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ITERATIVE SPLINE APPROXIMATION FOR RECTANGULAR FIN WITH TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY

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ABSTRACT

Fins are broadly used to enhance heat transfer between primary surface and the environment in lots of industrial programs. The rate of heat transfer, temperature distribution and fin efficiency have been discussed in this paper. The performance of fins with temperature based thermal conductivity for rectangular profile is analyzed using Spline collocation method. Obtained effects by way of spline collocation method are in comparison with outcomes received by differential transform method (DTM).

***Keywords**: Fins, Variable Thermal Conductivity, Rectangular profile, Exponential profile, Spline Collocation Method, Heat Transfer co-efficient.*

***2010Mathematics Subject Classification**: 34K10, 41A15.*

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1. INTRODUCTION

Heat conduction analysis of fin is used in air conditioning devices, heat exchangers, gas turbine blades and vehicle radiators in lots of industrial programs. Many researchers [9, 12, 15] have contributed to analyze the models on heat and mass transfer mechanism for different types of fins.Kraus, Aziz A. and Welty J. [8] extensively present the evaluation of extended surface, heat transfer. To study the temperature distribution and analytical expression for the fin performance, Arslanturk [1] used decomposition method. Coskun and Atay [5,6] used variation iteration method to study convective straight and radial fins with temperature structured thermal conductivity.

Most of the physical phenomena defined via nonlinear differential equations do not have analytical answer. Therefore, the numerical techniques are widely used to solve these equations. Rajabi [11] obtained performance and fin temperature distribution with temperature structured thermal conductivity through Adomian Decomposition Method and Homotopy Perturbation Method. Birkhoff and Garabesian H. [3] worked on investigation of errors bounds for spline interpolation. Carl de Boor [7] established the concept of existence and uniqueness of Bicubic Splines. Bickley [2] introduced ahead a beneficial factor of spline functions in mild that can be employed to solve a linear two-point boundary value problem. Blue [4] discussed the applicability of spline features to nonlinear differential equations. Shah P.M [13] applied the spline technique for exponential to investigate the effect of various parameters with temperature dependent thermal conductivity.

In this paper, an approximate solution of the governing equation of the straight fin for rectangular profile with temperature dependent thermal conductivity is obtained by spline collocation method. Spline collocation outcomes are compared with numerical effects received via differential transformation method.

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2. FORMULATION OF THE PROBLEM

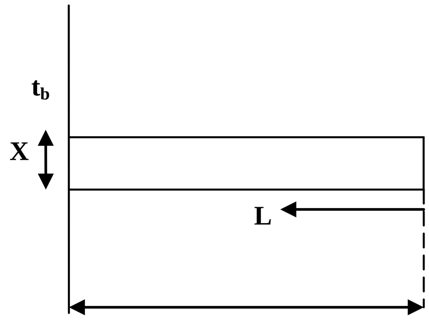

Figure -1: Rectangular Profile

The one- dimensional energy equation may be stated as: [10]

$$\frac{d}{dx}\left[k(T)A(x)\frac{dT}{dx}\right]-ph(T-T\infty)=0, \tag{2.1}$$

where

$$k(T)=k_b\left[1+\lambda\left(T-T_\infty\right)\right],\ \lambda \text{ is a constant} \tag{2.2}$$

The fin profile is described according to version of the fin thickness along its extended lengt has shown in figure-1.

The cross-section area of the fin can be expressed as,

$$A(x)=bt(x), \tag{2.3}$$

Using the subsequent dimensionless parameters:

$$\phi=\frac{T-T_\infty}{T-T_b},\quad \mathrm{X}=\frac{x}{L},\quad \mathrm{M}=\left(\frac{hpL^2}{K_bA_b}\right)^{\frac{1}{2}}, \tag{2.4}$$

The energy equation for rectangular profile is reduced into the following form.

$$\left(1+\beta\,\phi\right)\frac{\mathrm{d}^2\phi}{\mathrm{dX}^2}+\beta\left(\frac{\mathrm{d}\phi}{\mathrm{dX}}\right)^2-\mathrm{M}^2\phi=0\ , \tag{2.5}$$

where, $\beta=\lambda\left(T_b-T_\infty\right)$.

