International Journal of Mathematical Archive-12(8), 2021, 10-17
Available online through www.ijma.info ISSN 2229 – 5046

ON GENERALIZED CLOSED SETS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACE

SAGILI KARUNAKAR*[1], BABISHA JULIT R L[2]

[1]Department of Mathematics,
St. Xavier's College, Palayamkottai, Tamil Nadu, India.

[2]Department of Mathematics,
G. Venktaswamy Naidu College, Kovilpatti, Tamil Nadu, India.

(Received On: 21-07-21; Revised & Accepted On: 30-08-21)

ABSTRACT

*In this paper, we define and study new class of Intuitionistic fuzzy closed sets called Intuitionistic fuzzy g**p- closed sets in Intuitionistic fuzzy topological space. The study centres around general properties of IFg**p- closed sets. Furthermore, we study the relationships of IFg**p- closed set with already defined IFCSs in IFTS. We also introduce the concept of IFg**p- open sets.*

Key words: *IFg**p- closed sets, IFg**p- open sets.*

AMS Mathematics Subject Classification – *54A40.*

I. INTRODUCTION

L. A Zadeh [18] introduced the concept of fuzzy set. And C. L. Chang [4] established a generalization of Fuzzy Sets in the topological space as Fuzzy Topological Space. The concepts of intuitionistic fuzzy set and intuitionistic fuzzy topology were introduced by Atanassov [2] and Coker [5] respectively.

In this paper we, introduce the concept of Intuitionistic fuzzy g**p closed (open) sets in Intuitionistic fuzzy topological space and study some general properties of IFg**pC (IFg**pO) sets. Also, we discuss the relationship between these sets with other existing Intuitionistic fuzzy closed and open sets.

II. PRELIMINARIES

Definition 2.1: [2] An intuitionistic fuzzy set (IFS) A is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x\in X\}$where the functions $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x\in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x\in X$. An intuitionistic fuzzy set A in X is simply denoted by $A =\langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x\in X\}$.

Definition 2.2: [2] Let A and B be two IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x\in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x\in X\}$.Then,

(a) $A\subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$,
(b) $A = B$ if and only if $A\subseteq B$ and $A\supseteq B$,
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x\in X\}$,
(d) $A\cup B = \{\langle x, \mu_A(x)\vee\mu_B(x), \nu_A(x)\wedge\nu_B(x)\rangle : x\in X\}$,
(e) $A\cap B = \{\langle x, \mu_A(x)\wedge\mu_B(x), \nu_A(x)\vee\nu_B(x)\rangle : x\in X\}$.

The intuitionistic fuzzy sets $\tilde{\mathbf{0}} = \langle x, 0, 1\rangle$ and $\tilde{\mathbf{1}} = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Corresponding Author: Sagili Karunakar*[1],
[1]Department of Mathematics, St. Xavier's College, Palayamkottai, Tamil Nadu, India.

Definition 2.3: [5] Anintuitionistic fuzzy topology (IFT) on X is a family τ of IFSs in X satisfying the following axioms:

- $\tilde{\mathbf{0}}, \tilde{\mathbf{1}} \in \tau$,
- $M_1 \cap M_2 \in \tau$ for any $M_1, M_2 \in \tau$,
- $\cup M_i \in \tau$ for any family $\{M_i : i \in I\} \in \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS) in X. The complement A^c of an IFOS in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.4: [5] Let (X, τ) be an IFTS and $A = \langle x, \mu_A(x), \nu_A(x)\rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

- $Iint(A) = \cup \{G / G$ is an IFOS in X and $G \subseteq A\}$,
- $Icl(A) = \cap\{K / K$ is an IFCS in X and $A \subseteq K\}$.

It is to be noted that for any IFS A in (X, τ), we have $Icl(A^c) = (Iint(A))^c$ and $Iint(A^c) = (Icl(A))^c$.

Proposition 2.5: [5] For any IFSs A and B in (X, τ), we have

(1) $Iint(A) \subseteq A$
(2) $A \subseteq Icl(A)$
(3) A is an IFCS in X $\Leftrightarrow Icl(A) = A$
(4) A is an IFOS in X $\Leftrightarrow Iint(A) = A$
(5) $A \subseteq B \Rightarrow Iint(A) \subseteq Iint(B)$ and $Icl(A) \subseteq Icl(B)$
(6) $Iint(Iint(A)) = Iint(A)$
(7) $Icl(Icl(A)) = Icl(A)$
(8) $Icl(A \cup B) = Icl(A) \cup Icl(B)$
(9) $Iint(A \cap B) = Iint(A) \cap Iint(B)$

