International Journal of Mathematical Archive-13(5), 2022, 1-7
Available online through www.ijma.info ISSN 2229 – 5046

ARITHMETIC-COTRAHARMONIC AND CONTRAHARMONIC-ARITHMETIC INDICES

V. R. KULLI*

Department of Mathematics, Gulbarga University, Gulbarga - 585106, India.

(Received On: 11-04-21; Revised & Accepted On: 30-04-22)

ABSTRACT

Topological indices are applied to measure the chemical characteristics of chemical compounds. In this paper, we introduce the arithmetic-contraharmonic (AC) and contraharmonic-arithmetic (CA) indices of a graph and compute the exact values for some standard graphs and some families of nanotubes.

Keywords: *arithmetic-contraharmonic index, contraharmonic-arithmetic index, graph, nanotube.*

Mathematics Subject Classification: *05C05, 05C12, 05C35.*

1. INTRODUCTION

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. We refer [1], for other undefined notations and terminologies.

A molecular graph is a graph such that its vertices correspond to the atoms and edges to the bonds. Chemical Graph Theory is a branch of mathematical chemistry, which has an important effect on the development of Chemical Sciences. Several topological indices have been considered in Theoretical Chemistry and have found some applications.

The geometric-arithmetic index [2] of a graph G was defined as

$$GA(G)=\sum_{uv\in E(G)}\frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(u)}.$$

This index was studied, for example, in [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16].

Motivated by the definition of geometric-arithmetic index of a graph G, we define the arithmetic-contraharmonic index as

$$AC(G)=\sum_{uv\in E(G)}\frac{(d_G(u)+d_G(v))/2}{(d_G(u)^2+d_G(v)^2)/(d_G(u)+d_G(v))}$$
$$=\sum_{uv\in E(G)}\frac{(d_G(u)+d_G(v))^2}{2(d_G(u)^2+d_G(v)^2)}.$$

This equation consists from arithmetic mean of end vertex degrees of an edge uv, $\left(d_G(u)+d_G(v)/2\right)$ as numerator and contraharmonic mean of end vertex degrees of the edge uv, $(d_G(u)^2+d_G(v)^2)/(d_G(u)+d_G(v))$ as denominator.

Also we introduce the contraharmonic-arithmetic index of a graph G and defined it as

$$CA(G)=\sum_{uv\in E(G)}\frac{(d_G(u)^2+d_G(v)^2)/(d_G(u)+d_G(v))}{((d_G(u)+d_G(v))/2}$$
$$=\sum_{uv\in E(G)}\frac{2(d_G(u)^2+d_G(v)^2)}{(d_G(u)+d_G(v))^2}.$$

Corresponding Author: V. R. Kulli*
Department of Mathematics, Gulbarga University, Gulbarga - 585106, India.

Recently, some new indices were studied, for example, in [17, 18, 19, 20, 21, 22, 23, 24, 25].

In this paper, we compute these two newly defined novel graph indices for some standard graphs and certain families of nanotubes.. For nanotubes, see [26].

2. RESULTS FOR SOME STENDARD GRAPHS

Proposition 1: *Let* $K_{r,s}$ *be a complete bipartite graph with* $1 \le r \le s$ *and* $s \ge 2$ *vertices. Then*

$$AC\left(K_{r,s}\right) = \frac{rs(r+s)^2}{2(r^2+s^2)}.$$

Proof: Let $K_{r,s}$ be a complete bipartite graph with $r + s$ vertices and rs *edges* such that $|V_1|= r$, $|V_2| = s$, $V(K_{r,s}) = V_1 \cup V_2$ for $1 \le r \le s$, and $s \ge 2$. Every vertex of V_1 is incident with s edges and every vertex of V_2 is incident with r edges.

