}, \frac{c}{0.41}\right), \left(\frac{a}{0.61}, \frac{b}{0.71}, \frac{c}{0.53}\right) >$, H = $< x, \left(\frac{a}{0.48}, \frac{b}{0.43}, \frac{c}{0.42}\right), \left(\frac{a}{0.45}, \frac{b}{0.33}, \frac{c}{0.44}\right), \left(\frac{a}{0.52}, \frac{b}{0.7}, \frac{c}{0.5}\right) >$. Then τ_N is a NT and consider W= $< x, \left(\frac{a}{0.7}, \frac{b}{0.8}, \frac{c}{0.9}\right), \left(\frac{a}{0.7}, \frac{b}{0.9}, \frac{c}{0.8}\right), \left(\frac{a}{0.2}, \frac{b}{0.3}, \frac{c}{0.1}\right) >$. Then W is Nβω-open. But not NαG-open.

Example 3.11: Let X = {a, b, c}, $\tau_N = \{0_N, G, 1_N\}$ where G = $< x, \left(\frac{a}{0.66}, \frac{b}{0.66}, \frac{c}{0.76}\right), \left(\frac{a}{0.43}, \frac{b}{0.22}, \frac{c}{0.54}\right), \left(\frac{a}{0.56}, \frac{b}{0.46}, \frac{c}{0.56}\right) >$. Then τ_N is a NT and consider W = $< x, \left(\frac{a}{0.76}, \frac{b}{0.76}, \frac{c}{0.76}\right), \left(\frac{a}{0.74}, \frac{b}{0.64}, \frac{c}{0.66}\right), \left(\frac{a}{0.36}, \frac{b}{0.26}, \frac{c}{0.46}\right) >$. Then *W* is NαG-open but *W* is not Nβω-open.

Remark 3.9: The following examples show that Nβω-open set and NG-open are independent in (X, τ_N).

Example 3.12: Let X = {a, b, c}, $\tau_N = \{0_N, , 1_N\}$ where $G = < x, \left(\frac{a}{0.7}, \frac{b}{0.8}, \frac{c}{0.7}\right), \left(\frac{a}{0.6}, \frac{b}{0.7}, \frac{c}{0.6}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.3}\right) >$. Then τ_N is a NT and consider $W = < x, \left(\frac{a}{0.8}, \frac{b}{0.9}, \frac{c}{0.6}\right), \left(\frac{a}{0.7}, \frac{b}{0.8}, \frac{c}{0.7}\right), \left(\frac{a}{0.2}, \frac{b}{0.3}, \frac{c}{0.2}\right) >$. Then *W* is Nβω-open but *W* is not NG-open.

Example 3.13: Let X = {a, b, c}, $\tau_N = \{0_N, , 1_N\}$ where G = $< x, \left(\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}\right), \left(\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.4}\right) >$. Then τ_N is a NT and consider $W = < x, \left(\frac{a}{0.2}, \frac{b}{0.2}, \frac{c}{0.2}\right), \left(\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.4}\right), \left(\frac{a}{0.8}, \frac{b}{0.8}, \frac{c}{0.8}\right) >$. Then *W* is NG-open but *W* is not Nβω-open.

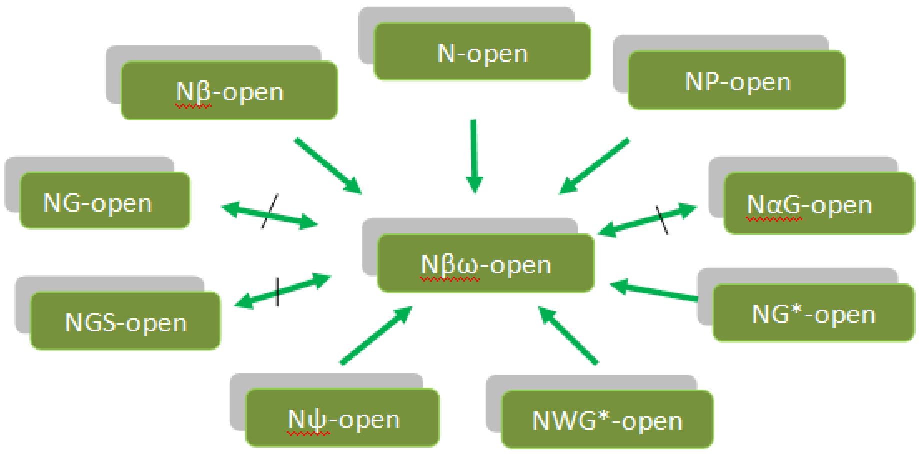

Figure-1: Implications of Nβω-open Set

where $A \longrightarrow B$ (resp. $A \nleftrightarrow B$) represents A implies B (resp. A and B are independent).