Boundary conditions are as follows:

$$X=0,\qquad \phi'=0, \tag{2.6}$$
$$\mathrm{X}=1,\qquad \phi=1.$$

3. SOLUTION OF THE PROBLEM

To obtain the solution, rewriting the differential equation (2.5) in the form,

$$\frac{\mathrm{d}^2\phi}{\mathrm{dX}^2}=-\frac{\beta\left(\frac{\mathrm{d}\theta}{\mathrm{dX}}\right)^2+M^2\phi}{\left(1+\beta\,\phi\right)} \tag{3.1}$$

with the boundary conditions (2.6).

Iterative spline collocation method is used to solve the above equation [14].

For an initial guess fit a straight line $y=mx+c$ through the boundary points, Here the straight line $\phi(x)=1$ can be fitted through the boundary conditions. Initially compute ϕ_i'' from equation (3.1) for $i=0,1,2,\ldots\mathrm{N}$ .Now the calculation of ϕ_i, $\mathrm{i}=0,1,2,..,\mathrm{N}$ is to be carried out through the solution of tridiagonal system of equations given by

$$\phi_0-\phi_1=\frac{h_1^2}{6}\left(2\phi_0^{''}+\phi_1^{''}\right)-h_i\phi_0^{'}$$

$$h_i\phi_{i+1}-(h_i+h_{i+1})\phi_i+h_{i+1}\phi_{i+1}\ =h_ih_{i+1}\left(\frac{h_i}{6}\phi_{i-1}^{''}+\frac{h_i+h_{i+1}}{3}\phi_i^{''}+\frac{h_{i+1}}{6}\phi_{i+1}^{''}\right),\ \ i=1,2,3,....n-1$$

$$\phi_{n-2}-2\phi_{n-1}=-\frac{h_n^2}{6}\left(\phi_{n-2}^{''}+4\phi_{n-1}^{''}+y_n''\right)-\phi_n$$

The coefficient matrix is a nonsingular matrix of dimension $(N+1)\,\mathrm{x}(\mathrm{N}+1)$.

Table-1: Comparison of the results obtained by spline with exact solution and results obtained by DTM of rectangular profile for $\beta=0$ and $M=1$, $M=0.5$ [10]

	$M=0.5,\beta=0$			$M=1,\beta=0$		
X	**Spline Collocation**	**Exact Solution**	**DTM**	**Spine Collocation**	**Exact Solution**	**DTM**
0	0.88681	0.88682	0.886819	0.648111	0.648054	0.648054
0.1	0.887919	0.88793	0.887928	0.651353	0.651297	0.651297
0.2	0.891248	0.89126	0.891257	0.66111	0.661059	0.661059
0.3	0.896806	0.89681	0.896815	0.67748	0.677436	0.677436
0.4	0.904606	0.90461	0.904615	0.700628	0.700594	0.700594
0.5	0.914669	0.91468	0.914677	0.730787	0.730763	0.730763
0.6	0.927019	0.92703	0.927026	0.76826	0.768246	0.768246
0.7	0.941688	0.94169	0.941694	0.813422	0.813418	0.813418
0.8	0.958711	0.95872	0.958716	0.866728	0.86673	0.86673
0.9	0.978132	0.97813	0.978135	0.928714	0.928718	0.928718
1	1	1	1	1	1	1