$$AC\left(K_{r,s}\right) = \frac{rs(r+s)^2}{2(r^2+s^2)}.$$

Corollary 1.1: ***Let*** $K_{r,r}$ *be a complete bipartite graph with* $r \ge 2$. *Then*

$$AC\left(K_{r,r}\right) = r^2.$$

Corollary 1.2: *Let* $K_{1,r-1}$ *be a star with* $r \ge 2$. *Then*

$$AC\left(K_{1,r-1}\right) = \frac{(r-1)r^2}{2(r^2-2r+2)}.$$

Proposition 2: *If G is r-regular with n vertices and* $r \ge 2$, *then*

$$AC(G) = \frac{nr}{2}.$$

Proof: Let G is r-regular with n vertices and $r \ge 2$ and $\frac{nr}{2}$ edges. Then

$$AC(G) = \frac{nr}{2}\frac{(r+r)^2}{2(r^2+r^2)} = \frac{nr}{2}.$$

Corollary 2.1: *Let* C_n *be a cycle with* $n \ge 3$ *vertices. Then*

$$AC\left(C_n\right) = n.$$

Corollary 2.2: *Let* K_n *be a complete graph with* $n \ge 3$ *vertices. Then*

$$AC\left(K_n\right) = \frac{n(n-1)}{2}.$$

Proposition 3: *If G is a path with* $n \ge 3$ *vertices, then*

$$AC\left(P_n\right) = n - 3 + \frac{9}{5}.$$

Proposition 4: *Let* $K_{r,s}$ *be a complete bipartite graph with* $1 \le r \le s$ *and* $s \ge 2$ *vertices. Then*

$$CA\left(K_{r,s}\right) = \frac{rs2(r^2+s^2)}{(r+s)^2}.$$

Proof: Let $K_{r,s}$ be a complete bipartite graph with $r + s$ vertices and rs *edges* such that $|V_1|= r$, $|V_2|= s$, $V(K_{r,s}) = V_1 \cup V_2$ for $1 \le r \le s$, and $s \ge 2$. Every vertex of V_1 is incident with s edges and every vertex of V_2 is incident with r edges.

$$CA\left(K_{r,s}\right) = \frac{rs2(r^2+s^2)}{(r+s)^2}.$$

Corollary 4.1: ***Let*** $K_{r,r}$ *be a complete bipartite graph with* $r \ge 2$. *Then*

$$CA\left(K_{r,r}\right) = r^2.$$

Corollary 4.2: *Let $K_{1,r-1}$ be a star with $r \geq 2$. Then*

$$CA\left(K_{1,r-1}\right)=\frac{2(\mathrm{r}-1)(r^2-2r+2)}{r^2}.$$

Proposition 5: *If G is r-regular with n vertices and $r \geq 2$, then*

$$CA(G)=\frac{nr}{2}.$$

Proof: Let G is r-regular with n vertices and $r \geq 2$ and $\frac{nr}{2}$ edges. Then

$$CA(G)=\frac{nr}{2}\frac{2(r^2+r^2)}{(r+r)^2}=\frac{nr}{2}.$$

Corollary 5.1: *Let C_n be a cycle with $n \geq 3$ vertices. Then*

$$CA\left(C_n\right)=n.$$

Corollary 5.2: *Let K_n be a complete graph with $n \geq 3$ vertices. Then*

$$CA\left(K_n\right)=\frac{n\left(n-1\right)}{2}.$$

Proposition 6: If G is a path with $n \geq 3$ vertices, then

$$CA\left(P_n\right)=n-3+\frac{20}{9}.$$

3. RESULTS FOR $HC_5C_7[p,q]$ NANOTUBES

In this section, we focus on the family of nanotubes, denoted by $HC_5C_7[p,q]$, in which p is the number of heptagons in the first row and q rows of pentagons repeated alternately. Let G be the graph of a nanotube $HC_5C_7[p,q]$.

