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FUZZY METRIC SPACE FOR WEAKLY COMPATIBLE OF TYPE (A)

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ABSTRACT

The aim of this paper is to obtain a common fixed point for weakly compatible of type (A) in Fuzzy metric space.

***Keywords**: common fixed point, weakly compatible of type (A), fuzzy metric space.*

1. INTRODUCTION AND PRELIMINARIES

In 1965 Zadeh [1] at the university of California U. S.A. proposed a way for mathematization of imprecisely described phenomena of introducing the concept of Fuzzy set.

Deng [6] defined a Fuzzy metric space by assigning a non negative real umber for every pair of Fuzzy point in anon empty set X and satisfying certain conditions. The idea of weak compatible maps of type (A) defined by Kramosil and Michalek [2], which was further modified by George and Veramani [3] as used by Pathak Kang and Beak [4] in Menger space. We extend the result of singh and chouhan [5] in Fuzzy metric space.

Definition 1.1: Self mappings P and Q of a Fuzzy metric space (X, T, ●) are said to be compatible of type (A), if $\lim_{n\to\infty} T(QPx_n, PPx_n, q) = 1$, $\lim_{n\to\infty} T(PQx_n, QQx_n, q) = 1$ for all q > 0, where $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = u$ for some u ε X.

Definition 1.2: Self mappings P and Q of a Fuzzy metric space (X, T, ●) are said to be weak compatible of type (A), if

$\lim_{n\to\infty} T(QPx_n, PPx_n, q) \geq \lim_{n\to\infty} T(PQx_n, PPx_n, q)$ and

$\lim_{n\to\infty} T(PQx_n, QQx_n, q) \geq \lim_{n\to\infty} T(QPx_n, QQx_n, q)$ for all q > 0

where $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n =$ u for some u ε X.

Preposition: Let P and Q be continuous, self maps of Fuzzy metric space (X, T ,●). Then commutativity implies weak compatible of type (A), but not conversely.

Proof: Let P and Q be self continuous and commuting maps of a Fuzzy metric space (X, T, ●). Now if $\{x_n\}$ is any sequence in X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n =$ u for some uε X, then by continuity of P,

$PPx_n, QQx_n \to P_u$ for q > 0.

$\lim_{n\to\infty} T(QPx_n, PPx_n) \geq \lim_{n\to\infty} T(QPx_n, PQx_n, q/2) \bullet \lim_{n\to\infty} T(PQx_n, PPx_n, q/2) = 1$

i. e. $\lim_{n\to\infty} T(QPx_n, PPx_n, q) \geq \lim_{n\to\infty} T(PQx_n, PPx_n, q)$

Similarly,

$\lim_{n\to\infty} T(PQx_n, QQx_n, q) \geq \lim_{n\to\infty} T(QPx_n, PPx_n, q)$

Hence P and Q are weak compatible maps of type (A).

For the converse part let (X, T, ●) is a Fuzzy metric space, where X = [0, 1]. Define self maps P and Q as $P_{x=\frac{x}{a}} Q_{x=\frac{x}{x+b}}$ for all x ε [0 ,1]

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Now PQ_x = P ($\frac{x}{x+b}$) = $\frac{x}{a\,(x+b)}$ and

QP$_x$ = Q($\frac{x}{a}$) = $\frac{x/a}{\frac{x}{a}+b}$ = $\frac{x}{x+ab}$

Hence PQ ≠ QP but, if x_r = $\frac{1}{r}$ r = 1, 2, 3...... then x_r → 0

Qx_r → 0, Px_r → 0 as r → ∞ then

PQx_n = ($\frac{x_n}{a(x_{n+b})}$) → 0 and QPx_n = $\frac{x_n}{x_{n+b}}$ → 0

PPx_n = P $\frac{x_n}{a}$ = $\frac{x_n}{a^2}$ → 0 and QQx_n = Q($\frac{x_n}{x_{n+b}}$) → 0 as n→ ∞ .

Hence

$\lim_{n\to\infty} T(PQx_n$,QQ$x_{n,}$q) = 1 = $\lim_{n\to\infty} T(QPx_n$,QQ$x_{n,}$q) and

$\lim_{n\to\infty} T(QPx_n$,PP$x_{n,}$q) = 1 = $\lim_{n\to\infty} T(PQx_n$,PP$x_{n,}$q).

Furthermore

Now P_x = x^4 and Q_x = 4x

P Q_x = 256 x^4, Q P_x = 4x^4

P P_x= x^{16}, Q Q_x = 16 x^2

Then for all q > 0

T(P Q_x , Q P_x , q) ≥ T(P_x, Q_x , q) is not true for all x

Now consider sequence x_r = $\frac{1}{r}$ r = 1, 2, 3 ......

then x_r → 0, Px_r, Qx_r → 0

and PQx_n, QPx_n , P P_x, Q Q_x → 0 as n →∞

Hence

$\lim_{n\to\infty} T(PQx_n$,QQ$x_{n,}$q) = 1 = $\lim_{n\to\infty} T(QPx_n$,QQ$x_{n,}$q) and

$\lim_{n\to\infty} T(QPx_n$,PP$x_{n,}$q) = 1 = $\lim_{n\to\infty} T(PQx_n$,PP$x_{n,}$q).

Result-1: Let P and Q be continuous self mappings of a fuzzy metric space (X, T, ●). Then P and Q are compatible if and only if they are weak compatible of type (A).

Result-2: Let P and Q be continuous self mappings of a fuzzy metric space (X, T, ●). Then P and Q are compatible of type (A) if and only if P and Q are weak compatible of type (A).

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