Proposition 2.6: [3] For any IFS A in (X, τ), we have

(1) $Iint(\tilde{\mathbf{0}}) = \tilde{\mathbf{0}}$ and $Icl(\tilde{\mathbf{0}}) = \tilde{\mathbf{0}}$
(2) $Iint(\tilde{\mathbf{1}}) = \tilde{\mathbf{1}}$ and $Icl(\tilde{\mathbf{1}}) = \tilde{\mathbf{1}}$
(3) $(Iint(A)))^C = Icl(A^C)$
(4) $(Icl(A))^C = Iint(A^C)$

Definition 2.7: [8] An IFS $A = \{\langle x, \mu_A(X), \nu_A(x)\rangle : x \in X\}$ in an IFTS (X, τ) is said to be an

1) intuitionistic fuzzy semi closed set (IFSCS) if $Iint(Icl(A)) \subseteq A$,
2) intuitionistic fuzzy pre closed set (IFPCS) if $Icl(Iint(A)) \subseteq A$,
3) intuitionistic fuzzy α-closed set (IFαCS) if $Icl(Iint(Icl(A)) \subseteq A$,
4) intuitionistic fuzzy regular closed set (IFRCS) if $Icl(Iint(A)) = A$,

Definition 2.8: [17] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized closed set (IFGCS) if $Icl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.

Definition 2.9: [11] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy α generalized closed set (IFαGCS) if $I\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.

Definition 2.10: [15] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized preclosed set (IFGPCS) if $IPcl(A) \subseteq U$, whenever $A \subseteq U$ and U is an IFOS in X.

Definition 2.11: [1] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi regular closed set (IFGPRCS) if $IPcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X.

Definition 2.12: [7] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized star closed set (IFG*CS) if $Icl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFGOS in X.

Definition 2.13: [8]An IFS A in an IFTS (X, τ) is said to be an

1) intuitionistic fuzzy semi open set (IFSOS) if $A \subseteq Icl(Iint(A))$,
2) intuitionistic fuzzy pre-open set (IFPOS) if $A \subseteq Iint(Icl(A))$,
3) intuitionistic fuzzy α open set (IFαOS) if $A \subseteq Iint(Icl(Iint(A)))$,
4) intuitionistic fuzzy regular open set (IFROS) if $A = Iint(Icl(A))$.

Definition 2.14: [13] Let $A = \{\langle x, \mu_A, \nu_A\rangle$ be IFS in an IFTS (X, τ). Then the pre- interior and pre- closure of A are defined as

$IPint(A) = \cup\{G / G$ is an IFPOS in X and $G \subseteq A\}$, $IPcl(A) = \cap\{K / K$ is an IFPCS in X and $A \subseteq K\}$.

© 2021, IJMA. All Rights Reserved

Result 2.15: [13] Let A be IFS in (X, τ), then

IPcl(A)=A∪*Icl*(*Iint*(A)) and *IPint*(A)=A∩ *Iint*(*Icl*(A))

III. INTUITIONISTIC FUZZY G**P-CLOSED SET

In this section we introduce Intuitionistic fuzzy g**pre-closed sets and study some of their Properties.

Definition 3.1: An intuitionistic fuzzy set K of an intuitionistic fuzzy topological space (X, ξ) is called an intuitionistic fuzzy g**p -closed if *IPcl* (K)⊆ Q whenever K⊆Q and Q is intuitionistic fuzzy g*-open in X.

Example 3.2: Let X = {a, b} and intuitionistic fuzzy set M is defined as follows
M = {< a, (0.6,0.3) >, <b, (0.7,0.2) >}. Let ξ = {$\tilde{\mathbf{0}}$, M, $\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X. Then,
P = {< a, (0.4, 0.6) >, < b, (0.5, 0.4) >} is intuitionistic fuzzy g**p- closed set.

Theorem 3.3: Every intuitionistic fuzzy pre- closed set is intuitionistic fuzzy g**p closed but not conversely.

Proof: Let K be intuitionistic fuzzy pre- closed set. Let K⊆U and U be intuitionistic fuzzy g*-open set in X. Since k is intuitionistic fuzzy pre- closed set, we have K = *IPcl*(K). Hence *IPcl*(K) ⊆ U, whenever K⊆ U and U is intuitionistic fuzzy g* open in X. Therefore,K is intuitionistic fuzzy g**p-closed set.

Example 3.4: Let X = {a, b} and intuitionistic fuzzy set P is defined as
P = {< a, (0.3, 0.7) >, < b, (0.2, 0.8) >}. Let ξ = {$\tilde{\mathbf{0}}$, P,$\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X.Then the intuitionistic fuzzy set Q = {< a, (0.5,0.4) >, < b, (0.9,0.1) >} is intuitionistic fuzzy g**p- closed set. But *Icl* (*Iint*(Q)) is not a subset of Q. Therefore, Q is not an intuitionistic fuzzy semi- closed set.

