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$\psi \hat{g}$ - CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduce and study the notion of $\psi \hat{g}$ -closed sets in topological spaces. Also we study some basic properties and applications of $\psi \hat{g}$ -closed sets. The relations between $\psi \hat{g}$ -closed sets with various closed sets are analyzed.

***Keywords**: ψ -closed sets, $\hat{g}$ -closed sets and $\psi \hat{g}$ -closed sets*

1. INTRODUCTION

Levine [3] introduced generalized closed (briefly g-closed) sets and studied their basic properties. Njastad[7] introduced pre-open sets, α-open sets respectively Bhattacharya and Lahiri[1], Maki et a [4,5], Dontchev and Ganster[2] introduced semi-generalized closed sets, generalized semi-closed sets, generalized α-closed sets, α-generalized closed sets and δ-generalized closed sets respectively. Veera Kumar [8] introduced $\hat{g}$ -closed sets in topological spaces.The purpose of this present paper is to define a new class of closed sets called $\psi \hat{g}$ -closed sets and also we obtain some basic properties of $\psi \hat{g}$ -closed sets in topological spaces.

2 PRELIMINARIES

Throughout this paper (X, τ) (or simply X) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, cl (A), int (A) and A ^{C} denote the closure of A, the interior of A and the complement of A respectively. Let us recall the following definitions which are useful in the sequel.

Definition: 2.1 A subset A of a topological space (X, τ) is called.
(i) semi-open set [4] if A $\subseteq$ cl(int(A)).
(ii) pre-open set [8] if A $\subseteq$ int(cl(A)).
(iii) α -open set [10] if A $\subseteq$ int(cl(int(A))).
(iv) regular open set [11] if A= int(cl(A)).

The complement of a semi-open (resp. pre-open, α -open, regular open) set is called semi-closed (resp. semi-closed, α -closed, regular closed).

Definition: 2.2 A subset A of a topological space (X, τ) is called
(1) a generalized closed set(briefly g-closed)[16] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X,τ).
(2) a generalized semi-closed set (briefly gs-closed) [3] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
(3) a ψ-closed set if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is sg-open of (X, τ).
(4) ψ -generalized closed set (briefly ψ g-closed) if ψ cl(A) $\subseteq$ A whenever A $\subseteq$ U and U is open in (X,τ).
(5) a ĝ-closed set[25] if cl(A) $\subseteq$ G whenever A $\subseteq$ G and G is semi-open in (X,τ).

The complements of the above mentioned sets are called their respective open sets.

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Theorem: 2.4 Every open set is $\hat{g}$ -open.

Proof: Let A be an open set in X. Then Ac is closed. Therefore, cl(A^C) = $A^C \subseteq$X whenever $A^C \subseteq$X and X is semi-open. This implies A^C is $\hat{g}$ -closed.

Hence A is $\hat{g}$ -open.

3 $\psi\ \hat{g}$ - CLOSED SETS IN TOPOLOGICAL SPACES

Definition: 3.1 A subset A of a space (X, τ) is called $\psi\ \hat{g}$ - closed if ψ cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\hat{g}$ -open in (X, τ).

Theorem: 3.2 Every closed set is $\psi\ \hat{g}$ - closed

Proof: Let A be a closed set of (X, τ).Let U be a $\hat{g}$ -open set of (X, τ) such that A $\subseteq$ U. Since A is closed ψ cl(A) $\subseteq$ cl(A)=A $\subseteq$ U. Therefore A is $\psi\ \hat{g}$ - closed.

The converse need not be true as seen from the following example

Example: 3.3 Let X ={a, b, c} with topology τ = { ϕ ,X, {a},{b},{a, c}}.closed={ ϕ ,X, {b},{b, c},{a, c}}, $\psi\ \hat{g}$ -closed={ ϕ ,X,{a},{b},{c},{b, c},{a, c}}.Here{a},{c} is $\psi\ \hat{g}$ -closed but not closed in (X, τ).

Theorem 3.4: Every $\hat{g}$ -closed set is $\psi\ \hat{g}$ -closed

Proof: A subset A of a topological space (X, τ) is $\hat{g}$ -closed. Let U be $\hat{g}$ -of set (X, τ) such that A $\subseteq$ U. Since A is $\hat{g}$ -closed ψ cl(A) $\subseteq$ cl(A)=A $\subseteq$ U. Therefore A is $\psi\ \hat{g}$ - closed.

The converse need not be true as seen from the following example

Example: 3.5 Let X ={a, b, c} with topology τ = { ϕ ,X, {a},{a, b}}. $\hat{g}$ -closed={ ϕ ,X, {b},{c},{a, b},{b, c}}, $\psi\ \hat{g}$ -closed={ ϕ ,X,{b},{c},{b, c},{a, c}}.Here{a, c} is $\psi\ \hat{g}$ -closed but not $\hat{g}$ -closed in (X, τ).

