International Journal of Mathematical Archive-2(10), 2011, Page: 1953-1962
Available online through www.ijma.info ISSN 2229 – 5046

UNION AND INTERSECTION OF FUZZY SETS AND FUZZY SOFT SETS: A GENERALIZED APPROACH

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(Received on: 07-10-11; Accepted on: 20-10-11)

ABSTRACT

In this paper, we generalize the notion of union and intersection of two fuzzy sets over the same universe initiated by H. K. Baruah and modified by Neog and Sut for two fuzzy sets over two universes respectively. Some related results have been proposed with supporting proofs and examples. We further generalize the notion of union and intersection of an arbitrary class of fuzzy sets taken over an arbitrary class of universes respectively. Finally we apply our views in case of fuzzy soft sets and verify certain properties according to our new notion. Our work is an attempt to generalize the notion of union and intersection of fuzzy sets and fuzzy soft sets.

Keywords: *Fuzzy Set, Fuzzy soft set, Union and intersection of fuzzy sets and fuzzy soft sets.*

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1. INTRODUCTION

H. K. Baruah [3] has reintroduced the theory of fuzzy sets from a new perspective. According to him, to represent a fuzzy set, two functions namely fuzzy membership function and fuzzy reference function are necessary. Accordingly, the notion of union and intersection of two fuzzy sets have been given a new direction which in turn gives us the Zadehian definitions [7] when we take the fuzzy reference function equal to zero. Neog and Sut [10] have further modified the definitions initiated by Baruah [3] so as to avoid degenerate cases. In this paper, an attempt has been made to generalize the notion of fuzzy union and intersection further to include fuzzy sets defined over different universes. We have compared our findings with the earlier works in case of usual fuzzy sets (where fuzzy reference function is equal to zero) put forward by Chakrabarty et al. [4].

In recent times, the theory of fuzzy soft set initiated by Maji et al [8] is a catching momentum in the context of many complicated problems arising in the fields of engineering, social science, economics, medical science etc involving uncertainties. The occurrence of union and intersection of two fuzzy sets in two fuzzy soft classes is very natural in many real life situations. We have already defined fuzzy soft union and intersection for two fuzzy soft sets defined over two fuzzy soft classes in [2]. In the present work, we have verified those results according to our new notion of fuzzy sets.

2. UNION AND INTERSECTION OF FUZZY SETS

In this section we furnish below some earlier works on union and intersection of two fuzzy sets defined over two universes in brief.

Chakrabarty et al. [4] generalized the notion of union and intersection of two fuzzy sets laid down by Zadeh [7]. In fact the concept of union of two fuzzy sets A and B in the universes X could be treated as a particular case of the following two fuzzy sets:
(i) Fuzzy set A in the universe X and
(ii) Fuzzy set B in the universe Y,
Where X and Y are two different universes, in general. Thus the definitions of union and intersection of two fuzzy sets as proposed in [4] are as follows:

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Definition: 2.1 [4] Let *A* be a fuzzy set of *X* with membership function μ_A and *B* be a fuzzy set of *Y* with membership function μ_B. Then the union of two fuzzy sets *A* and *B* denoted by $A \cup B$ is a fuzzy set of $X \cup Y$ with the membership function defined by

$$\mu_{A \cup B}(z) = \begin{cases} \mu_A(z) & \text{if } z \in X - Y \\ \mu_B(z) & \text{if } z \in Y - X \\ \max\{\mu_A(z), \mu_B(z)\} & \text{if } z \in X \cap Y \end{cases}$$

It may be observed that the fuzzy union defined by Zadeh [7] is a particular case of the above defined union when *X* = *Y*.

Definition: 2.2 [4] Let *A* be a fuzzy set of *X* with membership function μ_A and *B* be a fuzzy set of *Y* with membership function μ_B. Then the intersection of two fuzzy sets *A* and *B* denoted by $A \cap B$ is a fuzzy set of $X \cup Y$ with the membership function defined by

$$\mu_{A \cap B}(z) = \min\{\mu_A(z), \mu_B(z)\}, \qquad \forall z \in X \cup Y$$

It may be observed that the fuzzy intersection defined by Zadeh [7] is a particular case of the above defined union when *X* = *Y*.

It may also be observed that $\forall z \in (X \Delta Y), \mu_{A \tilde{\cap} B}(z) = 0$ and in this sense $A \cap B$ is a fuzzy set in $X \cap Y$.

Definition: 2.3 [5, 9] Let *U* be a set and *X* be a subset of *U*. Then for any fuzzy set *A* of *X*, the fuzzy set μ_A^U of *U* given by

$$\mu_A^U(x) = \begin{cases} \mu_A(x), & \forall\, x \in X \\ 0, & \text{otherwise} \end{cases}$$

is called the 'fuzzy set of *U* generated by μ_A'.

Chakrabarty et al [4] gave the following proposition:

Proposition: 2.1 [4] For any two fuzzy sets *A* and *B* of the sets *X* and *Y*, respectively, the following holds:
(i) If $A = \varphi, B = \varphi$, then $A \cap B = \varphi$ but not conversely;
(ii) $\varphi \cup \varphi = \varphi, A \cup \varphi = A$, $A \cap \varphi = \varphi$, where φ is the null fuzzy set.
(iii) $A \cup B = B \cup A$, $A \cap B = B \cap A$

Chakrabarty et al [6] put forward the following proposition for fuzzy sets *A*, *B, C* obtained from three different universes *X*, *Y* and *Z* respectively.