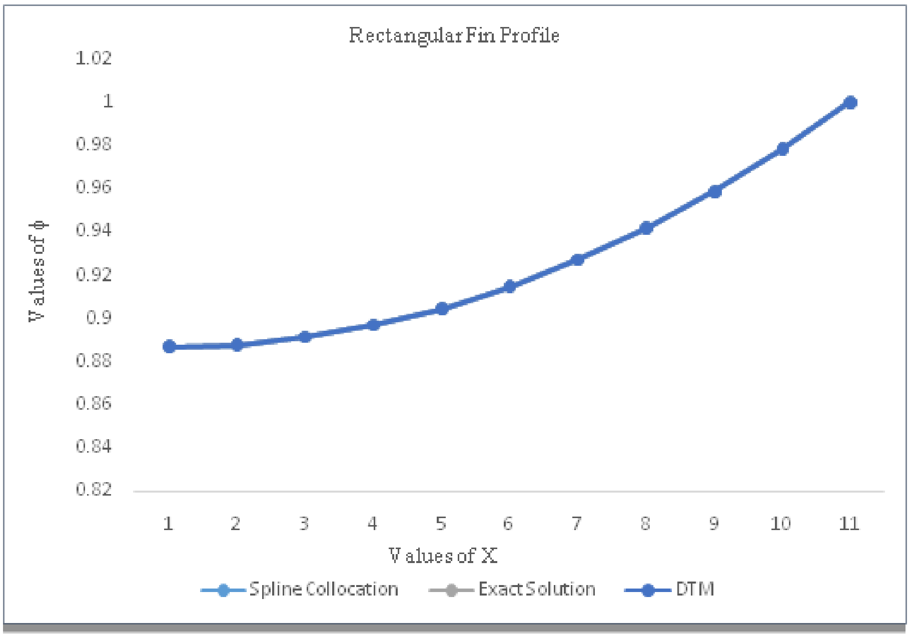

Figure-2: Temperature profile for $M=0.5,\beta=0$

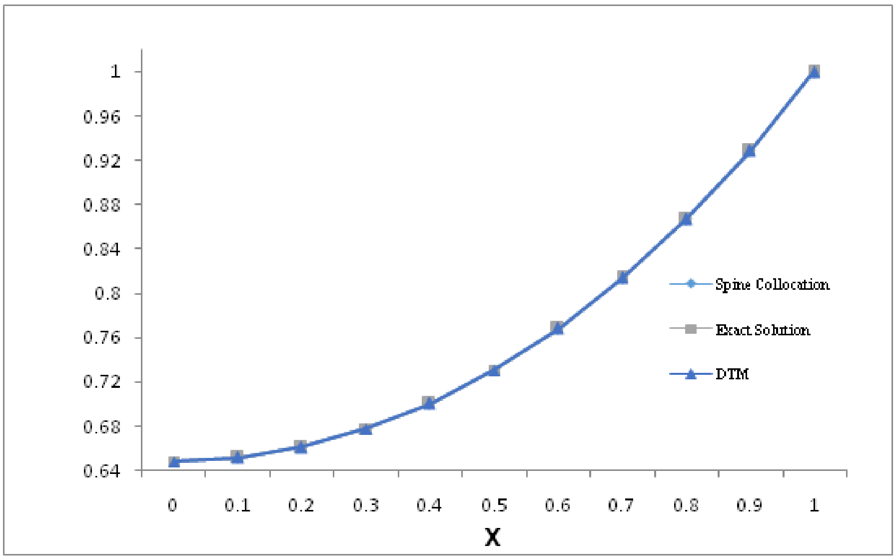

Figure-3: Temperature profile for $M = 1, \beta = 0$

Table-2: Spline Solution of Rectangular Profile Fin Problem at $M = 1, \beta = 0.5$

X	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$
0	0.666666	0.723466	0.729228	0.727678	0.728094	0.727982
0.1	0.669999	0.7261831	0.731887	0.730352	0.730764	0.730653
0.2	0.679999	0.734342	0.739874	0.738385	0.738785	0.738677
0.3	0.696666	0.747968	0.753214	0.751802	0.752181	0.752079
0.4	0.719999	0.767093	0.771940	0.770635	0.770985	0.770891
0.5	0.749999	0.791756	0.796092	0.794925	0.795238	0.795154
0.6	0.786666	0.822006	0.825716	0.824717	0.824985	0.824913
0.7	0.829999	0.857894	0.860862	0.860062	0.860277	0.860219
0.8	0.88	0.899480	0.901585	0.901017	0.901169	0.901129
0.9	0.936666	0.946826	0.947944	0.947642	0.947723	0.947701
1	1	1	1	1	1	1