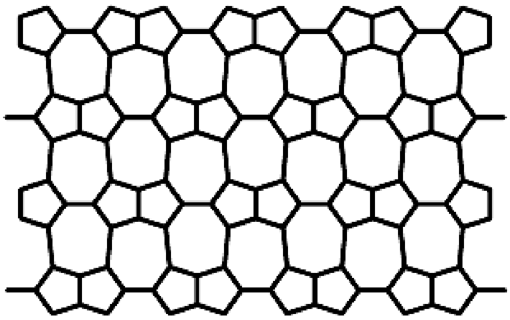

Figure-1: 2-*D* lattice of nanotubeHC_5C_7 [8, 4]

The 2-D lattice of nanotube $HC_5C_7[p, q]$ is shown in Fig. 1.By calculation, we obtain that G has $4pq$ vertices and $6pq - p$ edges. The graph G has two types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_1| = 4p$.

$E_2 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_2| = 6pq - 5p$.

Theorem 1: *Let G be the graph of a nanotube $HC_5C_7[p, q]$. Then*

$$AC\left(HC_5C_7\left[p,q\right]\right)=6pq-\frac{30}{26}.$$

Proof: From definition and by cardinalities of the edge partition of $HC_5C_7[p, q]$, we deduce

$$AC\left(HC_5C_7\left[p,q\right]\right)=\frac{4p(2+3)^2}{2(2^2+3^2)}+\frac{(6pq-5p)(3+3)^2}{2(3^2+3^2)}$$

$$=6pq-\frac{30}{26}.$$

Theorem 2: *Let G be the graph of a nanotube $HC_5C_7[p, q]$. Then*

$$CA\left(HC_5C_7\left[p,q\right]\right)=6pq-\frac{21}{25}.$$

Proof: From definitions and by cardinalities of the edge partition of $HC_5C_7[p, q]$, we deduce

$$CA\left(HC_5C_7\left[p,q\right]\right)=\frac{4p2(2^2+3^2)}{(2+3)^2}+\frac{(6pq-5p)2(3^2+3^2)}{(3+3)^2}$$

$$=6pq-\frac{21}{25}.$$

4. RESULTS FOR $SC_5C_7[p,q]$ NANOTUBES

In this section, we focus on the family of nanotubes, denoted by$SC_5C_7[p,q]$, in which p is the number of heptagons in the first row and q rows of vertices and edges are repeated alternately. The 2-D lattice of nanotube $SC_5C_7[p,q]$ is presented in Fig. 2.

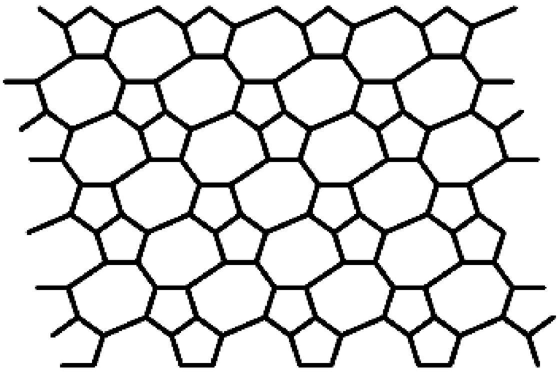

Figure-2: 2-*D* lattice of nanotube $SC_5C_7[p,q]$

Let G be the graph of $SC_5C_7[p,q]$. By calculation, we obtain that G has $4pq$ vertices and $6pq - p$ edges. Also by calculation, we get that G has three types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = q$.

$E_2 = \{uv\in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_2| = 6q$.

$E_2 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_3| = 6pq - p-7q$.

Theorem 3: *Let G be the graph of a nanotube $SC_5C_7[p, q]$. Then*

$$AC\left(SC_5C_7\left[p,q\right]\right)=6pq-p-\frac{3}{13}q.$$

Proof: From definition and by cardinalities of the edge partition of $SC_5C_7[p, q]$, we deduce

$$AC\left(SC_5C_7[p,q]\right)=\frac{q(2+2)^2}{2(2^2+2^2)}+\frac{6q(2+3)^2}{2(2^2+3^2)}+\frac{(6pq-p-7q)(3+3)^2}{2(3^2+3^2)}$$

