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FUZZY REGULAR SEMI GENERALIZED OPEN SETS IN FUZZY TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we have introduced the notion of fuzzy regular semi generalized open sets in fuzzy topological spaces. In this paper, we have investigated the relationship between the fuzzy regular semi generalized open sets and the already existed fuzzy open sets. Also their properties are discussed.

Key Words: *Fuzzy semi generalized open Sets, Fuzzy regular generalized open Sets, Fuzzy regular semi generalized closed Sets, fuzzy regular semi generalized open sets.*

1. INTRODUCTION:

The concept of a fuzzy set was initiated by L.AZadeh [21] in the year 1965. Later, the fuzzy set theory is widely studied and extended. In the year 1968, C.L.Chang [4] introduced the concept of fuzzy topological spaces as an application of fuzzy sets to topological spaces.

In this paper we offer a new class of fuzzy sets called fuzzy regular semi generalized open sets in fuzzy topological spaces (F.T. Space) and investigate certain basic properties of these fuzzy sets.

2. PRELIMINARIES:

Throughout this paper (X,τ) and (Y,) (or X and Y) represents fuzzy topological spaces (F.T. Spaces) on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a fuzzy topological space (F.T. Space) (X,), cl(A), int(A) and A^cdenote the fuzzy closure of A, the fuzzy interior of A and the fuzzy complement of A respectively. We recall the following basic definitions which are useful in the sequel.

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Definition 2.1: [21**]** Let X be a non empty set. A fuzzy subset A in X is characterized by a membership function: X→ $[O, 1]$ which associates with each point x in X, a real number μ_A(x) between 0 and 1which represents the degree or grade of membership to A .Thus a fuzzy subset A of a set X has the following representation A= {(x, μ_A(x)): x ∈ X}, where μ_A is the membership function.

Definition 2.2: [21**]** Let A and B be two fuzzy sets in F.T. Space (X, r). Then their union A V B, intersection A Λ B and complement A^c a real so fuzzy sets with membership functions defined as follows:

i) μ_{A^c}(x) =1- μ_A(x), ∀ x ∈ X.
ii) $\mu_{\text{AVB}}(x)$= max{μ_A(x), μ_B(x)}, ∀ x ∈ X.
iii) $_{\text{A}\Lambda\text{B}}(x)$= min{$\mu_A$(x), μ_B(x)}, ∀ x ∈ X.

Further

iv) A ⊆ B if and only if μ_A(x) ≤ μ_B(x), ∀ x ∈ X.
v) A = B if and only if μ_A(x) = μ_B(x), ∀ x ∈ X.

Definition2.3: [4**]** A family r of fuzzy sets is called fuzzy topology (F.T. in short) for X if it satisfies the three axioms:

i) Fuzzy sets 0 and1belongs to r
ii) $A\Lambda B \in r$, ∀ A, B ∈ r
iii) $\text{V}_{j\in J}A_j$∈,∀ $(A_j)_{j\in J}$

Hence the pair (X,τ) is called a fuzzy topological space (F.T. Space in short). The elements of $r$are called fuzzy open sets in X and the irrespective complements are called fuzzy closed sets of (X, τ).

Definition2.4: [4**]** For a fuzzy set λ of (X,τ), the fuzzy closure cl(λ)and fuzzy interior int(λ) is defined as follows:

Cl(λ) = Λ {μ: μ ≥ λ, μ is fuzzy closed set}
Int(λ) = ∨{μ:μ≤λ, μ is fuzzy open set}

Definition2.5: A fuzzy set λ in a fuzzy topological space (F.T. Space) (X, r) is called

(1) A fuzzy semi-open (fso) set [8] if $\lambda \leq$ cl(int(λ)) and a fuzzy semi closed set if int(cl(λ)) ≤ λ.
(2) A fuzzy semi regular open (fsro) set [19] if it is both fuzzy semi open and fuzzy semi closed.
(3) A fuzzy regular open (fro) set [8] if int(cl(λ)) = λ and a fuzzy regular closed set if cl(int(λ)) = λ.

Definition2.6: [18**]** Let λ be a fuzzy set in F.T. Space (X, τ).Then fuzzy semi closure (fscl) and fuzzy semi interior (fs int) of λ are defined as follows:

Scl(λ) =Λ{$\beta/\lambda\leq\beta$, β is fuzzy semi closed set}
Sint(λ) =∨{$\beta/\beta\leq\lambda$, β is fuzzy semi open set}

It is immediate that,

i) scl(λ) ≥ λ and Sint(λ) ≤ λ
ii) λ ≤ implies that scl(λ) ≤ scl(μ), sint(λ) ≤ sint(μ).