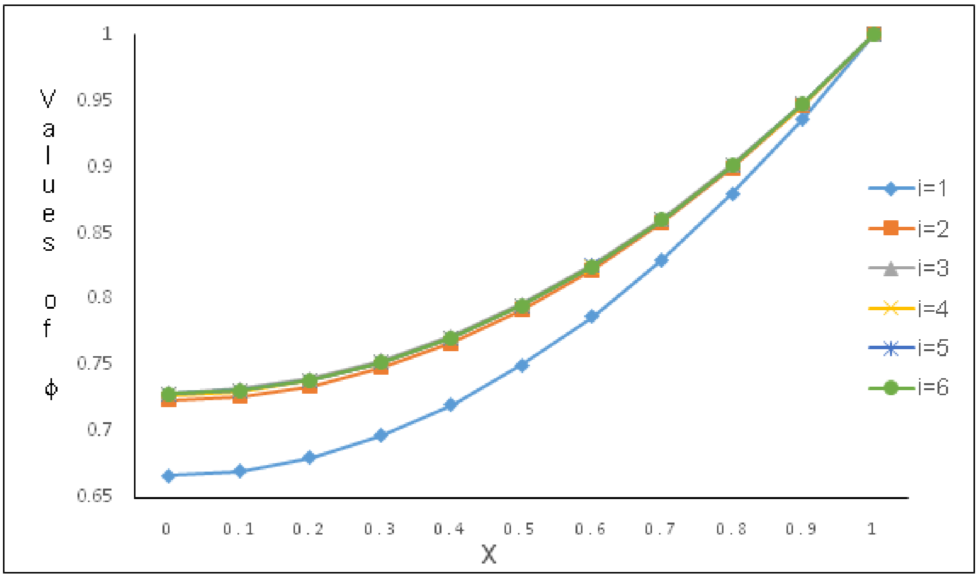

Figure-4: Iterative Spline Solution of Rectangular Profile Fin at $M = 1, \beta = 0.5$

Table-3: Iterative Spline Solution of Different Values of β and $M = 1$ for Rectangular Profile Fin Problem

X	$\beta = 1$	$\beta = 0.5$	$\beta = 0.3$	$\beta = 0.1$	$\beta = -0.1$	$\beta = -0.2$
0	0.781854	0.727982	0.702513	0.672916	0.642921	0.611112
0.1	0.784047	0.730653	0.705379	0.675971	0.646077	0.614492
0.2	0.790628	0.738677	0.713994	0.685163	0.655590	0.624682
0.3	0.801600	0.752079	0.728404	0.700574	0.671601	0.641846
0.4	0.816958	0.770892	0.748679	0.722335	0.694352	0.666266
0.5	0.836688	0.795154	0.774910	0.750629	0.724187	0.698358
0.6	0.860769	0.824913	0.80721	0.785688	0.761565	0.738684
0.7	0.889177	0.860219	0.845709	0.827796	0.807059	0.787970
0.8	0.921877	0.901129	0.890561	0.877287	0.861373	0.847135
0.9	0.958831	0.947701	0.941930	0.934543	0.925351	0.917332
1	1	1	1	1	1	1