$$=6pq-p-\frac{3}{13}q.$$

Theorem 4: *Let G be the graph of a nanotube $SC_5C_7[p, q]$. Then*

$$CA\left(SC_5C_7\left[p,q\right]\right)=6pq-p-\frac{6}{25}q.$$

Proof: From definitions and by cardinalities of the edge partition of $SC_5C_7[p, q]$, we deduce

$$CA\left(SC_5C_7[p,q]\right)=\frac{q2(2^2+2^2)}{(2+2)^2}+\frac{6q2(2^2+3^2)}{(2+3)^2}+\frac{(6pq-p-7q)2(3^2+3^2)}{(3+3)^2}$$

$$=6pq-p-\frac{6}{25}q$$

5. RESULTS FOR ARMCHAIR POLYHEX NANOTUBES

Carbon polyhex nanotubes are the nanotubes whose cylindrical surface is made up of entirely hexagons. These carbon nanotubes exist in nature with remarkable stability and possess very interesting electrical, thermal and mechanical properties, The armchair polyhex nanotube is denoted by $TUAC_6\,[p, q]$ is shown in Fig. 3.

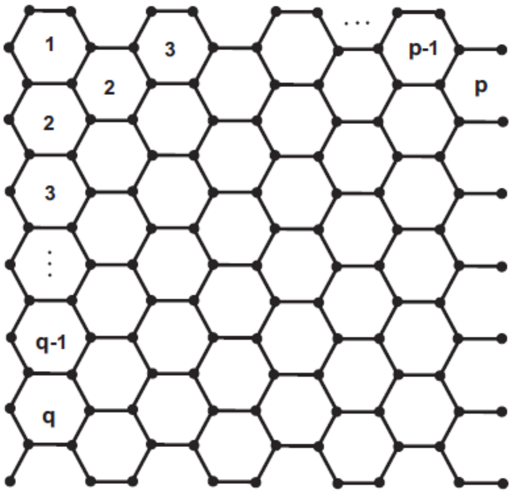

Figure-3: A 2-dimensional networks of $TUAC_6[p, q]$.

Let $G = TUAC_6\,[p, q]$. By calculation, G has $2p(q+1)$ vertices and $3pq + 2p$ edges. There are three types of edges based on degrees of end vertices of each edge. We present that the edge partition of G is given in Table 3.

$d_G(u), d_G(v) \setminus uv \in E(G)$	(2, 2)	(2, 3)	(3, 3)
Number of edges	p	$2p$	$3pq- p$

Table-3: Edge partition of $TUAC_6\,[p, q]$

Theorem 5: *Let G be the graph of an armchair polyhex nanotube $TUAC_6[p, q]$. Then*

$$AC\left(TUAC_6\left[p,\ q\right]\right)=3pq+\frac{25}{13}p.$$

Proof: From definition and by cardinalities of the edge partition of $TUAC_6[p, q]$, we deduce

$$AC\left(TUAC_6\left[p,\ q\right]\right)=\frac{p(2+2)^2}{2(2^2+2^2)}+\frac{2p(2+3)^2}{2(2^2+3^2)}+\frac{(3pq-p)(3+3)^2}{2(3^2+3^2)}$$

$$=3pq+\frac{25}{13}p.$$

Theorem 6: *Let G be the graph of an armchair polyhex nanotube $TUAC_6[p, q]$. Then*

$$CA\left(TUAC_6\left[p,\ q\right]\right)=3pq+\frac{52}{25}p.$$

Proof: From definition and by cardinalities of the edge partition of $TUAC_6[p, q]$, we deduce

$$CA\left(TUAC_6\left[p,\ q\right]\right)=\frac{p2(2^2+2^2)}{(2+2)^2}+\frac{2p2(2^2+3^2)}{(2+3)^2}+\frac{(3pq-p)2(3^2+3^2)}{(3+3)^2}$$

$$=3pq+\frac{52}{25}p.$$

6. RESULTS FOR ZIGZAG POLYHEX NANOTUBES

The zigzag polyhex nanotube is denoted by $TUZC_6\,[p, q]$, where p is the number of hexagons in a row whereas q is the number of hexagons in a column. A 2-dimensional networks of $TUZC_6\,[p, q]$ is depicted in FIG. 4.