Proposition: 2.2 [6] Let *A*,*B*,*C* be three fuzzy sets obtained from three different universes *X*,*Y* and *Z* respectively. Then the following holds.

(i) $A \cup (B \cup C) = (A \cup B) \cup C$
(ii) $A \cap (B \cap C) = (A \cap B) \cap C$
(iii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
(iv) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

H. K. Baruah [3] reintroduced the notion of fuzzy sets with the help of fuzzy membership function and fuzzy reference function. He pointed out that the fuzzy membership value is different from fuzzy membership function and put forward a new definition of union and intersection of two fuzzy sets. Neog and Sut [10] further modified the definition initiated by Baruah [3] so as to avoid degenerate cases as follows.

Definition: 2.4[10] Let $A(\mu_1,\mu_2)=\{x,\mu_1(x),\mu_2(x);x\in U\}$ and $B(\mu_3,\mu_4)=\{x,\mu_3(x),\mu_4(x);x\in U\}$ be two fuzzy sets defined over the same universe *U*. To avoid degenerate cases we assume that

$$\min(\mu_1(x),\mu_3(x))\geq \max(\mu_2(x),\mu_4(x))\forall x\in U\,.$$

Then the operation intersection is defined as

$$A(\mu_1,\mu_2)\cap B(\mu_3,\mu_4)=\{x,\min(\mu_1(x),\mu_3(x)),\max(\mu_2(x),\mu_4(x));x\in U\}$$

If for some $x\in U$, $\min(\mu_1(x),\mu_3(x))<\max(\mu_2(x),\mu_4(x))$, then our conclusion is that $A\cap B=\varphi$. If for some $x\in U$, $\min(\mu_1(x),\mu_3(x))=\max(\mu_2(x),\mu_4(x))$, then also $A\cap B=\varphi$.

Further, we define the operation union, with $\min(\mu_1(x),\mu_3(x))\geq \max(\mu_2(x),\mu_4(x))\forall x\in U$ as

$$A(\mu_1,\mu_2)\cup B(\mu_3,\mu_4)=\{x,\max(\mu_1(x),\mu_3(x)),\min(\mu_2(x),\mu_4(x));x\in U\}.$$

Also, our another conclusion is, that if for some $x\in U$, $\min(\mu_1(x),\mu_3(x))<\max(\mu_2(x),\mu_4(x))$, then the union of the fuzzy sets *A* and *B* cannot be expressed as one single fuzzy set.

The union, however, can be expressed in one single fuzzy set if

$$\min(\mu_1(x),\mu_3(x))=\max(\mu_2(x),\mu_4(x)).$$

Neog and Sut [10] put forward the definition of arbitrary fuzzy union and intersection in the same universe as follows.

Definition: 2.5 [10] Let $\Im=\{A_i(\mu_{i1},\mu_{i2})\mid i\in I\}$ be a family of fuzzy sets over the same universe *U*. To avoid degenerate cases we assume that $\min(\mu_{i1}(x))\geq \max(\mu_{i2}(x))\forall x\in U$. Then the union of fuzzy sets in $\Im$ is a fuzzy set given by

$$\bigcup_i A_i(\mu_{i1},\mu_{i2})=\{x,\max(\mu_{i1}(x)),\min(\mu_{i2}(x));x\in U\}.$$

And the intersection of fuzzy sets in $\Im$ is a fuzzy set given by

$$\bigcap_i A_i(\mu_{i1},\mu_{i2})=\{x,\min(\mu_{i1}(x)),\max(\mu_{i2}(x));x\in U\}$$

In the next section we are giving the definition of union and intersection of two fuzzy sets defined over two universes and then we generalize the same for an arbitrary class of fuzzy sets defined over an arbitrary class of universes respectively.

3. UNION AND INTERSECTION OF FUZZY SETS DEFINED OVER DIFFERENT UNIVERSES

Definition: 3.1 Let $A(\mu_1,\mu_2)=\{x,\mu_1(x),\mu_2(x);x\in X\}$ and $B(\chi_1,\chi_2)=\{x,\chi_1(x),\chi_2(x);x\in Y\}$ be two fuzzy sets defined over the universes *X* and *Y* respectively. Then the union of these two fuzzy sets $A(\mu_1,\mu_2)\cup B(\chi_1,\chi_2)$ is a fuzzy set $C(\eta_1,\eta_2)$ defined over the universe $X\cup Y$ defined as

$$C(\eta_1,\eta_2)=\begin{cases}\{x,\mu_1(x),\mu_2(x)\} & \text{if } x\in X-Y\\ \{x,\chi_1(x),\chi_2(x)\} & \text{if } x\in Y-X\\ \{x,\max(\mu_1(x),\chi_1(x)),\min(\mu_2(x),\chi_2(x))\} & \text{if } x\in X\cap Y\end{cases}$$

In order to avoid degenerate cases, we assume that

$$\min(\mu_1(x), \chi_1(x)) \geq \max(\mu_2(x), \chi_2(x)) \forall x \in X \cap Y$$