Theorem 3.5: Every intuitionistic fuzzy closed set is intuitionistic fuzzy g**p-closedbut not conversely.

Proof: Let A be intuitionistic fuzzy closed set. Let A ⊆ U and U be intuitionistic fuzzy g*-open set in X. Since A is intuitionistic fuzzy-closed set we have A= *Icl*(A).But *IPcl*(A) ⊆*Icl*(A), therefore, *IPcl*(A) ⊆ U, whenever A ⊆ U and U is intuitionisticfuzzy g*-open in X. Hence A is intuitionistic fuzzy g**p-closed set.

Example 3.6: Let X = {a, b} and intuitionistic fuzzy set P is defined as P = {< a, (0.3, 0.7) >, < b, (0.2, 0.8) >}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X.
Then the intuitionistic fuzzy set Q = {< a, (0.2, 0.8) >, < b, (0.2, 0.85) >}. is intuitionistic fuzzy g**p –closed set. But *Icl*(Q) ≠ Q, therefore, it is not intuitionistic fuzzy closed.

Theorem 3.7: Every intuitionistic fuzzy α-closed set is intuitionistic fuzzy g**p closed but not conversely.

Proof: Let A be intuitionistic fuzzy α-closed set. Let A ⊆ U and U be intuitionistic fuzzy g*-open set in X. We know that every α closed set is pre closed set. By Theorem 3.3, Every intuitionistic fuzzy pre closed set is IFg**p closed set.Therefore, A is intuitionistic fuzzy g**p-closed set.

Example 3.8: Let X = {a, b} and ξ = {$\tilde{\mathbf{0}}$, P,$\tilde{\mathbf{1}}$} be an IFTS on X, where P = {< a, (0.3, 0.6) >, < b, (0.2, 0.7) >}. Then, the IFS Q = {< a, (0.8, 0.2) >, < b, (0.9, 0.1) >} is an IF g**p-closed set. But *Icl* (*Iint*(*Icl*(Q)))⊈Q. Therefore, Q is not IF α-closed set.

Theorem 3.9: Every intuitionistic fuzzy regular-closed set is intuitionistic fuzzy g**p-closed but not conversely.

Proof: Let A be intuitionisticfuzzy regular closed set. We know that every intuitionistic fuzzy regular closed set is intuitionistic fuzzy closed set. Then, by Theorem 3.5, we get A is intuitionistic fuzzy g**p- closed set.

Example 3.10: Let X = {a, b} and intuitionistic fuzzy set P is defined as P = {< a, (0.3, 0.7) >, < b, (0.2, 0.7) >}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X.
Then, the intuitionistic fuzzy set A = {< a, (0.85, 0.15) >, < b, (0.8, 0.2) >} is intuitionistic fuzzy g**p- closed set. But *Icl*(*Iint* (A)) ≠ A, therefore, it is not intuitionisticfuzzy regular-closed set .

Theorem 3.11: Every intuitionistic fuzzy g*-closed set is intuitionistic fuzzy g**p-closed but converse is not true.

Proof: Let A be intuitionistic fuzzy g*-closed set. Let A⊆ U and U be intuitionistic fuzzy g*-open set in X.By definition of intuitionistic fuzzy g*-closed set, cl(A) ⊆ U. Note that *IPcl*(A) ⊆ cl(A) is always true. Now we have *IPcl*(A) ⊆ U, whenever A⊆ U, U is IFg*- closed set. Hence A is intuitionistic fuzzy g**p-closed set.

© 2021, IJMA. All Rights Reserved

Example 3.12: Let X = {a, b} and intuitionistic fuzzy set G is defined as G = {< a, (0.5, 0.3) >, < b, (0.7, 0.2) >}. Let ξ = {$\tilde{\mathbf{0}}$, G, $\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X. Then the IFS S = {< a, (0.3, 0.7) >, < b, (0.4, 0.6) >} is intuitionistic fuzzy g**p –closed set. Then take an intuitionistic fuzzy g*-open set M= {< a, (0.7, 0.3) >, < b, (0.8, 0.2) >}. Now *Icl*(S) ⊈ M. Therefore, S is not an intuitionistic fuzzy g*-closed.

Theorem 3.13: Every intuitionistic fuzzy g**p-closed set is IFGP-closed but converse is not true.

Proof: Let A be intuitionistic fuzzy g**p-closed set. Let A⊆ U and U be intuitionistic Fuzzyopen set in X. Since every intuitionistic fuzzy open set is intuitionistic fuzzy g*- open set, U is intuitionistic fuzzy g*- open set such that A⊆ U. Now by definition of intuitionistic fuzzy g**p-closed set *IPcl*(A) ⊆ U. We have *IPcl*(A) ⊆U whenever A⊆U and U is intuitionistic fuzzy open in X. Therefore, A is IFGP-closed set.