Remark: 3.6 Every ψ - closed set is semi closed

Theorem: 3.7 Every $\psi\ \hat{g}$ - closed set is gs-closed

Proof: Let A be an $\psi\ \hat{g}$ - closed and U be any open set containing A in (X, τ).Since every open set is $\hat{g}$ -open, ψ cl(A) $\subseteq$ U for every subset A of X. Since scl(A) $\subseteq$ ψ cl(A) $\subseteq$ U, scl(A) $\subseteq$ U and hence A is gs-closed.

The converse need not be true as seen from the following example

Example: 3.8 Let X = {a, b, c} with topology τ = { ϕ , X, {a}, {c}, {b, c}}. gs-closed ={ ϕ ,X, ,{a}, {b}, {c},{a, b}, {b, c}}, $\psi\ \hat{g}$ -closed={ ϕ ,X,{a},{b},{c},{b, c},{a, c}}. Then the set {c} is gs-closed but not $\psi\ \hat{g}$ -closed in (X, τ).

Theorem: 3.9 Every $\psi\ \hat{g}$ -closed set is ψ g-closed

Proof: Let A be a $\psi\ \hat{g}$ -closed and U be any open set containing A.Since every open set is $\hat{g}$ -open, ψ cl(A) $\subseteq$U, whenever A$\subseteq$U and U is $\hat{g}$ -open. Therefore ψ cl(A) $\subseteq$U and U is open. Hence A is ψ g-closed.

The converse need not be true as seen from the following example

Example: 3.10 Let X = {a, b, c} with topology τ={ϕ,X,{b}}. ψ g-closed= {ϕ,X,{b},{a, b}, {b, c}}, $\psi\ \hat{g}$ -closed={ϕ,X,{a},{c},{a, c}}Then the set {b} is ψ g -closed but not $\psi\ \hat{g}$ -closed in (X,τ).

Theorem: 3.11 The union of two $\psi\ \hat{g}$ - closed subsets of X is also an $\psi\ \hat{g}$ - closed subset of X.

Proof: Assume that A and B are $\psi\ \hat{g}$ - closed sets in X. Let U be $\hat{g}$ -open in X such that A$\cup$B$\subseteq$U. Then A$\subseteq$U and B$\subseteq$U. Since A and B are $\psi\ \hat{g}$ - closed, ψ cl(A) $\subseteq$U and ψ cl(B) $\subseteq$U .Hence ψ cl(A$\cup$B) =ψ cl(A) $\cup$ ψ cl(B) $\subseteq$U.That is ψ cl(A$\cup$B) $\subseteq$U. Therefore A$\cup$B is $\psi\ \hat{g}$ - closed set in X.

Theorem: 3.12 Let A be a $\psi\ \hat{g}$ - closed set of (X,τ).Then ψ cl(A)-A does not contain a non-empty $\hat{g}$ - closed set.

Proof: Suppose that A is $\psi\ \hat{g}$ - closed set, let F be a $\hat{g}$ -closed set contained in ψ cl(A)-A. Now F^C is $\hat{g}$ -open set of (X,τ) such that A$\subseteq$ F^C.Since A is $\psi\ \hat{g}$ - closed set of (X,τ),then ψ cl(A) $\subseteq$ F^C.Thus F $\subseteq$(ψ cl(A))^C.Also F$\subseteq$ ψ cl(A)-A. Therefore F$\subseteq$ (ψ cl(A))^C $\cap$ (ψ cl(A))=ϕ .Hence F=ϕ.

Theorem: 3.13 If A is $\psi\ \hat{g}$ - closed set in (X,τ) and A$\subseteq$B$\subseteq$ ψ cl(A).Then B is $\psi\ \hat{g}$ - closed set.

Proof: Let U be a $\hat{g}$ -open set of (X,τ) such that B$\subseteq$U. Then A$\subseteq$U. Since A is $\psi\ \hat{g}$ - closed set, ψ cl(A) $\subseteq$U. Also since B$\subseteq$ ψ cl(A), ψ cl(B) $\subseteq$ ψ cl(ψ cl(A))= ψ cl(A).Henceψ cl(B) $\subseteq$U.

Therefore B is also a $\psi\ \hat{g}$ - closed set.

Theorem: 3.14 For each x$\in$ X either {x} is $\hat{g}$ - closed or $\{x\}^C$ is $\psi\ \hat{g}$ - closed set.

