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ON GENERALIZED CLOSED SETS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACE

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ABSTRACT

*In this paper, we define and study new class of Intuitionistic fuzzy closed sets called Intuitionistic fuzzy g**p- closed sets in Intuitionistic fuzzy topological space. The study centres around general properties of IFg**p- closed sets. Furthermore, we study the relationships of IFg**p- closed set with already defined IFCSs in IFTS. We also introduce the concept of IFg**p- open sets.*

Key words: *IFg**p- closed sets, IFg**p- open sets.*

AMS Mathematics Subject Classification – *54A40.*

I. INTRODUCTION

L. A Zadeh [18] introduced the concept of fuzzy set. And C. L. Chang [4] established a generalization of Fuzzy Sets in the topological space as Fuzzy Topological Space. The concepts of intuitionistic fuzzy set and intuitionistic fuzzy topology were introduced by Atanassov [2] and Coker [5] respectively.

In this paper we, introduce the concept of Intuitionistic fuzzy g**p closed (open) sets in Intuitionistic fuzzy topological space and study some general properties of IFg**pC (IFg**pO) sets. Also, we discuss the relationship between these sets with other existing Intuitionistic fuzzy closed and open sets.

II. PRELIMINARIES

Definition 2.1: [2] An intuitionistic fuzzy set (IFS) A is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x\in X\}$where the functions $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x\in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x)\leq 1$ for each $x\in X$. An intuitionistic fuzzy set A in X is simply denoted by $A =\langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x\in X\}$.

Definition 2.2: [2] Let A and B be two IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x\in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle: x\in X\}$.Then,

(a) $A\subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
(b) $A = B$ if and only if $A\subseteq B$ and $A\supseteq B$,
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle: x\in X\}$,
(d) $A\cup B = \{\langle x, \mu_A(x)\vee\mu_B(x), \nu_A(x)\wedge\nu_B(x)\rangle: x\in X\}$,
(e) $A\cap B = \{\langle x, \mu_A(x)\wedge\mu_B(x), \nu_A(x)\vee\nu_B(x)\rangle: x\in X\}$.

The intuitionistic fuzzy sets $\tilde{0} = \langle x, 0, 1\rangle$ and $\tilde{1} = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

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Definition 2.3: [5] Anintuitionistic fuzzy topology (IFT) on X is a family τ of IFSs in X satisfying the following axioms:

- $\tilde{\mathbf{0}}, \tilde{\mathbf{1}} \in \tau$,
- $M_1 \cap M_2 \in \tau$ for any $M_1, M_2 \in \tau$,
- $\cup M_i \in \tau$ for any family $\{M_i : i \in I\} \in \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS) in X. The complement A^c of an IFOS in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.4: [5] Let (X, τ) be an IFTS and $A = \langle x, \mu_A(x), \nu_A(x)\rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

- $Iint(A) = \cup \{G / G$ is an IFOS in X and $G \subseteq A\}$,
- $Icl(A) = \cap\{K / K$ is an IFCS in X and $A \subseteq K\}$.

It is to be noted that for any IFS A in (X, τ), we have $Icl(A^c) = (Iint(A))^c$ and $Iint(A^c) = (Icl(A))^c$.

Proposition 2.5: [5] For any IFSs A and B in (X, τ), we have

1. $Iint(A) \subseteq A$
2. $A \subseteq Icl(A)$
3. A is an IFCS in X $\Leftrightarrow Icl(A) = A$
4. A is an IFOS in X $\Leftrightarrow Iint(A) = A$
5. $A \subseteq B \Rightarrow Iint(A) \subseteq Iint(B)$ and $Icl(A) \subseteq Icl(B)$
6. $Iint(Iint(A)) = Iint(A)$
7. $Icl(Icl(A)) = Icl(A)$
8. $Icl(A \cup B) = Icl(A) \cup Icl(B)$
9. $Iint(A \cap B) = Iint(A) \cap Iint(B)$

Proposition 2.6: [3] For any IFS A in (X, τ), we have

1. $Iint(\tilde{\mathbf{0}}) = \tilde{\mathbf{0}}$ and $Icl(\tilde{\mathbf{0}}) = \tilde{\mathbf{0}}$
2. $Iint(\tilde{\mathbf{1}}) = \tilde{\mathbf{1}}$ and $Icl(\tilde{\mathbf{1}}) = \tilde{\mathbf{1}}$
3. $(Iint(A)))^C = Icl(A^C)$
4. $(Icl(A))^C = Iint(A^C)$

Definition 2.7: [8] An IFS $A = \{\langle x, \mu_A(X), \nu_A(x)\rangle : x \in X\}$ in an IFTS (X, τ) is said to be an

1. intuitionistic fuzzy semi closed set (IFSCS) if $Iint(Icl(A)) \subseteq A$,
2. intuitionistic fuzzy pre closed set (IFPCS) if $Icl(Iint(A)) \subseteq A$,
3. intuitionistic fuzzy α-closed set (IFαCS) if $Icl(Iint(Icl(A)) \subseteq A$,
4. intuitionistic fuzzy regular closed set (IFRCS) if $Icl(Iint(A)) = A$,

Definition 2.8: [17] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized closed set (IFGCS) if $Icl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.