Example 3.14: Let X = {a, b} and intuitionistic fuzzy set P is defined as P = {< a, (0.3, 0.7) >, < b, (0.2, 0.7) >}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set M = {< a, (0.5, 0.4) >, < b, (0.4, 0.6) >} is IFGP- closed but it is not intuitionistic fuzzy g**p-closed.

Theorem 3.15: Every intuitionistic fuzzy g**p-closed set is IFGPR –closed but converse is not true.

Proof: Let A be intuitionistic fuzzy g**p-closed set. Let A ⊆ U and U is intuitionistic fuzzyregularopen sets in X. Since every intuitionistic fuzzy regular open set is intuitionistic fuzzy g*- open set, U is intuitionistic fuzzy g*- open set such that A ⊆U. By definition of intuitionistic fuzzy g**p-closed sets *IPcl*(A) ⊆ U. We have *IPcl*(A)⊆ U, whenever A ⊆ U and U is intuitionistic fuzzy regular open in X. Therefore, A is IFGPR- closed set.

Example 3.16: Let X = {a, b} and intuitionistic fuzzy set P is defined as P = {< a, (0.6, 0.3) >, < b, (0.7, 0.2) >}. Let ξ= {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X. Q = {< a, (0.7, 0.2) >, < b, (0.7, 0.2) >} is IFGPRCS.
Let P = {< a, (0.7, 0.2) >, < b, (0.8, 0.2) >}. Then P is an IFg*OS. Also, Q⊆P,*IPcl*(Q) =$\tilde{\mathbf{1}}$⊈ P. Hence Q is not IFg**pCS.

Theorem 3.17: Every intuitionistic fuzzy g**p-closed set is intuitionistic fuzzy GSP-closed but converse is not true.

Proof: Let A be intuitionistic fuzzy g**p-closed set. Let A⊆ U and U be intuitionistic fuzzyopen set in X. Since every intuitionistic fuzzy open set is intuitionistic fuzzy g*-open, U is intuitionistic fuzzy g*-open set. Now by definition of intuitionistic fuzzy g**p-closed set, *IPcl*(A) ⊆U. Note that *ISPcl*(A)⊆*IPcl*(A) is always true. We have *ISPcl*(A)⊆U whenever A ⊆ U and U is intuitionisticfuzzy open in X. Therefore, A is intuitionistic fuzzy GSP-closed set.

Example 3.18: Let X = {a, b} and ξ ={$\tilde{\mathbf{0}}$, P,$\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X, where, P = {< a, (0.6, 0.4) >, < b, (0.75, 0.2) >}. Then, Q = {< a, (0.7, 0.2) >, < b, (0.7, 0.2) >} is IFGSP-closed but it is not intuitionistic fuzzy g**p- closed.

Remark 3.19: IFSCS and IFg**pCS are independent

Example 3.20: Let X={a,b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.65) >, < b, (0.2, 0.7) >}. Consider an IFS W= {< a, (0.4, 0.6) >, < b, (0.3, 0.65) >}. Then W is IFSCS but not IFg**pCS.

Example 3.21: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.65) >, < b, (0.2, 0.7) >}. Consider an IFS W= {< a, (0.2, 0.8) >, < b, (0.2, 0.85) >}. Then W is IFg**pCS but not IFSCS.

Remark 3.22: IFGCS and IFg**pCS are independent.

Example 3.23: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.6) >, < b, (0.2, 0.8) >}. Consider an IFS W= {< a, (0.2, 0.8) >, < b, (0.1, 0.9) >}. Then W is IFg**pCS but not IFGCS.

Example 3.24: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.65) >, < b, (0.2, 0.7) >}. Consider an IFS W= {< a, (0.5, 0.5) >, < b, (0.6, 0.4) >}. Then W is IFGCS but not IFg**pCS.

Remark 3.25: IFGαCS and IFg**pCS are independent.

Example 3.26: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P,$\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.2, 0.7) >, < b, (0.3, 0.6) >}. Consider an IFS W= {< a, (0.15, 0.8) >, < b, (0.1, 0.9) >}. Then W is IFg**pCS but *Iαcl* (W)=P^c⊈P. Therefore, W is not IFGαCS.

Example 3.28: Let X= {a, b}. Let $\xi = \{\tilde{0}, P, \tilde{1}\}$ be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.7) >, < b, (0.2, 0.8) >}. Consider an IFS W= {< a, (0.5, 0.4) >, < b, (0.7, 0.3) >}. Then W is IFGαCS but not IFg**pCS.

Remark 3.29: From the above discussion. we have the following diagram of implications.