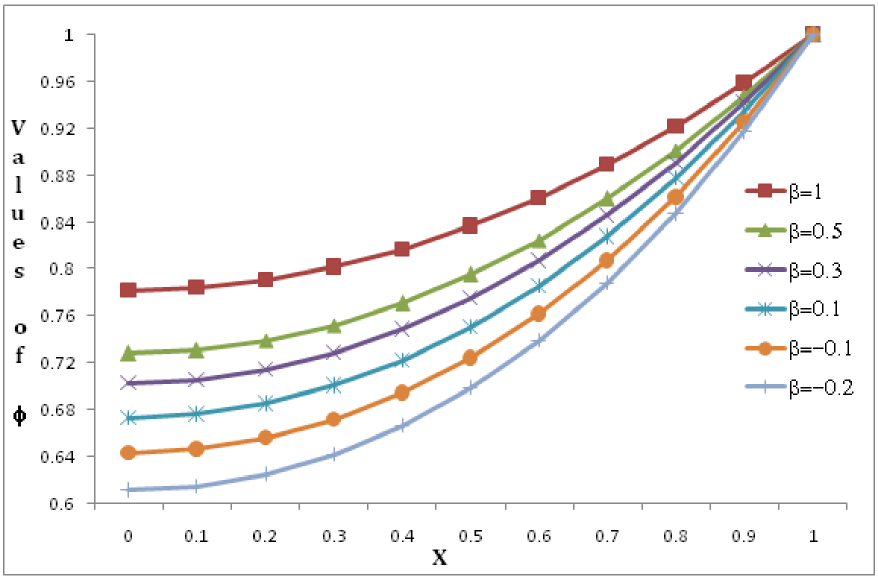

Figure-5: Temperature Distribution of Rectangular Profile Fin at Different Values of β and $M = 1$

4. RESULTS AND DISCUSSION

The comparison between the exact, differential transform method and results of spline collocation iterative method for rectangular profile fin at $\beta = 0$ and $M = 1, \beta = 0.5$ is shownintable-1.The convective heat transfer rate increases and fin tip temperature decreases, when M increases, it indicates that there is an oblique relation among the tip temperature and the value of M. The differential equation remains linear in the absence of parameter; on the other hand, the little contribution of this parameter leads to the non-linear differential equation. The convergent nature of the iterative method is visualized through aspects of representing results in table 2, for $\beta = 0.5$ and $M = 1$. The pictorial representation of this result is shown in figure 4.|For rectangular profile,the temperature distributions are presented for different values of β for $M = 1$,in figure 5. The obtained results show that, for rectangular profile in straight fin, with decreasing β, the fin base temperature decreases.

The consequences are compared with analytical answer and differential transform method. The consequences show that spline collocation approach has an excellent approximation. This approach delivers dependable effects in the form of numerical statistics and graph. Results of spline collocation method shows a good agreement with differential transform method.

5. CONCLUSION

From table-1 and table-2, comparison of the outcomes received with the aid of DTM and Spline answer and precise solution shows that the analytical approach and numerical facts are in a very good agreement with every other. The obtained results show that the solutions received via spline collocation method are also very closed to actual answer and the solutions acquired with the aid of DTM. The analysis is presented without doing any linearization for solving highly non-linear governing equations. The class of differential equations is simply too huge and obtaining their solutions is an essential task of such differential equations. In case, where an analytical function as the solution of a differential equation is not possible or rather too difficult to obtain, one has to move for obtaining approximate or numerical answers to the equation, which requires minimal variety of steps, consumes the shortest computing time and yet does no longer produce any excessive errors. Hence we conclude that spline is an effective method for solving differential equations. The results indicate that spline collocation method gives the best approximation for the linear and non-linear engineering problems without any assumption and linearization. It indicates that answers of spline collocation are much closer to the precise analytical solution.

NOMENCLATURE

L-Length of a straight fin
$A(x)$-Cross-section area
$k\ (T)$ - Thermal conductivity
k_b - Fin thermal conductivity at ambient temperature
T_∞ - Convective environment at temperature
h-Local heat transfer coefficient
p-Periphery of the fin,
T_∞ -Ambient temperature
b-Width of the fin,
$t(x)$-Fin thickness along the length.
A_b -Base area,
T_b -Base temperature

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