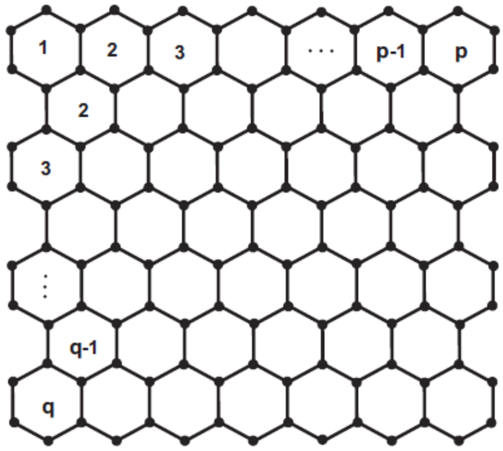

Figure-4: A 2-dimensional networks of $TUZC_6[p, q]$

Let G be a graph of a (p, q) dimensional zigzag polyhex nanotube. The graph G has $2p(q+1)$ vertices and $3pq + 2p$ edges. In G, there are two types of edges based on degrees of end vertices of each edge. By calculation, the edge partition of G is given in Table 4.

$d_G(u), d_G(v) \setminus uv \in E(G)$	(2, 3)	(3, 3)
Number of edges	$4p$	$3pq - 2p$

Table-4: Edge partition of $TUZC_6\,[p, q]$

Theorem 7: *Let G be the graph of a zigzag polyhex nanotube* $TUZC_6[p, q]$. *Then*

$$AC\left(TUZC_6\left[p,\ q\right]\right) = 3pq + \frac{24}{13}p.$$

Proof: From definition and by cardinalities of the edge partition of $TUZC_6[p, q]$, we deduce

$$AC\left(TUZC_6\left[p,\ q\right]\right) = \frac{4p(2+3)^2}{2(2^2+3^2)} + \frac{(3pq-2p)(3+3)^2}{2(3^2+3^2)}$$

$$= 3pq + \frac{24}{13}p.$$

Theorem 8: *Let G be the graph of a zigzag polyhex nanotube* $TUZC_6[p, q]$. *Then*

$$CA\left(TUZC_6\left[p,\ q\right]\right) = 3pq + \frac{54}{25}p.$$

Proof: From definition and by cardinalities of the edge partition of $TUZC_6[p, q]$, we deduce

$$CA\left(TUZC_6\left[p,\ q\right]\right) = \frac{p2(2^2+2^2)}{(2+2)^2} + \frac{2p2(2^2+3^2)}{(2+3)^2} + \frac{(3pq-p)2(3^2+3^2)}{(3+3)^2}$$

$$= 3pq + \frac{54}{25}p.$$

7. CONCLUSION

In this study, we have introduced the arithmetic-contraharmonic index and contraharmonic-arithmetic index of a graph. Furthermore we have computed these indices for some standard graphs and certain families of nanotubes.

Many questions are suggested by this research, among them are the following:

1. Characterize the AC and CA indices in terms of other degree based topological indices.
2. Obtain the extremal values and extremal graphs of AC and CA indices.
3. Compute the exact values of these two indices for other chemical nanostructures.