Proof: Suppose that {x} is not $\hat{g}$ - closed in X. Then only $\{x\}^C$ is not $\hat{g}$ -open and the only $\hat{g}$ -open set containng $\{x\}^C$ is the space X itself. That is $\{x\}^C$ $\subseteq$X. Therefore ψ cl(A) ($\{x\}^C$)$\subseteq$X and so $\{x\}^C$ is $\psi\ \hat{g}$ - closed set.

Theorem: 3.15 Let x be a $\psi\ \hat{g}$ - closed set in X. Then A is ψ - closed set iffψ cl(A)-A is closed.

Proof: Necessity: Let A be an $\psi\ \hat{g}$ - closed subset of X. Then ψ cl(A)=A and so ψ cl(A)-A=ϕ which is closed.

Sufficiency: Since A is $\psi\ \hat{g}$ - closed set by theorem 3.12 ψ cl(A)-A contains no nonempty closed set. But ψ cl(A)-A is closed. This implies ψ cl(A)-A= ϕ.That is ψ cl(A)=A. Hence A is ψ -closed.

Definition: 3.16 The intersection of all $\hat{g}$ -open subsets of (X, τ) containing A is called the $\hat{g}$ -kernel of A and is denoted by $\hat{g}$ -ker(A) .

Theorem: 3.17 A subset A of (X, τ) is $\psi\hat{g}$ - closed set iff ψ cl(A) $\subseteq \hat{g}$ -ker(A)

Proof: Suppose that A is $\psi\hat{g}$ - closed set in X. Then ψ cl(A) $\subseteq$U whenever A $\subseteq$U and U is $\hat{g}$ -open in (X, τ).Let x$\in \psi$ cl(A).If x$\notin \hat{g}$ -ker(A),then there is a $\hat{g}$ -open set U such that x$\notin$ U. Since U is a $\hat{g}$ -open set containing A, we have x$\notin \psi$ cl(A),which is a contradiction.

Conversely let ψ cl(A) $\subseteq \hat{g}$ -ker(A).If u is any $\hat{g}$ -open set containing A ,then ψ cl(A) $\subseteq \hat{g}$ -ker(A) $\subseteq$U. Therefore A is $\hat{g}\,\psi\hat{g}$ - closed set.

Definition: 3.18 A space (X, τ) is called $T_{\psi\hat{g}}$ if every $\psi\hat{g}$ - closed set in it is ψ -closed.

Theorem: 3.19 For a topological space (X, τ), the following conditions are equivalent.

(i) (X, τ) is a $T_{\psi\hat{g}}$ -space.

(ii) Every singleton {x} is either $\hat{g}$ -closed or ψ -open.

Proof: (i)$\Rightarrow$(ii)

Let x $\in$ X. Suppose {x} is not a $\hat{g}$ -closed set of (X, τ).Then X-{x} is not a $\hat{g}$ -open set. Thus X-{x} is an $\psi\hat{g}$ - closed set of (X, τ).Since (X, τ) is $T_{\psi\hat{g}}$ -closed, x-{x} is an ψ -closed set of (X, τ), that is {x} is ψ -open set of (X, τ).

(ii) $\Rightarrow$ (i)

Let A be an $\psi\hat{g}$ - closed set of (X, τ) .Let x$\in \psi$ cl(A) .By(ii) {x} is either $\hat{g}$ - closed or ψ -open.

Case (i): Let {x} be $\hat{g}$ - closed. If we assume that x$\notin$ A, then we would have x$\in \psi$ cl(A)-A, which cannot happen according to theorem3.8. Hence x$\in$ A.

Case (ii): Let {x} be ψ -open. Since X$\in \psi$ cl(A),then{x}$\cap$A=ϕ .This shows that x$\in$ A.

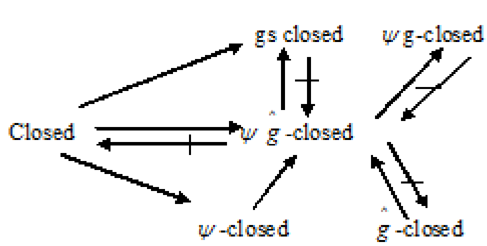
So in both the cases we have A $\subseteq \psi$ cl(A).Therefore A=ψ cl(A) or equivalently A is closed. Hence (X, τ) is a $T_{\psi\hat{g}}$ -space.

Theorem: 3.20 If A is $\hat{g}$ -open and $\psi\hat{g}$ - closed subset of (X, τ), then A is a ψ -closed subset of (X, τ).

Proof: Since A is $\hat{g}$ -open and $\psi\hat{g}$ - closed set, ψ cl(A) $\subseteq$A. Hence A is ψ -closed.

Remark: 3.21

The following diagram shows the relationships of $\psi\hat{g}$-closed sets with some other sets discussed in this section.



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