Definition 3.2 Let $A(\mu_1, \mu_2) = \{x, \mu_1(x), \mu_2(x); x \in X\}$ and $B(\chi_1, \chi_2) = \{x, \chi_1(x), \chi_2(x); x \in Y\}$ be two fuzzy sets defined over the universes *X* and *Y* respectively. Then the intersection of these two fuzzy sets $A(\mu_1, \mu_2) \cap B(\chi_1, \chi_2)$ is a fuzzy set $C(\eta_1, \eta_2)$ defined over the universe $X \cup Y$ defined as

$$C(\eta_1, \eta_2) = \{x, \min(\mu_1(x), \chi_1(x)), \max(\mu_2(x), \chi_2(x))\} \forall x \in X \cup Y \text{ with}$$
$$\mu_1(x) = \mu_2(x) = \chi_2(x) \text{ if } x \notin X \text{ and } \chi_1(x) = \chi_2(x) = \mu_2(x) \text{ if } x \notin Y$$

Naturally, in order to avoid degenerate cases, we assume that

$$\min(\mu_1(x), \chi_1(x)) \geq \max(\mu_2(x), \chi_2(x)) \forall x \in X \cap Y$$

It can be seen that our definitions give the definitions of union and intersection of two fuzzy sets proposed by Baruah [3] and modified by Neog and Sut [10] whenever *X* = *Y*. Also the definitions of union and intersection of two fuzzy sets in two universes initiated by Chakrabarty et al [4] can be obtained as particular cases of ours if we take $\mu_2(x) = \eta_2(x) = 0 \forall x$.

Example: 3.1 Let $X = \{a, b, c\}$ and $Y = \{b, d, e\}$ be two universes. We take two fuzzy sets *A* and *B* over *X* and *Y* respectively as –

$$A = \{(a, 0.6, 0.1), (b, 0.5, 0.2), (c, 0.7, 0.3)\} \text{ and } B = \{(b, 0.7, 0.3), (d, 0.7, 0.2), (e, 0.6, 0.1)\} .$$

Then $A \cup B \quad = \{(a, 0.6, 0.1), (b, 0.7, 0.2), (c, 0.7, 0.3), (d, 0.7, 0.2), (e, 0.6, 0.1)\}$

And $A \cap B \quad = \{(a, 0.1, 0.1), (b, 0.5, 0.3), (c, 0.3, 0.3), (d, 0.2, 0.2), (e, 0.1, 0.1)\}$

Remark: 3.1 It may be observed that $\forall x \in X \Delta Y$, fuzzy membership value of *x* is 0 and in this sense $A(\mu_1, \mu_2) \cap B(\chi_1, \chi_2)$ is a fuzzy set over $X \cap Y$. Above example makes this clear.

We now give the following De Morgan Laws for two fuzzy sets $A(\mu_1, \mu_2) = \{x, \mu_1(x), \mu_2(x); x \in X\}$ and $B(\chi_1, \chi_2) = \{x, \chi_1(x), \chi_2(x); x \in Y\}$ defined over the universes *X* and *Y* respectively.

Proposition: 3.1 Let $A(\mu_1, \mu_2) = \{x, \mu_1(x), \mu_2(x); x \in X\}$ and $B(\chi_1, \chi_2) = \{x, \chi_1(x), \chi_2(x); x \in Y\}$ be two fuzzy sets defined over the universes *X* and *Y* respectively. Then the following De Morgan Laws are valid.

(i) $\quad (A(\mu_1, \mu_2) \cup B(\chi_1, \chi_2))^c = (A(\mu_1, \mu_2))^c \cap (B(\chi_1, \chi_2))^c$

(ii) $\quad (A(\mu_1, \mu_2) \cap B(\chi_1, \chi_2))^c = (A(\mu_1, \mu_2))^c \cup (B(\chi_1, \chi_2))^c$

Where $(A(\mu_1, \mu_2) \cup B(\chi_1, \chi_2))^c$ and $(A(\mu_1, \mu_2) \cap B(\chi_1, \chi_2))^c$ are the complements of $A(\mu_1, \mu_2) \cup B(\chi_1, \chi_2)$ and $A(\mu_1, \mu_2) \cap B(\chi_1, \chi_2)$ in $X \cup Y$ respectively, $(A(\mu_1, \mu_2))^c$ is the complement of $A(\mu_1, \mu_2)$ in *X* and $(B(\chi_1, \chi_2))^c$ is the complement of $B(\chi_1, \chi_2)$ in *Y*.

