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NON-HOMOGENEOUS BIQUADRATIC EQUATION WITH FOUR UNKNOWNS $xy\left(x+y\right)+4zw^{3}=0$

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ABSTRACT

The non-homogeneous bi-quadratic equation with four unknowns given by $xy(x+y)+4zw^{3}=0$ *is analysed for finding its non-zero distinct integer solutions through different methods. A few interesting relations among the solutions are presented.*

Keywords: *Non- homogeneous bi-quadratic, Bi-quadratic with four unknowns, Integer solutions.*

INTRODUCTION

The theory of Diophantine equations offers a rich variety of fascinating problems. In particular, biquadratic diophantine equations, homogeneous and non-homogeneous have aroused the interest of numerous mathematicians since antiquity [1-5]. In this context, one may refer [6-17] for various problems on the biquadratic diophantine equations with four variables. However, often we come across non-homogeneous biquadratic equations and as such one may require its integral solution in its most general form.

It is towards this end, this paper concerns with the problem of determining non-trivial integral solutions of the non-homogeneous equation with four unknowns given by $xy(x+y)+4zw^{3}=0$

NOTATIONS

$t_{m,n}$: polygonal number of rank n with size m.

P_m^n : pyramidal number of rank n with size m.

METHOD OF ANALYSIS

The non- homogeneous bi-quadratic equation with four unknowns to be solved is

$$xy(x+y)+4zw^{3}=0 \tag{1}$$

Substitution of the linear transformations

$$x=u+v\,,\ y=u-v\,,\ z=u,\left(u\neq v\neq 0\right) \tag{2}$$

in (1) leads to

$$v^{2}-u^{2}=2w^{3} \tag{3}$$

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Solving (3) through different ways for u, v, w and using (2), one obtains different sets of solutions to (1) that are illustrated below:

WAY 1:
Write (3) as the system of double equations as shown in Table 1 below:

Table-1: System of Double Equations

SYSTEM	1	2	3	4	5	6
v+u	w^3	$2w^2$	w^2	$2w$	w	2
v-u	2	w	$2w$	w^2	$2w^2$	w^3

Solving each of the system of equations in Table 1, the corresponding values of u, v and w are obtained. Substituting the values of u and v in (2), the respective values of x and y are determined. The integer solutions to (1) obtained through solving each of the above system of equations are exhibited.

Solutions through SYSTEM 1: $x = 8k^3, y = -2, z = 4k^3 - 1, w = 2k$

OBSERVATIONS:
- $x + w^3 = 16P_k^{\,6}$
- $x^3 + y^3 + 6xyz = 8z^3$
- $z + w = 6P_k^{\,4} + y + 1$
- $x - w = 6P_k^{\,8}$
- $2x - 5w = 6P_k^{\,16}$

Solutions through SYSTEM 2: $x = 8k^2, y = -2k, z = 4k^2 - k, w = 2k$

OBSERVATIONS:
- $y^3 + 12P_k^{\,4} = 2w$
- $xy + 5w + 6P_k^{\,16} = 0$
- $x + w^3 + 2k = 6P_k^{\,4} + 2z$
- $3w + 2t_{10,k} = x$
- $2x + 7y = 2t_{18,k}$

Solutions through SYSTEM 3: $x = 4k^2, y = -4k, z = 2k^2 - 2k, w = 2k$

OBSERVATIONS:
- 3(z-w-y) is a nasty number.
- $z = 2k^2 + w + y$
- $x - w = 2t_{6,k}$
- $5y^2 = 32t_{7,k} + 24w$
- $2z + w^2 = 8t_{3,k-1} + x$
- $7w^3 + 8y = 24P_k^{\,14}$

Solutions through SYSTEM 4: $x = 4k, y = -4k^2, z = 2k - 2k^2, w = 2k$

OBSERVATIONS:

- $13w^3 - 14x = 48P_k^{13}$
- $3y + 2x + 4t_{8,k} = 0$
- $2y + 3w + 2t_{10,k} = 0$
- $2z + w^3 = 12P_k^{4} + y$
- $x + y + 8t_{3,k-1} = 0$

Solutions through SYSTEM 5: $x = 2k, y = -8k^2, z = k - 4k^2, w = 2k$

OBSERVATIONS:

- $z^2 + w + 6P_k^{8} = x^4 + k^2$
- $6\left(z^2 + w - x^4 + 6P_k^{8}\right)$ is a nasty number.
- $yw + 5x + 6P_k^{16} = 0$
- $5y + 12w + 16t_{7,k} = 0$

Solutions through SYSTEM 6: $x = 2, y = -8k^3, z = 1 - 4k^3, w = 2k$

OBSERVATIONS:

- $y + 4w + 6P_k^{8} = 6k$
- $w + 1 = z + 6P_k^{4}$
- $y + 8P_k^{6} = 0$
- $x + y + 6P_k^{5} + 2P_k^{9} = 2$
- $y + z + 6P_k^{12} + 6k = 1$

WAY 2:
Taking the linear transformations

$$x = u + w, y = u - w, z = u, (u \neq w \neq 0) \tag{4}$$

in (1), it is written as

$$u^2 = w^2(1 - 2w) \tag{5}$$

After some algebra, it is seen that (5) is satisfied by the following two choices of u and w:

Choice 1: $w = -2(k^2 + k), u = 2(k^2 + k)(2k + 1)$

Choice 2: $w = -2(k^2 + k), u = -2(k^2 + k)(2k + 1)$

In view of (4), the corresponding sets of integer solutions to (1) are represented below:

Solutions to choice 1:

$$x = 4k(k^2 + k)$$
$$y = 4(k^2 + k)(k + 1)$$
$$z = 2(k^2 + k)(2k + 1)$$
$$w = -2(k^2 + k)$$

Solutions to choice 2:

$$x = -4(k^2 + k)(k+1)$$
$$y = -4k(k^2 + k)$$
$$z = -2(k^2 + k)(2k+1)$$
$$w = -2(k^2 + k)$$

CONCLUSION

In this paper, an attempt has been made to determine non-zero distinct integer solutions to the non-homogeneous bi-quadratic equation with four unknowns given by $xy(x+y)+4zw^3 = 0$. The researchers in this field may search for other sets of integer solutions to the equation under consideration and other forms of bi-quadratic equations with four or more variables.

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