Definition 2.9: [11] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy α generalized closed set (IFαGCS) if $I\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.

Definition 2.10: [15] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized preclosed set (IFGPCS) if $IPcl(A) \subseteq U$, whenever $A \subseteq U$ and U is an IFOS in X.

Definition 2.11: [1] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi regular closed set (IFGPRCS) if $IPcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X.

Definition 2.12: [7] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized star closed set (IFG*CS) if $Icl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFGOS in X.

Definition 2.13: [8]An IFS A in an IFTS (X, τ) is said to be an

1. intuitionistic fuzzy semi open set (IFSOS) if $A \subseteq Icl(Iint(A))$,
2. intuitionistic fuzzy pre-open set (IFPOS) if $A \subseteq Iint(Icl(A))$,
3. intuitionistic fuzzy α open set (IFαOS) if $A \subseteq Iint(Icl(Iint(A)))$,
4. intuitionistic fuzzy regular open set (IFROS) if $A = Iint(Icl(A))$.

Definition 2.14: [13] Let $A = \{\langle x, \mu_A, \nu_A\rangle$ be IFS in an IFTS (X, τ). Then the pre- interior and pre- closure of A are defined as

$IPint(A) = \cup\{G / G$ is an IFPOS in X and $G \subseteq A\}$, $IPcl(A) = \cap\{K / K$ is an IFPCS in X and $A \subseteq K\}$.

Result 2.15: [13] Let A be IFS in (X, τ), then
IPcl(A)=A∪*Icl*(*Iint*(A)) and *IPint*(A)=A∩ *Iint*(*Icl*(A))

III. INTUITIONISTIC FUZZY G**P-CLOSED SET

In this section we introduce Intuitionistic fuzzy g**pre-closed sets and study some of their Properties.

Definition 3.1: An intuitionistic fuzzy set K of an intuitionistic fuzzy topological space (X, ξ) is called an intuitionistic fuzzy g**p -closed if *IPcl* (K)⊆ Q whenever K⊆Q and Q is intuitionistic fuzzy g*-open in X.

Example 3.2: Let X = {a, b} and intuitionistic fuzzy set M is defined as follows
M = {< a, (0.6,0.3) >, <b, (0.7,0.2) >}. Let ξ = {$\tilde{\mathbf{0}}$, M, $\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X. Then,
P = {< a, (0.4, 0.6) >, < b, (0.5, 0.4) >} is intuitionistic fuzzy g**p- closed set.

Theorem 3.3: Every intuitionistic fuzzy pre- closed set is intuitionistic fuzzy g**p closed but not conversely.

Proof: Let K be intuitionistic fuzzy pre- closed set. Let K⊆U and U be intuitionistic fuzzy g*-open set in X. Since k is intuitionistic fuzzy pre- closed set, we have K = *IPcl*(K). Hence *IPcl*(K) ⊆ U, whenever K⊆ U and U is intuitionistic fuzzy g* open in X. Therefore,K is intuitionistic fuzzy g**p-closed set.

Example 3.4: Let X = {a, b} and intuitionistic fuzzy set P is defined as
P = {< a, (0.3, 0.7) >, < b, (0.2, 0.8) >}. Let ξ = {$\tilde{\mathbf{0}}$, P,$\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X.Then the intuitionistic fuzzy set Q = {< a, (0.5,0.4) >, < b, (0.9,0.1) >} is intuitionistic fuzzy g**p- closed set. But *Icl* (*Iint*(Q)) is not a subset of Q. Therefore, Q is not an intuitionistic fuzzy semi- closed set.

Theorem 3.5: Every intuitionistic fuzzy closed set is intuitionistic fuzzy g**p-closedbut not conversely.

Proof: Let A be intuitionistic fuzzy closed set. Let A ⊆ U and U be intuitionistic fuzzy g*-open set in X. Since A is intuitionistic fuzzy-closed set we have A= *Icl*(A).But *IPcl*(A) ⊆*Icl*(A), therefore, *IPcl*(A) ⊆ U, whenever A ⊆ U and U is intuitionisticfuzzy g*-open in X. Hence A is intuitionistic fuzzy g**p-closed set.