Proof: (i) $A(\mu_1, \mu_2) \cup B(\chi_1, \chi_2)$ is a fuzzy set $C(\eta_1, \eta_2)$ defined over the universe $X \cup Y$ defined as

$$C(\eta_1, \eta_2) = \begin{cases} \{x, \mu_1(x), \mu_2(x)\} & \text{if } x \in X - Y \\ \{x, \chi_1(x), \chi_2(x)\} & \text{if } x \in Y - X \\ \{x, \max(\mu_1(x), \chi_1(x)), \min(\mu_2(x), \chi_2(x))\} & \text{if } x \in X \cap Y \end{cases}$$

In order to avoid degenerate cases, we assume that

$$\min(\mu_1(x), \chi_1(x)) \geq \max(\mu_2(x), \chi_2(x)) \forall x \in X \cap Y$$

Case I. When $x \in X - Y$

$$\begin{aligned}
(A(\mu_1, \mu_2) \cup B(\chi_1, \chi_2))^c &= \{x, \mu_1(x), \mu_2(x)\}^c \\
&= \{x, \mu_2(x), 0\} \cup \{x, 1, \mu_1(x)\}
\end{aligned}$$

$$\begin{aligned}
(A(\mu_1, \mu_2))^c \cap (B(\chi_1, \chi_2))^c &= \{x, \mu_1(x), \mu_2(x)\}^c \cap \{x, \mu_2(x), \mu_2(x)\}^c \\
&= \{\{x, \mu_2(x), 0\} \cup \{x, 1, \mu_1(x)\}\} \cap X \\
&= \{x, \mu_2(x), 0\} \cup \{x, 1, \mu_1(x)\}
\end{aligned}$$

Case II. When $x \in Y - X$

$$\begin{aligned}
(A(\mu_1, \mu_2) \cup B(\chi_1, \chi_2))^c &= \{x, \chi_1(x), \chi_2(x)\}^c \\
&= \{x, \chi_2(x), 0\} \cup \{x, 1, \chi_1(x)\}
\end{aligned}$$

$$\begin{aligned}
(A(\mu_1, \mu_2))^c \cap (B(\chi_1, \chi_2))^c &= \{x, \chi_1(x), \chi_2(x)\}^c \cap \{x, \chi_2(x), \chi_2(x)\}^c \\
&= \{\{x, \chi_2(x), 0\} \cup \{x, 1, \chi_1(x)\}\} \cap Y \\
&= \{x, \chi_2(x), 0\} \cup \{x, 1, \chi_1(x)\}
\end{aligned}$$

Case III. When $x \in X \cap Y$

$$\begin{aligned}
(A(\mu_1, \mu_2) \cup B(\chi_1, \chi_2))^c &= \{x, \max(\mu_1(x), \chi_1(x)), \min(\mu_2(x), \chi_2(x))\}^c \\
&= \{x, \min(\mu_2(x), \chi_2(x)), 0\} \cup \{x, 1, \max(\mu_1(x), \chi_1(x))\}
\end{aligned}$$

$$\begin{aligned}
(A(\mu_1, \mu_2))^c \cap (B(\chi_1, \chi_2))^c &= \{x, \mu_1(x), \mu_2(x)\}^c \cap \{x, \chi_1(x), \chi_2(x)\}^c \\
&= \{\{x, \mu_2(x), 0\} \cup \{x, 1, \mu_1(x)\}\} \cap \{\{x, \chi_2(x), 0\} \cup \{x, 1, \chi_1(x)\}\} \\
&= [\{\{x, \mu_2(x), 0\} \cup \{x, 1, \mu_1(x)\}\} \cap \{x, \chi_2(x), 0\}] \\
&\quad \cup [\{\{x, \mu_2(x), 0\} \cup \{x, 1, \mu_1(x)\}\} \cap \{x, 1, \chi_1(x)\}] \\
&= [\{\{x, \mu_2(x), 0\} \cap \{x, \chi_2(x), 0\}\} \cup \{\{x, 1, \mu_1(x)\} \cap \{x, \chi_2(x), 0\}\}] \\
&\quad \cup [\{\{x, \mu_2(x), 0\} \cap \{x, 1, \chi_1(x)\}\} \cup \{\{x, 1, \mu_1(x)\} \cap \{x, 1, \chi_1(x)\}\}] \\
&= [\{x, \min(\mu_2(x), \chi_2(x)), 0\} \cup \varphi] \cup [\varphi \cup \{x, 1, \max(\mu_1(x), \chi_1(x))\}] \\
&= \{x, \min(\mu_2(x), \chi_2(x)), 0\} \cup \{x, 1, \max(\mu_1(x), \chi_1(x))\}
\end{aligned}$$

Thus in all the three cases, $(A(\mu_1, \mu_2) \cup B(\mu_3, \mu_4))^c = (A(\mu_1, \mu_2))^c \cap (B(\mu_3, \mu_4))^c$

We have assumed that

$\min(\mu_1(x), \chi_1(x)) \geq \max(\mu_2(x), \chi_2(x)) \forall x \in X \cap Y$, so
$\chi_2(x) = \min(1, \chi_2(x)) \leq \max(\mu_1(x), 0) = \mu_1(x) \forall x \in X \cap Y$ and as such $\{x, 1, \mu_1(x)\} \cap \{x, \chi_2(x), 0\} = \varphi$.

Similarly
$\mu_2(x) = \min(\mu_2(x), 1) \leq \max(0, \chi_1(x)) = \chi_1(x) \forall x \in X \cap Y$ and hence $\{x, \mu_2(x), 0\} \cap \{x, 1, \chi_1(x)\} = \varphi$

Proof of (ii) is similar to this.

Proposition: 3.2 Let $A(\mu_1,\mu_2)=\{x,\mu_1(x),\mu_2(x);x\in X\}$ and $B(\chi_1,\chi_2)=\{x,\chi_1(x),\chi_2(x);x\in Y\}$ be two fuzzy sets defined over the universes *X* and *Y* respectively. If $A(\mu_1,\mu_2)=\varphi$, $B(\chi_1,\chi_2)=\varphi$ then $A(\mu_1,\mu_2)\cap B(\chi_1,\chi_2)=\varphi$ but the converse result is not always true.