4. BETA OMEGA INTERIOR

Definition 4.1: For any $G \in (X, \tau)$, $\beta\omega int_N(G)$ is defined as the union of all Nβω-open sets contained in G. That is, $\beta\omega int_N(G) = \cup \{H : H \subseteq G \text{ and } H \in N\beta\omega O(X, \tau)\}$

Theorem 4.1: Let G be any subset of (X, τ). Then
1. $(\beta\omega int_N(G))^C = \beta\omega cl_N(G^C)$
2. $\beta\omega int_N(G) = (\beta\omega cl_N(G^C))^C$
3. $\beta\omega cl_N(G) = (\beta\omega int_N(G^C))^C$

Lemma 4.1: For any set $G \in (X, \tau)$, $int_N(G) \subseteq \beta\omega int_N(G)$.

Proof: The proof follows from that every N-open set is Nβω-open set.

Theorem 4.1: For any two subsets G and H of (X, τ), the following statements are true:
1. $\beta\omega int_N(1_N) = 1_N$ and $\beta\omega int_N(0_N) = 0_N$
2. $\beta\omega int_N(G) \subseteq G$
3. If H is any Nβω-open set contained in G, then $H \subseteq \beta\omega int_N(G)$
4. If $G \subseteq H$, then $\beta\omega int_N(G) \subseteq \beta\omega int_N(H)$
5. If G and H are subsets of (X, τ), then $\beta\omega int_N(G) \cup \beta\omega int_N(H)=\beta\omega int_N(G\cup H)$.
6. If G and H are subsets of (X, τ), then $\beta\omega int_N(G\cap H) \subseteq \beta\omega int_N(G) \cap \beta\omega int_N(H)$.

Proof:
1. Since 1_N and 0_N are Nβω-open sets, $\beta\omega int_N(1_N) = \cup \{G : G \subseteq 1_N \text{ and } G \in N\beta\omega O(X, \tau)\} = 1_N$. Similarly, since 0_N is the only Nβω-open set contained in 0_N, $N\beta\omega int(0_N) = 0_N$.
2. By the definition of Nβω-interior of G, it is obvious that $\beta\omega int_N(G) \subseteq G$
3. Let H be any Nβω-open set contained in G. Since $\beta\omega int_N(G)$ is the union of all Nβω-open sets contained in G, $\beta\omega int_N(G)$ is containing every Nβω-open set containing G. Hence $H \subseteq \beta\omega int_N(G)$
4. Follows from the definition 4.1.
5. Since $G \subseteq G \cup H$ and $H \subseteq G \cup H$, we get $\beta\omega int(G) \subseteq \beta\omega int_N(G \cup H)$ and $\beta\omega int_N(H) \subseteq \beta\omega int_N(G \cup H)$ which implies that $\beta\omega int_N(G) \cup \beta\omega int_N(H) \subseteq \beta\omega int_N(G \cup H)$. Also $\beta\omega int_N(G \cup H) = \cup\{V : V \subseteq G \cup H, V \in N\beta\omega O(X, \tau)\} \subseteq \cup \{V : V \subseteq G, V \in N\beta\omega O(X, \tau)\}\cup\{V : V \subseteq H, V \in N\beta\omega O(X, \tau)\}= \beta\omega int_N(G) \cup \beta\omega int_N(H)$. Hence $\beta\omega int_N(G\cup H) = \beta\omega int_N(G) \cup \beta\omega int_N(H)$.
6. Since $G\cap H\subseteq G$ and $G\cap H \subseteq H$, by theorem 4.1(4), $\beta\omega int_N(G\cap H) \subseteq \beta\omega int_N(G)$ and $\beta\omega int_N(G\cap H) \subseteq \beta\omega int_N(H)$. Hence $\beta\omega int_N(G\cap H) \subseteq \beta\omega int_N(G) \cap \beta\omega int_N(H)$.

Remark 4.3: The following example shows that the reverse inclusion of theorem 4.1(6) is not true.