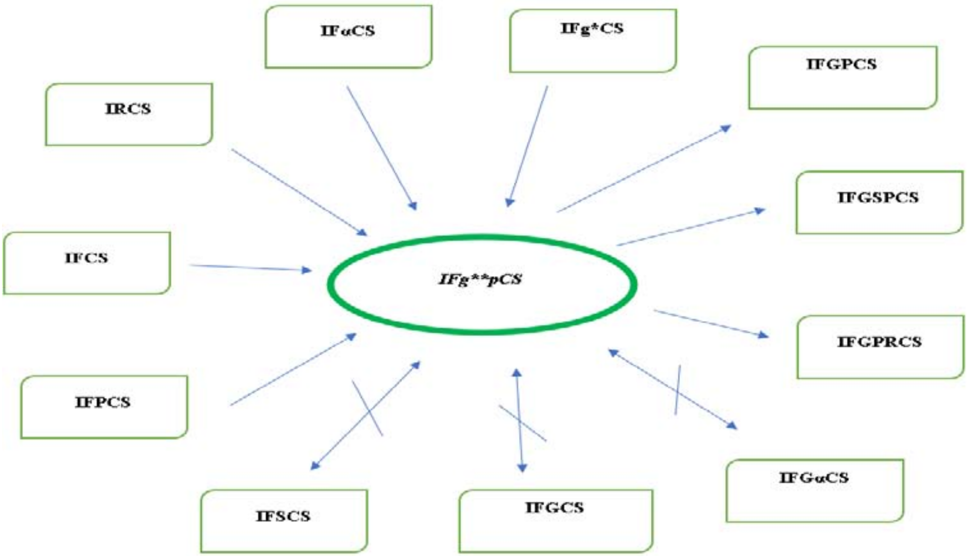

Figure 1: Relationship between intuitionistic fuzzy g**p closed set and other existing intuitionistic fuzzy closed sets.

Where A ⟶ B represents A implies B and A↮ B represents A and B are independent.

Theorem 3.30: Union of two IFg**p- closed sets is again an IFg**p- closed set.

Proof: Let A and B be two IFg**p closed sets in IFTS (X, ξ).And let Q be an IFg* OS in X, Such that A U B⊆ Q. Since A and B are IFg**pCSs we have, *IPcl*(A) ⊆ Q &*IPcl*(B) ⊆ Q. Therefore *IPcl*(A) U *IPcl*(B) ⊆*IPcl*(A U B) ⊆ Q. Hence A U B is an IFg**p closed set.

Remark 3.31: Intersection of two IFg**p closed sets need not be an IFg**p closed set.

Example 3.32: Let X= {a, b}. Let $\xi = \{\tilde{0}, P, \tilde{1}\}$ be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.7) >, < b, (0.2, 0.8) >}. Consider two IFg**pCSs W= {< a, (0.5, 0.4) >, < b, (0.9, 0.1) >}, V= {< a, (0.7, 0.3) >, < b, (0.8, 0.2) >}. Then, W ∩ V = {< a, (0.5, 0.4) >, < b, (0.8, 0.2) >} is not an IFg**pCS.

Lemma 3.33: If A is IFg*-open and IFg**p-closed set in X, then A is Intuitionistic Fuzzy pre-closed set.

Proof: Since A is IFg*-open and IFg**p-closed, then *IPcl*(A) ⊆ A, we know that A ⊆*IPcl*(A).Therefore *IPcl*(A)=A. Hence, A is IF pre closed set.

Theorem 3.34: Let A be intuitionistic fuzzy g**p-closed set in an intuitionistic fuzzy topological space (X, τ) and A ⊆ B ⊆*IPcl* (A).Then, B is intuitionistic fuzzy g**p-closed in X.

Proof: Let V be an intuitionistic fuzzy g*-open set in X such that B ⊆ V. Then A ⊆V and since A is intuitionistic fuzzy g**p-closed, *IPcl*(A) ⊆ V. By hypothesis B ⊆*IPcl* (A) then *IPcl* (B) ⊆*IPcl*(*IPcl* (A)).Thus *IPcl* (B) ⊆*IPcl*(A). This implies *IPcl* (B) ⊆ V. Hence B is intuitionistic fuzzy g**p-closed set.

Theorem 3.35: Let (X, τ) be an IFTS. Then IFC(X) = IF g**pC(X) if every IFS in (X, τ) is an IFg*OS in X, where IFC(X)denotes the collection of IFCSs of an IFTS (X, τ).

Proof: Suppose that every IFS in (X, τ) is an IFg*OS in X. Let A ∈ IF g**pC(X). Then *IPcl*(A) ⊆ U whenever A ⊆ U and U is an IFg*OS in X. Since every IFS is an IFg*OS, A is also an IFg*OS andA ⊆ A. Therefore *IPcl*(A) ⊆ A. Hence *IPcl*(A) = A. Therefore A ∈ IFC(X). Hence IF g**pC(X) ⊆ IFC(X) → (1) Let A ∈ IFC(X). Then by Theorem 3.5, A ∈ IF g**pC(X). Hence IFC(X) ⊆IFg**pC(X) → (2). From (1) and (2), we have IFC(X) = IFg**pC(X).