Proof: The proof is straight forward. For the converse part we cite the following example.

Example: 3.2 Let $X=\{a,b,c\}$, $Y=\{b,c,d,e\}$. We take the fuzzy sets

$A(\mu_1,\mu_2)=\{(a,0.6,0.1),(b,0.7,0.2),(c,0.2,0)\}\neq\varphi$,

$B(\chi_1,\chi_2)=\{(b,0.2,0.1),(c,0.8,0.2),(d,0.7,0.3),(e,0.5,0)\}\neq\varphi$ over X,Y respectively .

$$A(\mu_1,\mu_2)\cap B(\chi_1,\chi_2)=\{(a,0.1,0.1),(b,0.2,0.2),(c,0.2,0.2),(d,0.3,0.3),(e,0,0)\}$$
$$=\varphi$$

Proposition: 3.3 Let $A(\mu_1,\mu_2)=\{x,\mu_1(x),\mu_2(x);x\in X\}$, $B(\chi_1,\chi_2)=\{x,\chi_1(x),\chi_2(x);x\in Y\}$ and $C(\eta_1,\eta_2)=\{x,\eta_1(x),\eta_2(x);x\in Z\}$ be three fuzzy sets defined over the universes *X*,*Y* and *Z* respectively. Then the following results are valid.

(i) $$A(\mu_1,\mu_2)\cup B(\chi_1,\chi_2)=B(\chi_1,\chi_2)\cup A(\mu_1,\mu_2)$$
$$A(\mu_1,\mu_2)\cap B(\chi_1,\chi_2)=B(\chi_1,\chi_2)\cap A(\mu_1,\mu_2)$$

(ii) $$A(\mu_1,\mu_2)\cup(B(\chi_1,\chi_2)\cup C(\eta_1,\eta_2))=(A(\mu_1,\mu_2)\cup B(\chi_1,\chi_2))\cup C(\eta_1,\eta_2)$$
$$A(\mu_1,\mu_2)\cap(B(\chi_1,\chi_2)\cap C(\eta_1,\eta_2))=(A(\mu_1,\mu_2)\cap B(\chi_1,\chi_2))\cap C(\eta_1,\eta_2)$$

(iii) $$A(\mu_1,\mu_2)\cup(B(\chi_1,\chi_2)\cap C(\eta_1,\eta_2))=(A(\mu_1,\mu_2)\cup B(\chi_1,\chi_2))\cap(A(\mu_1,\mu_2)\cup C(\eta_1,\eta_2))$$
$$A(\mu_1,\mu_2)\cap(B(\chi_1,\chi_2)\cup C(\eta_1,\eta_2))=(A(\mu_1,\mu_2)\cap B(\chi_1,\chi_2))\cup(A(\mu_1,\mu_2)\cap C(\eta_1,\eta_2))$$

Definition: 3.3 Let *U* be an initial universe and $\{X_i\}$ be a class of subsets of *U i.e.* $X_i\subseteq U\ \forall i$ with $\bigcup_i X_i=U$. Let $A_i(\mu_{i1},\mu_{i2})$ be fuzzy sets defined over X_i respectively. Then the fuzzy set of *U* generated by A_i would be denoted by $A_i(\mu_{i1},\mu_{i2})^U$ and defined as

$$A_i(\mu_{i1},\mu_{i2})^U=\begin{cases}\{x,\mu_{i1}(x),\mu_{i2}(x)\} & \text{if } x\in X_i\\ \left\{x,\max\limits_{\substack{k\\ x\in X_k}}\mu_{k2}(x),\max\limits_{\substack{k\\ x\in X_k}}\mu_{k2}(x)\right\} & \text{if } x\notin X_i, i\neq k\end{cases}$$

Example: 3.3 Let $U=\{a,b,c,d,e,f\}$ be an initial universe. Let $X_1=\{a,b,c\}$, $X_2=\{c,d,e\}$ and $X_3=\{a,c,e,f\}$ be three subsets of *U*. We take three fuzzy sets

$A_1(\mu_{11},\mu_{12})=\{(a,0.6,0.1),(b,0.7,0.2),(c,0.9,0)\}$,
$A_2(\mu_{21},\mu_{22})=\{(c,0.8,0.1),(d,0.7,0.3),(e,0.5,0)\}$ and
$A_3(\mu_{31},\mu_{32})=\{(a,0.7,0.2),(c,0.7,0.2),(e,0.4,0.1),(f,0.6,0.2)\}$ over X_1,X_2 and X_3 respectively . Then
$A_1(\mu_{11},\mu_{12})^U=\{(a,0.6,0.1),(b,0.7,0.2),(c,0.9,0),(d,0.3,0.3),(e,0.1,0.1),(f,0.2,0.2)\}$
$A_2(\mu_{21},\mu_{22})^U=\{(a,0.2,0.2),(b,0.2,0.2),(c,0.8,0.1),(d,0.7,0.3),(e,0.5,0),(f,0.2,0.2)\}$ and