Further, A fuzzy set λ in F. T. Space (X, r) is

i) Fuzzy semi closed (fs-closed) iff int(cl(λ)) ≤ λ [respectively, scl(λ) = λ]
ii) Fuzzy semi open (fs-open) iff λ≤ cl(int(λ)) [respectively, sint(λ) = λ

Definition 2.7: A fuzzy set λ in a fuzzy topological space (F.T. Space) (X, r) is called

1. A fuzzy generalized closed set (briefly, fg-closed) [5] if cl(λ) ≤ μ, whenever $\lambda \leq \mu$ and μ is a fuzzy open set in X.
2. A fuzzy regular generalized closed set (briefly, frg-closed) [7] if cl(λ) ≤ μ, whenever $\lambda \leq \mu$ and μ is a fuzzy regular open set in X.
3. A fuzzy semi generalized closed set (briefly, fsg-closed) [6] if scl(λ) ≤ μ, whenever $\lambda \leq \mu$ and μ is a fuzzy semi open set in X.
4. A fuzzy generalized semi-closed set (briefly, fgs-closed) [15] if scl(λ) ≤ μ, whenever $\lambda \leq \mu$ and is a fuzzy open set in X.
5. A fuzzy regular semi open set (briefly, frs-open) [2] if $\mu \leq \lambda \leq$ cl(μ), whenever μ is fuzzy regular open set in X.

3. FUZZY REGULAR SEMI GENERALIZED OPEN SETS (FRSGO)

In this section, the notion of fuzzy regular semi generalized open sets in F.T. Space is introduced.

Definition 3.1: A fuzzy set λ in a F.T. Space (X, τ) is called a fuzzy regular semi generalized open set if X-λ is fuzzy regular semi generalized closed (frsgc) set.

Example 3.2: Let X = {a, b} and let A = {< a, 0.6 >, < b, 0.7>}, B = {< a, 0.4 >, < b, 0.0>}. Then τ = {0, A, B, 1} be F.T.S. Consider C = {< a, 0.6 >, < b, 1>}. Here C is fuzzy regular semi generalized open set.

Theorem 3.3: A fuzzy set λ in a F.T. Space (X, τ) is fuzzy regular semi generalized open (frsgo) set if and only if $\mu \leq$int(λ) whenever $\mu \leq \lambda$ for every fuzzy semi regular open(fsrc) set μ in X.

Proof: Let λ be a fuzzy regular semi generalized open (frsgo) set and μ be a fuzzy semi regular open set such that $\mu \leq \lambda$. Then X- λ is fuzzy regular semi generalized closed (frsgc) set and X- $\lambda \leq$ X-μ. Since X-μ is fuzzy semi regular open set in X, Cl(X- λ) ≤ X-μ and hence X-int(λ) ≤ X- μ. Hence $\mu \leq int(\lambda)$.

Conversely, let $\mu \leq int(\lambda)$ whenever $\mu \leq \lambda$ and μ is fuzzy semi regular open in X. This implies that X-int(λ) = Cl(X-λ) ≤ X-μ whenever X- $\lambda \leq$ X-μ and X-μ is fuzzy semi regular open in X. This proves that X- λ is frsg-closed set in X and hence λ is fuzzy regular semi generalized open (frsgo) set in X.

Theorem 3.4: Every fuzzy open set is fuzzy regular semi generalized open (frsgo) set.

Proof: Let λ be a fuzzy open set in (X, τ). Then 1-λ is fuzzy closed set in X. Also 1-λ is fuzzy regular semi generalized closed (frsgc) set in X, Since every fuzzy closed set is fuzzy regular semi generalized closed (frsgc) set. Hence λ is fuzzy regular semi generalized open (frsgo) set in X.

Remark 3.5: The converse of the above theorem is not true.

Example 3.6: Let X = {a, b} and let A = {< a, 0.6 >, < b, 0.7>}, B = {< a, 0.2 >, < b, 0.0>}, C = {< a, 0.4 >, < b, 0.3>} and D = {< a, 0.8>, < b, 1> }, Then τ = {0, A, B, C, 1 } be F.T. Space on X. Consider D ={< a, 0.6 >, < b, 1> }. Here D is fuzzy regular semi generalized open (frsgo) set but not fuzzy open set.

Remark 3.7: Every fuzzy regular open (fr-open) set is fuzzy regular semi generalized open (frsgo) set.

Example 3.8: Let X = {a, b} and let A = {< a, 0.6 >, < b, 0.7>}, B = {< a, 0.4 >, < b, 0.0>}. Then τ = {0, A, B, 1 } be F.T. Space on X. Here A is fuzzy regular open (fro) set Since int (cl(A)) = A. Also A is fuzzy regular semi generalized open (frsgo) set.

Remark 3.9: Every fuzzy regular generalized open set (frg-open) is fuzzy regular semi generalized open (frsg-open) set.

