

FUZZY METRIC SPACE FOR WEAKLY COMPATIBLE OF TYPE (A)

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ABSTRACT

The aim of this paper is to obtain a common fixed point for weakly compatible of type (A) in Fuzzy metric space.

Keywords: common fixed point, weakly compatible of type (A), fuzzy metric space.

1. INTRODUCTION AND PRELIMINARIES

In 1965 Zadeh [1] at the university of California U. S.A. proposed a way for mathematization of imprecisely described phenomena of introducing the concept of Fuzzy set.

Deng [6] defined a Fuzzy metric space by assigning a non negative real number for every pair of Fuzzy point in an empty set X and satisfying certain conditions. The idea of weak compatible maps of type (A) defined by Kramosil and Michalek [2], which was further modified by George and Veramani [3] as used by Pathak Kang and Beak [4] in Menger space. We extend the result of Singh and Chouhan [5] in Fuzzy metric space.

Definition 1.1: Self mappings P and Q of a Fuzzy metric space (X, T, \bullet) are said to be compatible of type (A), if $\lim_{n \rightarrow \infty} T(QPx_n, PPx_n, q) = 1$, $\lim_{n \rightarrow \infty} T(PQx_n, QQx_n, q) = 1$ for all $q > 0$, where $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} Px_n = \lim_{n \rightarrow \infty} Qx_n = u$ for some $u \in X$.

Definition 1.2: Self mappings P and Q of a Fuzzy metric space (X, T, \bullet) are said to be weak compatible of type (A), if $\lim_{n \rightarrow \infty} T(QPx_n, PPx_n, q) \geq \lim_{n \rightarrow \infty} T(PQx_n, PPx_n, q)$ and $\lim_{n \rightarrow \infty} T(PQx_n, QQx_n, q) \geq \lim_{n \rightarrow \infty} T(QPx_n, QQx_n, q)$ for all $q > 0$

where $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} Px_n = \lim_{n \rightarrow \infty} Qx_n = u$ for some $u \in X$.

Proposition: Let P and Q be continuous, self maps of Fuzzy metric space (X, T, \bullet) . Then commutativity implies weak compatible of type (A), but not conversely.

Proof: Let P and Q be self continuous and commuting maps of a Fuzzy metric space (X, T, \bullet) . Now if $\{x_n\}$ is any sequence in X such that $\lim_{n \rightarrow \infty} Px_n = \lim_{n \rightarrow \infty} Qx_n = u$ for some $u \in X$, then by continuity of P ,

$$PPx_n, QQx_n \rightarrow P_u \text{ for } q > 0.$$

$$\lim_{n \rightarrow \infty} T(QPx_n, PPx_n, q) \geq \lim_{n \rightarrow \infty} T(QPx_n, PQx_n, q/2) \bullet \lim_{n \rightarrow \infty} T(PQx_n, PPx_n, q/2) = 1$$

$$\text{i. e. } \lim_{n \rightarrow \infty} T(QPx_n, PPx_n, q) \geq \lim_{n \rightarrow \infty} T(PQx_n, PPx_n, q)$$

Similarly,

$$\lim_{n \rightarrow \infty} T(PQx_n, QQx_n, q) \geq \lim_{n \rightarrow \infty} T(QPx_n, PPx_n, q)$$

Hence P and Q are weak compatible maps of type (A).

For the converse part let (X, T, \bullet) is a Fuzzy metric space, where $X = [0, 1]$. Define self maps P and Q as $P_x = \frac{x}{a}$, $Q_x = \frac{x}{x+b}$ for all $x \in [0, 1]$

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Now $PQ_x = P\left(\frac{x}{x+b}\right) = \frac{x}{a(x+b)}$ and
 $QP_x = Q\left(\frac{x}{a}\right) = \frac{x/a}{x/a+b} = \frac{x}{x+ab}$

Hence $PQ \neq QP$ but, if $x_r = \frac{1}{r}$ $r = 1, 2, 3, \dots$ then $x_r \rightarrow 0$
 $Qx_r \rightarrow 0, Px_r \rightarrow 0$ as $r \rightarrow \infty$ then
 $PQx_n = \left(\frac{x_n}{a(x_n+b)}\right) \rightarrow 0$ and $QPx_n = \frac{x_n}{x_n+ab} \rightarrow 0$
 $PPx_n = P\frac{x_n}{a} = \frac{x_n}{a^2} \rightarrow 0$ and $QQx_n = Q\left(\frac{x_n}{x_n+b}\right) \rightarrow 0$ as $n \rightarrow \infty$.

Hence
 $\lim_{n \rightarrow \infty} T(PQx_n, QQx_n, q) = 1 = \lim_{n \rightarrow \infty} T(QPx_n, QQx_n, q)$ and
 $\lim_{n \rightarrow \infty} T(QPx_n, PPx_n, q) = 1 = \lim_{n \rightarrow \infty} T(PQx_n, PPx_n, q)$.

Furthermore

Now $P_x = x^4$ and $Q_x = 4x$
 $PQ_x = 256x^4, QP_x = 4x^4$
 $PP_x = x^{16}, QQ_x = 16x^2$

Then for all $q > 0$
 $T(PQ_x, QP_x, q) \geq T(P_x, Q_x, q)$ is not true for all x

Now consider sequence $x_r = \frac{1}{r}$ $r = 1, 2, 3, \dots$
then $x_r \rightarrow 0, Px_r, Qx_r \rightarrow 0$
and $PQx_n, QPx_n, PP_x, QQ_x \rightarrow 0$ as $n \rightarrow \infty$

Hence
 $\lim_{n \rightarrow \infty} T(PQx_n, QQx_n, q) = 1 = \lim_{n \rightarrow \infty} T(QPx_n, QQx_n, q)$ and
 $\lim_{n \rightarrow \infty} T(QPx_n, PPx_n, q) = 1 = \lim_{n \rightarrow \infty} T(PQx_n, PPx_n, q)$.

Result-1: Let P and Q be continuous self mappings of a fuzzy metric space (X, T, ●). Then P and Q are compatible if and only if they are weak compatible of type (A).

Result-2: Let P and Q be continuous self mappings of a fuzzy metric space (X, T, ●). Then P and Q are compatible of type (A) if and only if P and Q are weak compatible of type (A).

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