$A_3(\mu_{31},\mu_{32})^U = \{(a,0.7,0.2),(b,0.2,0.2),(c,0.7,0.2),(d,0.3,0.3),(e,0.4,0.1),(f,0.6,0.2)\}$

Definition: 3.4 Let $\Im = \{A_i(\mu_{i1},\mu_{i2}) \mid i \in I\}$ be a class of fuzzy sets defined over universes X_i respectively. To avoid degenerate cases we assume that

$$\min(\mu_{j1}(x),\mu_{k1}(x)) \geq \max(\mu_{j2}(x),\mu_{k2}(x)) \ \forall x \in X_j \cap X_k .$$

Then their union $\underset{i}{\cup} A_i(\mu_{i1},\mu_{i2})$ is a fuzzy set over $\underset{i}{\cup} X_i$ defined as

$$\underset{i}{\cup} A_i(\mu_{i1},\mu_{i2}) = \{x, \max_i \mu_{i1}(x), \min_i \mu_{i2}(x)\} \text{ with } \mu_{i1}(x) = \mu_{i2}(x) = \min_{\substack{k \\ x \in X_k}} \mu_{k2}(x) \text{ if } x \notin X_i$$

Also the intersection $\underset{i}{\cap} A_i(\mu_{i1},\mu_{i2})$ is a fuzzy set over $\underset{i}{\cup} X_i$ defined as

$$\underset{i}{\cap} A_i(\mu_{i1},\mu_{i2}) = \{x, \min_i \mu_{i1}(x), \max_i \mu_{i2}(x)\} \text{ with } \mu_{i1}(x) = \mu_{i2}(x) = \max_{\substack{k \\ x \in X_k}} \mu_{k2}(x) \text{ if } x \notin X_i$$

We see that this definition yields our earlier definition [10] if we take $X_i = X_j \ \forall i, j$

Example: 3.4 Let $X_1 = \{a,b,c\}$, $X_2 = \{c,d,e\}$ and $X_3 = \{a,c,e,f\}$. We take three fuzzy sets

$A_1(\mu_{11},\mu_{12}) = \{(a,0.6,0.1),(b,0.7,0.2),(c,0.2,0)\}$,

$A_2(\mu_{21},\mu_{22}) = \{(c,0.8,0.1),(d,0.7,0.3),(e,0.5,0)\}$ and

$A_3(\mu_{31},\mu_{32}) = \{(a,0.7,0.2),(c,0.7,0.2),(e,0.4,0.1),(f,0.6,0.2)\}$ over X_1, X_2 and X_3 respectively . Then

$A_1(\mu_{11},\mu_{12}) \cup A_2(\mu_{21},\mu_{22}) \cup A_3(\mu_{31},\mu_{32})$

$= \{(a,0.7,0.1),(b,0.7,0.2),(c,0.8,0),(d,0.7,0.3),(e,0.5,0.1),(f,0.6,0.2)\}$

And

$A_1(\mu_{11},\mu_{12}) \cap A_2(\mu_{21},\mu_{22}) \cap A_3(\mu_{31},\mu_{32})$

$= \{(a,0.2,0.2),(b,0.2,0.2),(c,0.2,0.2),(d,0.3,0.3),(e,0.1,0.1),(f,0.2,0.2)\}$

$= \varphi$

It is clear from this example that $A_1(\mu_{11},\mu_{12}) \cap A_2(\mu_{21},\mu_{22}) \cap A_3(\mu_{31},\mu_{32}) = \varphi$ does not necessarily mean that $A_1(\mu_{11},\mu_{12}) = A_2(\mu_{21},\mu_{22}) = A_3(\mu_{31},\mu_{32}) = \varphi$

Proposition: 3.4 Let $\Im = \{A_i(\mu_{i1},\mu_{i2}) \mid i \in I\}$ be a class of fuzzy sets defined over universes X_i respectively. To avoid degenerate cases we assume that

$$\min(\mu_{j1}(x),\mu_{k1}(x)) \geq \max(\mu_{j2}(x),\mu_{k2}(x)) \ \forall x \in X_j \cap X_k .$$

Then the following De Morgan Laws are valid.

(i) $\{\underset{i}{\cup} A_i(\mu_{i1},\mu_{i2})\}^c = \underset{i}{\cap}\{A_i(\mu_{i1},\mu_{i2})\}^c$

(ii) $\{\underset{i}{\cap} A_i(\mu_{i1},\mu_{i2})\}^c = \underset{i}{\cup}\{A_i(\mu_{i1},\mu_{i2})\}^c$

4. UNION AND INTERSECTION OF TWO FUZZY SOFT SETS IN TWO FUZZY SOFT CLASSES

In view of our discussion in the preceding sections and our earlier definition of union and intersection of two fuzzy soft sets in two fuzzy soft classes [2], in this section, we proceed to apply these notions of fuzzy sets in the context of fuzzy soft sets. Our definition of union and intersection of two fuzzy soft sets in two fuzzy soft classes are as follows.