Theorem 3.36: In an intuitionistic fuzzy topological space (X, τ), every IFg*-open set is Intuitionistic Fuzzy pre closed iff every subset of X is IFg**p- closed set.

Proof:
Necessity: Suppose that every IFg*-open set is pre closed. Let A be a subset of X such that A⊆ U whenever U is IFg*-open. But *IPcl*(A) ⊆*IPcl*(U)=U. Therefore, A is IFg**p closed set.

Sufficiency: Suppose that every subset of X is IFg**p closed. Let U be IFg*-open. Since U is g**p-closed, we have *IPcl*(U) ⊆ U. Therefore, *IPcl*(U)=U. Hence the proof.

IV. INTUITIONISTIC FUZZY g**p -OPEN SET

In this section we introduce and studied Intuitionistic fuzzy g**p- open set and its properties.

Definition 4.1: An IFS A is said to be an Intuitionistic fuzzy g**p open set (IFg**pOS in short) in (X, ξ) if the complement A^c is an IFg**pCS in X. The family of all IFg**pOSs of IFTS (X, ξ) is denoted by IFg**pO(X)

Example 4.2: Let X = {a, b} and intuitionistic fuzzy set M is defined as M = {< a, (0.6, 0.3) >, < b, (0.7, 0.2) >}. Let ξ = {$\tilde{\mathbf{0}}$, M, $\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X.
Then P = {< a, (0.6, 0.4) >, < b, (0.4, 0.5) >} is intuitionistic fuzzy g**p- open set.

Theorem 4.3: For any IFTS (X, ξ), we have the following:

- Every IFOS is an IFg**pOS.
- Every IFPOS is an IFg**pOS.
- Every IFαOS is an IFg**pOS.
- Every IFROS is an IFg**pOS.
- Every IFg*OS is an IFg**pOS.

Proof: Straight forward.

Corollary 4.4: Every IFg**pOS need not be an IFPOS in (X,ξ). It is shown in the following example.

Example 4.5: Let X = {a, b} and intuitionistic fuzzy set P is defined as P = {< a, (0.3, 0.7) >, < b, (0.2, 0.8) >}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X.Then the intuitionistic fuzzy set Q = {< a, (0.4,0.5) >, < b, (0.1,0.9) >} is intuitionistic fuzzy g**p open set but not an intuitionistic fuzzy pre- open set.

Corollary 4.6: Every IFg**pOS need not be an IFOS in (X,ξ). It is shown in the following example.

Example 4.7 Let X = {a, b} and ξ = {$\tilde{\mathbf{0}}$, P,$\tilde{\mathbf{1}}$} be an IFTS on X, where P = {< a, (0.3, 0.7) >, < b, (0.2, 0.8) >}. Then the IFS Q = {< a, (0.8, 0.2) >, < b, (0.85, 0.1) >} is an IF g**p-open set. But *Iint*(Q) ≠ Q, therefore, it is not intuitionistic fuzzy open set.

Corollary 4.8: Every IFg**pOS need not be an IFαOS in (X,ξ). It is shown in the following example.

Example 4.9: Let X = {a, b} and ξ = {$\tilde{\mathbf{0}}$, P,$\tilde{\mathbf{1}}$} be an IFTS on X, where P = {< a, (0.3, 0.6) >, < b, (0.2, 0.7) >}. Then, the IFS S = {< a, (0.2, 0.8) >, < b, (0.1, 0.9) >} is an IF g**p- closed set. But S ⊈ *Iint(Icl(Iint(S)))* = A. Therefore, S is not IFα-open.

Corollary 4.10: Every IFg**pOS need not be an IFROS in (X, ξ). It is shown in the following example.

Example 4.11: Let X = {a, b} and intuitionistic fuzzy set P is defined as P = {< a, (0.3, 0.7) >, < b, (0.2, 0.7) >}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X.

Then the intuitionistic fuzzy set A = {< a, (0.15, 0.85) >, < b, (0.2, 0.8) >} is intuitionistic fuzzy g**p- open set. But *Iint*(*Icl*(A)) ≠ A, therefore it is not intuitionistic fuzzy regular-open set.

Corollary 4.12: Every IFg**pOS need not be an IFg*OS in (X,ξ). It is shown in the following example.

Example 4.13: Let X = {a, b} and intuitionistic fuzzy set G is defined as G = {< a, (0.5, 0.3) >, < b, (0.7, 0.2) >}. Let ξ = {$\tilde{\mathbf{0}}$, G, $\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X. Then the IFS S = {< a, (0.7, 0.3) >, < b, (0.6, 0.4) >} is intuitionistic fuzzy g**p – open set but not IF g*-open set.