REFERENCES

1. V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
2. D.Vukicevic and B.Furtula, Topological index based on the ratios of geometrical and arithmetical means of end vertex degrees of edges, *Journal of Mathematical Chemistry,* 46(2) (2009) 1369-1376.
3. G.Fath-Tabar, B.Furtula, I.Gutman, A new geometric-arithmetic index, *Journal of Mathematical Chemistry,* 47(10 (2010) 477-486.
4. V.R.Kulli, Some new multiplicative geometric-arithmetic indices, *Journal of Ultra Scientist of Physical Sciences,* 29(2) (2017) 52-57.
5. V.R.Kulli, M.H.Akhabari, Multiplicative atom bond connectivity and multiplicative geometric-arithmetic indices of dendrimer nanostars, *Annals of Pure and Applied Mathematics,* 16(2) (2018) 429-436.
6. K.C.Das, I.Gutman and B.Furtula, Survey on geometric-arithmetic indices of graphs, *MATCH Commun. Math. Comput. Chem.* 65 (2011) 595-644.
7. A.Graovac, M.Ghorbani and M.A.Hosseinzadeh, Computing fifth geometric-arithmetic index for nanostar dendrimers, *Journal of Mathematical Nanoscience,* 1(1) (2011) 33-42.
8. N.M.Husin, R.Hasni and N.E.Arif, Atom bond connectivity and geometric-arithmetic indices of dendrimer nanostars, *Australian Journal of Basic and Applied Sciences,* 7(9) (2013) 10-14.
9. V.R.Kulli, New arithmetic-geometric indices, *Annals of Pure and Applied Mathematics,* 13(2) (2017) 165-172.
10. V.R.Kulli, Computing fifth arithmetic-geometric index of certain nanostructures, *Journal of Computer and Mathematical Sciences,* 8(5) (2017) 196-201.
11. V.R.Kulli, Two new arithmetic-geometric ve-degree indices, *Annals of Pure and Applied Mathematics,* 17(1) (2018) 107-112.
12. V.R.Kulli, On two arithmetic-geometric Banhatti indices of of certain dendrimer nanostars, *International Journal of Fuzzy Mathematical Archive,* 16(1) (2018) 7-12.
13. V.R.Kulli, Computation of arithmetic-geometric Revan indicesof certain benzenoid systems, *Journal of Global Research in Mathematical Archives,* 5(8) (2018) 8-13.
14. V.R.Kulli, Different versions of multiplicative arithmetic geometric indices of some chemical structures, *International Journal of Engineering Sciences and Research Technology*, 10(6) (2021) 34-44.
15. A.Madanshekaf and M.Moradi, The first geometric-arithmetic index of some nanostar dendrimers, *Iranian Journal of Mathematical Chemistry,* 5 (2014) 1-6.
16. Y.Yuan, B.Zhou and N.Trinajstic, On geometric-arithmetic index, *Journal of Mathematical Chemistry,* 47(2) (2010) 833-841.
17. S.Ediz and K.Yamac, Quadratic-contraharmonic index of graphs, *MATI* 3(2) (2021) 22-28.
18. V.R.Kulli, K1 and K2 indices, *International Journal of Mathematics Trends and Technology,* 68(1) (2022).
19. V.R.Kulli, Geometric-quadratic and quadratic-geometric indices, *Annals of Pure and Applied Mathematics,* 25(1) (2022) 1-5.
20. V.R.Kulli, Contraharmonic-quadratic index of certain nanostar dendrimers, *International Journal of Mathematical Archive,* 13(1) (2022) 1-7.
21. V.R.Kulli and I.Gutman, On some mathematical properties of Nirmala index, *Annals of Pure and Applied Mathematics*, 23(2) (2021) 93-99.
22. V.R.Kulli, Computation of status neighborhood indices of graphs, *International Journal of Recent Scientific Research,* 11(4) (2020) 38079-38085.
23. V.R.Kulli, On multiplicative leap Gourava indices of graphs, *International Journal of Engineering Sciences and Research Technology*, 8(10) (2019) 22-30.
24. V.R.Kulli, Leap Gourava indices of certain windmill graphs, *International Journal of Mathematical Archive,* 10(11) (2019) 7-14.
25. V.R.Kulli, The (*a*, *b*)-Kulli-Basava index of graphs, *International Journal of Engineering Sciences and Research Technology*, 8(12) (2019) 103-112.
26. I.Iranmanesh, M.Zeraatkar, Computing GA index for nanotubes, *Opteolectron Adv. Mater. Rapid Commun.* 4(11) (2010) 1852-1855.

Source of support: Nil, Conflict of interest: None Declared.