Definition: 4.1 Union of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in the fuzzy soft classes (*X*, *E*) and (*Y*, *E'*) respectively, is a fuzzy soft set (*H*, *C*) in the fuzzy soft class $(X \cup Y, E \cup E')$ where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon)=\begin{cases}[F(\varepsilon)]^{X\cup Y}, & \text{if } x\in A-B\\ [G(\varepsilon)]^{X\cup Y}, & \text{if } x\in B-A\\ F(\varepsilon)\cup G(\varepsilon), & \text{if } x\in A\cap B\end{cases}$$

and is written as $(F,A)\tilde{\cup}(G,B)=(H,C)$, where $[F(\varepsilon)]^{X\cup Y}$ is the fuzzy set of $X\cup Y$ generated by $F(\varepsilon)$ and $[G(\varepsilon)]^{X\cup Y}$ is the fuzzy set of $X\cup Y$ generated by $G(\varepsilon)$. It is to be noted that whenever $X = Y$, $[F(\varepsilon)]^{X\cup Y}=F(\varepsilon)$, which is a fuzzy set of X and $[G(\varepsilon)]^{X\cup Y}=G(\varepsilon)$, which is a fuzzy set of Y and then the above definition reduces to the form

$$H(\varepsilon)=\begin{cases}[F(\varepsilon)]^{X\cup X}, & \text{if } x\in A-B\\ [G(\varepsilon)]^{X\cup X}, & \text{if } x\in B-A\\ F(\varepsilon)\cup G(\varepsilon), & \text{if } x\in A\cap B\end{cases}$$

$$=\begin{cases}[F(\varepsilon)]^{X}, & \text{if } x\in A-B\\ [G(\varepsilon)]^{X}, & \text{if } x\in B-A\\ F(\varepsilon)\cup G(\varepsilon), & \text{if } x\in A\cap B\end{cases}$$

$$=\begin{cases}F(\varepsilon), & \text{if } x\in A-B\\ G(\varepsilon), & \text{if } x\in B-A\\ F(\varepsilon)\cup G(\varepsilon), & \text{if } x\in A\cap B\end{cases}$$

Which is nothing but the definition of union of two fuzzy soft sets in the same fuzzy soft class as laid down by Maji et al [8].

Definition: 4.2 Intersection of two fuzzy soft sets (F, A) and (G, B) in the fuzzy soft classes (X, E) and (Y, E') respectively, is a fuzzy soft set (H, C) in the fuzzy soft class $(X\cup Y, E\cup E')$ where $C=A\cap B$ with $A\cap B\neq\varphi$ and $\forall\varepsilon\in C$, $H(\varepsilon)=F(\varepsilon)\cap G(\varepsilon)$ and is written as $(F,A)\tilde{\cap}(G,B)=(H,C)$, where $F(\varepsilon)$ is a fuzzy set in X and $G(\varepsilon)$ is a fuzzy set in Y. It is to be noted that whenever $X = Y$, the above definition gives the intersection of two fuzzy soft sets in the same fuzzy soft class as laid down in [1].

We take one example below with fuzzy reference function function 0.

Example: 4.1 Let $X=\{c_1,c_2,c_3,c_4\}$ be the set of four cars manufactured by a certain Indian car manufacturer and $Y=\{c_3,c_4,c_5,c_6,c_7\}$ be the set of five sedan cars manufactured by the Indian car manufacturers.

Let $E=\{e_1(\text{costly}), e_2(\text{Beautiful}), e_3(\text{Fuel Efficient}), e_4(\text{Modern Technology}), e_5(\text{Luxurious})\}$ and

$E'=\{e_1(\text{costly}), e_2(\text{Beautiful}), e_6(\text{Sporty look})\}$ be the set of parameters

$A = \{e_1, e_2, e_3\}\subseteq E$ and $B = \{e_2, e_6\}\subseteq E'$.

We consider the fuzzy soft sets

$(F, A) = \{\, F(e_1) = \{ (c_1,0.9,0), (c_2,0.1,0), (c_3,0.4,0), (c_4,0.6,0) \},$

$F(e_2) = \{ (c_1,1,0), (c_2,0,0), (c_3,0.9,0), (c_4,0.5,0) \},$

$F(e_3) = \{ (c_1,0.8,0), (c_2,0.2,0), (c_3,0.7,0), (c_4,0.6,0) \}\,\}$ and

$(G, B) = \{ G(e_2) = \{(c_3,0.7,0), (c_4,0.2,0), (c_5,0.2,0), (c_6,0.7,0), (c_7,0.4,0) \},$
$G(e_6) = \{ (c_3,0.9,0), (c_4,0.6,0), (c_5,0.5,0), (c_6,1,0), (c_7,0.6,0) \}$ in the fuzzy soft classes (X, E) and (Y, E') respectively.