Example 3.10: Let X = {a, b} and let A = {< a, 0.6 >, < b, 0.7>}, B = {< a, 0.2 >, < b, 0.0>} .C = {< a, 0.4 >, < b, 0.3>} Then τ = {0, A, B, C, 1} be F.T. Space on X. Consider D = {< a, 0.8 >, < b, 0.1>}. Here D is fuzzy regular generalized open set. Also D is fuzzy regular semi generalized open set.

Remark 3.11: Every fuzzy generalized open set is not necessarily fuzzy regular semi generalized open (frsg-open) set.

Example 3.12: Let X = {a, b, c} and let A = {< a, 0.3 >, < b, 0.5 >, {< c, 0.2 >}, B = {< a, 0.8 >, < b, 0.6 >, {< c, 0.9 >}. Then τ = {0, A, 1} be F.T. Space on X. Consider C = {< a, 0.7 >, < b, 0.5 >, {< c, 0.8 >}. Here C is fuzzy generalized open set but not fuzzy regular semi generalized open set (frsg-open).

Remark 3.13: Fuzzy semi open sets and fuzzy regular semi generalized open sets are independent of each other.

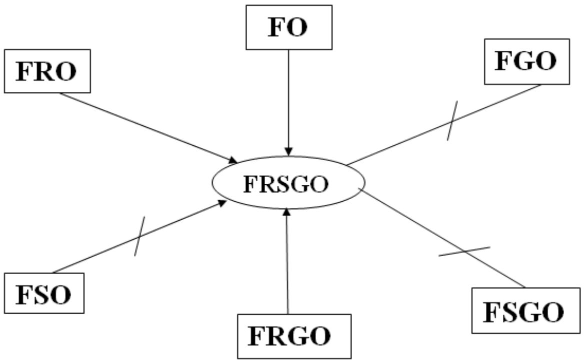
Example 3.14:

1) Let X = {a, b, c} and let λ_1 = {< a, 0.3 >, < b, 0.5 >, < c, 0.2 >}, λ_2 = {< a, 0.2 >, <b, 0.4>, < c, 0.1>} be fuzzy subsets of X. Let τ = {0, λ_1, 1} be F.T. Space. Here λ_1 is fuzzy semi open set but not fuzzy regular semi generalized open set.
2) Let X = {a, b} and let A = {< a, 0.6 >, < b, 0.7>}, B = {< a, 0.4 >, < b, 0.0>}. Then τ = {0, A, B, 1} be F.T. Space X. Consider C = {< a, 0.6 >, < b, 1>}. Here C is fuzzy regular semi generalized open set but not fuzzy semi open set.

Remark 3.15: Fuzzy semi generalized open sets and fuzzy regular semi generalized open sets are independent of each other.

Example 3.16: Let X = {a, b} and let A = {< a, 0.6 >, < b, 0.7>}, B = {< a, 0.4 >, < b, 0.0>}. Then τ = {0, A, B, 1} be F.T. Space X. Consider C = {< a, 0.6 >, < b, 1>}. Here C is fuzzy regular semi generalized open set but not fuzzy semi generalized open set.

The following diagram shows the relationships of fuzzy regular semi generalized open sets with some other open sets discussed in this sect



Where A -> B means A implies B But not conversely.
A —/→ B means A and B are independent.

Remark 3.17: The union of two fuzzy regular semi generalized open (frsg-open) sets is not necessarily fuzzy regular semi generalized open set (frsg-open).

Example 3.18: Let A = {< a, 0.8 >, < b, 1>} and B = {< a, 0.6 >, < b, 0.7>} be two fuzzy regular semi generalized open sets in X. But their union, A V B = {< a, 0.8 >, < b, 1>} is not necessarily fuzzy regular semi generalized open set (frsg-open).

Remark 3.19: The intersection of two fuzzy regular semi generalized open sets is again a fuzzy regular semi generalized open set.

Example 3.20: Let A = {< a, 0.8 >, < b, 1>} and B = {< a, 0.6 >, < b, 0.7>} be two fuzzy regular semi generalized open sets in X. Then their intersection, A Λ B = {< a, 0.6 >, < b, 0.7>} is again a fuzzy regular semi generalized open set.

Theorem 3.21: If A and B are fuzzy regular semi generalized open sets, then AΛB is fuzzy regular semi generalized open.

Proof: If $\mu \leq$ AΛB and μ is semi regular set, then $\mu \leq$ int (A) and $\mu \leq$ int (B). Hence $\mu \leq$ int(A) Λ int(B) = int(AΛB). Hence $\mu \leq$ int (AΛB). Hence by theorem 3.4, AΛB is fuzzy regular semi generalized open set.

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