Theorem 4.14: For any IFTS (X, ξ), we have the following:

- Every IF g**p-open set IF GP-open.
- Every IF g**p-open set IFGPR- open.
- Every IF g**p-open set IFGSP- open.

Corollary 4.15: Every IFGPOS need not be an IFg**pOS in (X, ξ). It is shown in the following example.

Example 4.16: Let X = {a, b} and intuitionistic fuzzy sets P is defined as P = {< a, (0.3, 0.7) >, < b, (0.2, 0.7) >}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set M = {< a, (0.4, 0.5) >, < b, (0.6, 0.4) >} is intuitionistic fuzzy GP-open but it is not intuitionistic fuzzy g**p-open.

Corollary 4.17: Every IFGPROS need not be an IFg**pOS in (X, ξ). It is shown in the following example.

Example 4.18: Let X = {a, b} and intuitionistic fuzzy sets P is defined as P = {< a, (0.6, 0.3) >, < b, (0.7, 0.2) >}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X. Q = {< a, (0.2, 0.7) >, < b, (0.2, 0.7) >} is IFGPROS but not IFg**pOS.

Corollary 4.19: Every IFGSPOS need not be an IFg**pOS in (X,ξ). It is shown in the following example.

Example 4.20: Let X = {a, b} and intuitionistic fuzzy sets P is defined as P = {< a, (0.6, 0.4) >, < b, (0.75, 0.2) >}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X. Q = {< a, (0.2, 0.7) >, < b, (0.2, 0.7) >} is IFGSP-open but it is not intuitionistic fuzzy g**p- open.

Remark 4.21: IFSOS and IFg**pOS are independent.

Example 4.22: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.65) >, < b, (0.2, 0.7) >}. Consider an IFS W= {< a, (0.6, 0.4) >, < b, (0.65, 0.3) >}. Then W is IFSOS but not IFg**pOS.

Example 4.23: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.65) >, < b, (0.2, 0.7) >}. Consider an IFS W= {< a, (0.8, 0.2) >, < b, (0.85, 0.15) >}. Then W is IFg**pOS but not IFSOS.

Remark 4.24: IFGOS and IFg**pOS are independent.

Example 4.25: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.6) >, < b, (0.2, 0.8) >}. Consider an IFS W= {< a, (0.8, 0.2) >, < b, (0.9, 0.1) >}. Then W is IFg**pOS but not IFGOS.

Example 4.26: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.65) >, < b, (0.2, 0.7) >}. Consider an IFS W= {< a, (0.5, 0.5) >, < b, (0.4, 0.6) >}. Then W is IFGOS but not IFg**pOS.

Remark 4.27: IFGαOS and IFg**pOS are independent.

Example 4.28: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.2, 0.7) >, < b, (0.3, 0.6) >}. Consider an IFS W= {< a, (0.8, 0.15) >, < b, (0.9, 0.1) >}. Then W is IFg**pOS but not IFGαOS.

Example 4.29: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.7) >, < b, (0.2, 0.8) >}. Consider an IFS W= {< a, (0.4, 0.5) >, < b, (0.3, 0.7) >}. Then W is IFGαOS but not IFg**pOS.
.

Theorem 4.30: If A and B are IFg**p-open sets in an IFTS (X, ξ), then (A∩B) is an IFg**p-open sets in (X, ξ).

Proof: Let A and B be IFg**p-open sets in IFTS (X, ξ). Therefore, A^c and B^c are IFg**p-closed sets in (X, ξ).
By Theorem 3.30, ($A^c \cup B^c$) is IFg**p-closed sets in (X, ξ).Since ($A^c \cup B^c$) = $(A\cap B)^c$, (A∩B) is IFg**p open sets in (X, ξ).

Remark 4.31: Union of two IFg**p-open sets need not be an IFg**p-open set.

Example 4.32: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.7, 0.3) >, < b, (0.8, 0.2) >}. Consider two IFg**pOSs W= {< a, (0.4, 0.5) >, < b, (0.1, 0.9) >}, V= {< a, (0.3, 0.7) >, < b, (0.2, 0.8) >}. Then, W ∪ V = {< a, (0.4, 0.5) >, < b, (0.2, 0.9) >} is not an IFg**pOS.

© 2021, IJMA. All Rights Reserved

Theorem 4.33: An IFsubset B of a IFTS (X,ξ) is IFg**p- open if and only if K⊆*IPint*(B) whenever K is IFg*-closed and K⊆B.

Proof: Suppose that B is IFg**p - open in X, K is IFg*-closed and K⊆B. Then K^c is IFg*-open and $B^c \subseteq K^c$. Since, B^c is IFg**p- closed, then $IPcl(B^c) \subseteq K^c$. But, $IPcl\ (B^c) = [\ IPint(B)]^c \subseteq K^c$. Hence K⊆*IPint*(B).