Example 4.1: Let X = {a, b, c}, $\tau_N = \{0_N, G, 1_N\}$ where $G = < x, \left(\frac{a}{0.8}, \frac{b}{0.8}, \frac{c}{0.7}\right), \left(\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}\right), \left(\frac{a}{0.4}, \frac{b}{0.6}, \frac{c}{0.5}\right) >$. Then τ_N is a NT and consider $W_N = < x, \left(\frac{a}{0.8}, \frac{b}{0.8}, \frac{c}{0.9}\right), \left(\frac{a}{0.8}, \frac{b}{0.7}, \frac{c}{0.9}\right), \left(\frac{a}{0.9}, \frac{b}{0.8}, \frac{c}{0.9}\right) >$ and $V_N = < x, \left(\frac{a}{0.2}, \frac{b}{0.2}, \frac{c}{0.2}\right), \left(\frac{a}{0.2}, \frac{b}{0.2}, \frac{c}{0.2}\right), \left(\frac{a}{0.2}, \frac{b}{0.2}, \frac{c}{0.2}\right) >$. Here, $\beta\omega int_N(W) \cap \beta\omega int_N(V) = W \cap V$ and $\beta\omega int_N(W \cap V) = 0_N$. Hence $\beta\omega int_N(W) \cap \beta\omega int_N(V) \nsubseteq \beta\omega int_N(W \cap V)$.

Proposition 4.1: Let G be any subset of (X, τ). If G is Nβω-open in (X, τ) then $\beta\omega int_N(G) = G$.

Proof: Let G be Nβω-open in (X, τ). We know that $\beta\omega int_N(G) \subseteq G$. Also G is a Nβω-open set contained in G. From theorem 4.1., $G \subseteq \beta\omega int_N(G)$. Hence $\beta\omega int_N(G) = G$.

Corollary 4.1: $\beta\omega int_N(\beta\omega int_N(G)) = \beta\omega int_N(G)$

Proof: By (2) and (4) of theorem 4.1., $\beta\omega int_N(\beta\omega int_N(G)) \subseteq \beta\omega int_N(G)$.

5. PROPERTIES OF NEUTROSOPHIC Nβω-OPEN SET

Theorem 5.1: G is any Nβω-open iff $H \subseteq \beta int_N(G)$ where H is Nω-closed and $H \subseteq G$.

Proof: Let G be any Nβω-open set and H be Nω-closed such that $H \subseteq G$. Then $G^C \subseteq H^C$ which implies $\beta cl_N(G^C) \subseteq H^C$, since G^C is Nβω-closed and H^C is Nω-open. Therefore, we have $H \subseteq \beta int_N(G)$. Conversely, assume that $H \subseteq \beta int_N(G)$ whenever H is Nω-closed and $H \subseteq G$. Let I be any Nω-open such that $I^C \subseteq G$. Then I^C is Nω-closed. Therefore by assumption, $I^C \subseteq \beta int_N(G)$ which implies $\beta cl_N(G^C) \subseteq I_N$. Hence G is Nβω–open.

Theorem 5.2: If $\beta int_N(G) \subseteq H \subseteq G$ and G is Nβω-open, then H is Nβω-open.

Proof: $\beta int_N(G) \subseteq H \subseteq G$ implies $G^C \subseteq H^C \subseteq \beta cl_N(G)^C$. Since G is Nβω-open, G^C is Nβω-closed. Therefore, H is Nβω-closed. Hence H is Nβω-open.

Theorem 5.3: The union of the Nβω-open sets is Nβω-open.

Proof: Let G and H_N be Nβω-open sets in (X, τ_N). By theorem 4.1., $\beta\omega int_N(G \cup H) = \beta\omega int_N(G) \cup \beta\omega int_N(H) = G \cup H$. However, $\beta\omega int_N(G \cup H) \subseteq G \cup H$. Therefore $G \cup H = \beta\omega cl_N(G \cup H)$. Hence G∪H is Nβω-open set.

Remark 5.1: The intersection of two Nβω-open sets need not be Nβω-open.

Example 5.1: Let X = {a, b, c}, $\tau_N = \{0_N, G, 1_N\}$ where G = $< x, \left(\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.4}\right), \left(\frac{a}{0.7}, \frac{b}{0.6}, \frac{c}{0.6}\right), \left(\frac{a}{0.4}, \frac{b}{0.6}, \frac{c}{0.5}\right) >$. Then τ_N is a NT and consider W = $< x, \left(\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.1}\right), \left(\frac{a}{0.2}, \frac{b}{0.2}, \frac{c}{0.2}\right), \left(\frac{a}{0.2}, \frac{b}{0.2}, \frac{c}{0.2}\right) >$ and V = $< x, \left(\frac{a}{0.9}, \frac{b}{0.8}, \frac{c}{0.7}\right), \left(\frac{a}{0.8}, \frac{b}{0.7}, \frac{c}{0.9}\right), \left(\frac{a}{0.8}, \frac{b}{0.8}, \frac{c}{0.8}\right) >$. Then W and V are Nβω-open. But W∩V is not Nβω-open.

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