Then $(F,A)\tilde{\cup}(G,B)=(H,C)$, where $C = A \cup B = \{e_1,e_2,e_3,e_6\}$ and
$(H, C) = \{ H(e_1) = \{ (c_1,0.9,0), (c_2,0.1,0), (c_3,0.4,0), (c_4,0.6,0), (c_5,0,0), (c_6,0,0), (c_7,0,0)\},$
$H(e_2) = \{ (c_1,1,0), (c_2,0,0), (c_3,0.9,0), (c_4,0.5,0), (c_5,0.2,0), (c_6,0.7,0), (c_7,0.4,0)\},$
$H(e_3) = \{ (c_1,0.8,0), (c_2,0.2,0), (c_3,0.7,0), (c_4,0.6,0), (c_5,0,0), (c_6,0,0), (c_7,0,0) \},$
$H(e_6) =\{ (c_1,0,0), (c_2,0,0), (c_3,0.9,0), (c_4,0.6,0), (c_5,0.5,0), (c_6,1,0), (c_7,0.6,0) \}\}$

Again $(F,A)\tilde{\cap}(G,B)=(H,C)$, where $C = A \cap B = \{ e_2 \}$ and
$(H, C) = \{ H(e_2) = \{ (c_1,0,0), (c_2,0,0), (c_3,0.7,0), (c_4,0.2,0), (c_5,0,0), (c_6,0,0), (c_7,0,0) \}\}$

In [2], we have given the following propositions related to the union and intersection of two fuzzy soft sets in two fuzzy soft classes.

Proposition: 4.1 For two fuzzy soft sets (F, A) and (G, B) in the fuzzy soft classes (X, E) and (Y, E') respectively, the following holds:

(i) If $(F,A)=\tilde{\varphi}$, $(G,B)=\tilde{\varphi}$, then $(F,A)\tilde{\cap}(G,B)=\tilde{\varphi}$ but not conversely.

(ii) $\tilde{\varphi}\tilde{\cup}\tilde{\varphi}=\tilde{\varphi}, (F,A)\tilde{\cup}\tilde{\varphi}=(F,A), (F,A)\tilde{\cap}\tilde{\varphi}=\tilde{\varphi}$

(iii) $(F,A)\tilde{\cup}(G,B)=(G,B)\tilde{\cup}(F,A)$
$(F,A)\tilde{\cap}(G,B)=(G,B)\tilde{\cap}(F,A)$

One can verify that these results are valid under our new notions. However for the converse part of (i), we put forward the following example.

Example: 4.2
Let $X=\{c_1,c_2,c_3,c_4\}$ and $Y=\{c_3,c_4,c_5,c_6,c_7\}$.

Let $E=\{e_1,e_2,e_3,e_4,e_5\}$ and $E'=\{e_1,e_2,e_6\}$ be the set of parameters and $A = \{e_1, e_2, e_3\} \subseteq E$, $B = \{ e_2, e_6\} \subseteq E'$.
We consider the fuzzy soft sets

$(F, A) = \{ F(e_1) = \{ (c_1,0.9,0), (c_2,0.1,0), (c_3,0.4,0), (c_4,0.6,0) \},$
$F(e_2) = \{ (c_1,1,0), (c_2,0,0), (c_3,0.9,0.2), (c_4,0.5,0.2) \},$
$F(e_3) = \{ (c_1,0.8,0), (c_2,0.2,0), (c_3,0.7,0), (c_4,0.6,0) \}\}$ and

$(G, B) = \{ G(e_2) = \{(c_3,0.2,0), (c_4,0.2,0), (c_5,0.2,0), (c_6,0.7,0), (c_7,0.4,0) \},$
$G(e_6) = \{ (c_3,0.9,0), (c_4,0.6,0), (c_5,0.5,0), (c_6,1,0), (c_7,0.6,0) \}$ in the fuzzy soft classes (X, E) and (Y, E') respectively.
Here $(F,A)\tilde{\cap}(G,B)=(H,C)$, where $C = A \cap B = \{ e_2 \}$ and
$(H, C) = \{ H(e_2) = \{ (c_1,0,0), (c_2,0,0), (c_3,0.2,0.2), (c_4,0.2,0.2), (c_5,0,0), (c_6,0,0), (c_7,0,0) \}\}$

Thus $\forall \varepsilon \in C, H(\varepsilon)=\varphi$. It follows that $(H, C) =\tilde{\varphi}$

Proposition: 4.2 For three fuzzy soft sets (F, A), (G, B) and (H, C) in the fuzzy soft classes (X_1, E_1), (X_2, E_2) and (X_3, E_3) respectively with $A\cap B\cap C\neq\varphi$, the following holds:

(i) $(F,A)\tilde{\cup}((G,B)\tilde{\cup}(H,C))=((F,A)\tilde{\cup}(G,B))\tilde{\cup}(H,C)$

(ii) $(F,A)\tilde{\cap}((G,B)\tilde{\cap}(H,C))=((F,A)\tilde{\cap}(G,B))\tilde{\cap}(H,C)$

(iii) $(F,A)\tilde{\cap}((G,B)\tilde{\cup}(H,C)) = ((F,A)\tilde{\cap}(G,B))\tilde{\cup}((F,A)\tilde{\cap}(H,C))$

(iv) $(F,A)\tilde{\cup}((G,B)\tilde{\cap}(H,C)) = ((F,A)\tilde{\cup}(G,B))\tilde{\cap}((F,A)\tilde{\cup}(H,C))$

It can be verified that these results are valid under our new notions.

5. CONCLUSION

We have generalized the idea of union and intersection of fuzzy sets over the same universe proposed by Baruah [3] to an arbitrary class of fuzzy sets over an arbitrary class of universes respectively. We have compared our results with the one found in [4,10]. We have applied our new notions in case of fuzzy soft sets and verified a few propositions given in [2].We hope that our findings would help enhancing this study in fuzzy sets and fuzzy soft sets.

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