Conversely, suppose that K⊆*IPint*(B) whenever K⊆B and K is IFg*-closed. If H is an IF g*-open set in X containing B^c, then H^c is a IFg*-closed setcontained in B. Hence by hypothesis, $H^c \subseteq IPint(B)$, then by taking the complements, we have, $IPcl(B^c) \subseteq H$. Therefore B^c is IFg**p − closed in X and hence B is IFg**p-open in X.

Corollary 4.34: If B is IFg**p -open in IFTS (X,ξ), then H=X, whenever H is IFg*-open and $IPint(B)$ U $(B^c) \subseteq H$.

Proof: Assume that H is IFg*-open and $IPint(B) \cup (B^c) \subseteq H$. Hence $H^c \subseteq IPcl(B^c) \cap B = IPcl(B^c) - (B^c)$. Since, H^c is IFg*-closed and B^c is IFg**p −closed, then by Theorem 3.34, $H^c = \emptyset$ and hence, H=X.

Theorem 4.35: Let (X, ξ) be an IFTS. Then IFO(X) = IFg**pO(X) if every IFS in (X, ξ) is an IFg*-open in X, where IFO(X) denotes the collection of IFOSs of an IFTS(X, ξ).

Proof: Suppose that every IFS in (X, ξ) is an IFg*-open in X. Then by theorem 3.35, we have IFC(X) = IFg**pC(X). Therefore IFO(X) = IFg**pO(X).

REFERENCES

1. Anitha S, Mohana K., IFgsr closed sets in Intuitionistic Fuzzy Topological Spaces, International Journal Of Innovative Research In Technology, July 2018, Volume 5
2. Atanassov, K., Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 12, 1986, 87-96.
3. Bhattacharjee, A. Bhaumik, R. N. Pre- Semi Closed Sets &Presemi separation Axioms in Intuitionistic Fuzzy Topological Spaces, Wen Math Notes, (2012), VOl.8, pp. 11-17.
4. Chang, C.L. Fuzzy topological spaces**,** J.Math.Anal.Appl, 24(1968), 182-190.
5. Coker, D., An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, 88, 1997, 81-89.
6. Coker, D., and Demirci, M., On intuitionistic fuzzy points, Notes on Intuitionistic Fuzzy Sets, 1, 1995, 79-84.
7. Gandhimathi, T. Rameshkumar, M. g*-closed sets in intuitionistic fuzzy topological spaces, International Journal of Business Intelligence and Data Mining, (2020) Vol.16 No.4, pp.445 – 458.
8. Gurcay, H., Coker, D., and Hayder, Es. A., On fuzzy continuity in intuitionistic fuzzy topological spaces, The Journal of Fuzzy Mathematics, 5, 1997, 365-378.
9. Jyoti Pandey Bajpai, Thakur. S.S., Intuitionistic Fuzzy Strongly G*-Closed Sets, International Journal of Innovative Research in Science and Engineering, VOl.2, Issue 12 December 2016, 19-30, 166-179.
10. N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo, 19(1970), 89-96.
11. Pious Missier S, Anto M, g ^*S – closed sets in topological Spaces, International Journal of Modern Engineering Research, Vol. 4, Iss. 11, 2014, 32- 38.
12. Pious Missier S, BabishaJulit R L, Intuitinistic Fuzzy Generalized semi star closed sets, International Journal of Mathematic Archive, Vol.12, 2021, no. 2, 47-55.
13. Sakthivel, K., Intuitionistic fuzzy alpha generalized continuous mappings and intuitionistic fuzzy alpha generalized irresolute mappings, Applied Mathematical Sciences, 37, 2010, 1831-1842.
14. Santhi R, Arun Prakash K, On Intuitionistic Fuzzy Semi-Generalized Closed Sets and its ApplicationsInt. J. Contemp. Math. Sciences, Vol. 5, 2010, no. 34, 1677 - 1688.
15. Santhi, R., and Jayanthi. D., Generalized semi-pre connectedness in intuitionistic fuzzy topological spaces, Annals of Fuzzy Mathematics and Informatics, 3, 2012, 243-253.
16. Thakur, S. S., and Rekha Chaturvedi, Regular generalized closed sets in intuitionistic fuzzy topological spaces, Universitatea Din Bacau, Studii Si CercetariStiintifice, Seria: Mathematica, 16, 2006, 257-272.
17. Thakur, S. S., and Rekha Chaturvedi, Generalized closed sets in intuitionistic fuzzy topology, The Journal of Fuzzy Mathematics, 16, 2008, 559-570.
18. Zadeh, L.A. Fuzzy sets, Information control, 8 (1965), pp. 338-353.

Source of support: Nil, Conflict of interest: None Declared.
[Copy right © 2021. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]