Example 3.6: Let X = {a, b} and intuitionistic fuzzy set P is defined as P = {< a, (0.3, 0.7) >, < b, (0.2, 0.8) >}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X.
Then the intuitionistic fuzzy set Q = {< a, (0.2, 0.8) >, < b, (0.2, 0.85) >}. is intuitionistic fuzzy g**p –closed set. But *Icl*(Q) ≠ Q, therefore, it is not intuitionistic fuzzy closed.

Theorem 3.7: Every intuitionistic fuzzy α-closed set is intuitionistic fuzzy g**p closed but not conversely.

Proof: Let A be intuitionistic fuzzy α-closed set. Let A ⊆ U and U be intuitionistic fuzzy g*-open set in X. We know that every α closed set is pre closed set. By Theorem 3.3, Every intuitionistic fuzzy pre closed set is IFg**p closed set.Therefore, A is intuitionistic fuzzy g**p-closed set.

Example 3.8: Let X = {a, b} and ξ = {$\tilde{\mathbf{0}}$, P,$\tilde{\mathbf{1}}$} be an IFTS on X, where P = {< a, (0.3, 0.6) >, < b, (0.2, 0.7) >}. Then, the IFS Q = {< a, (0.8, 0.2) >, < b, (0.9, 0.1) >} is an IF g**p-closed set. But *Icl* (*Iint*(*Icl*(Q)))⊈Q. Therefore, Q is not IF α-closed set.

Theorem 3.9: Every intuitionistic fuzzy regular-closed set is intuitionistic fuzzy g**p-closed but not conversely.

Proof: Let A be intuitionisticfuzzy regular closed set. We know that every intuitionistic fuzzy regular closed set is intuitionistic fuzzy closed set. Then, by Theorem 3.5, we get A is intuitionistic fuzzy g**p- closed set.

Example 3.10: Let X = {a, b} and intuitionistic fuzzy set P is defined as P = {< a, (0.3, 0.7) >, < b, (0.2, 0.7) >}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X.
Then, the intuitionistic fuzzy set A = {< a, (0.85, 0.15) >, < b, (0.8, 0.2) >} is intuitionistic fuzzy g**p- closed set. But *Icl*(*Iint* (A)) ≠ A, therefore, it is not intuitionisticfuzzy regular-closed set .

Theorem 3.11: Every intuitionistic fuzzy g*-closed set is intuitionistic fuzzy g**p-closed but converse is not true.

Proof: Let A be intuitionistic fuzzy g*-closed set. Let A⊆ U and U be intuitionistic fuzzy g*-open set in X.By definition of intuitionistic fuzzy g*-closed set, cl(A) ⊆ U. Note that *IPcl*(A) ⊆ cl(A) is always true. Now we have *IPcl*(A) ⊆ U, whenever A⊆ U, U is IFg*- closed set. Hence A is intuitionistic fuzzy g**p-closed set.

Example 3.12: Let X = {a, b} and intuitionistic fuzzy set G is defined as G = {< a, (0.5, 0.3) >, < b, (0.7, 0.2) >}. Let ξ = {$\tilde{\mathbf{0}}$, G, $\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X. Then the IFS S = {< a, (0.3, 0.7) >, < b, (0.4, 0.6) >} is intuitionistic fuzzy g**p –closed set. Then take an intuitionistic fuzzy g*-open set M= {< a, (0.7, 0.3) >, < b, (0.8, 0.2) >}. Now *Icl*(S) ⊈ M. Therefore, S is not an intuitionistic fuzzy g*-closed.

Theorem 3.13: Every intuitionistic fuzzy g**p-closed set is IFGP-closed but converse is not true.

Proof: Let A be intuitionistic fuzzy g**p-closed set. Let A⊆ U and U be intuitionistic Fuzzyopen set in X. Since every intuitionistic fuzzy open set is intuitionistic fuzzy g*- open set, U is intuitionistic fuzzy g*- open set such that A⊆ U. Now by definition of intuitionistic fuzzy g**p-closed set *IPcl*(A) ⊆ U. We have *IPcl*(A) ⊆U whenever A⊆U and U is intuitionistic fuzzy open in X. Therefore, A is IFGP-closed set.

Example 3.14: Let X = {a, b} and intuitionistic fuzzy set P is defined as P = {< a, (0.3, 0.7) >, < b, (0.2, 0.7) >}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set M = {< a, (0.5, 0.4) >, < b, (0.4, 0.6) >} is IFGP- closed but it is not intuitionistic fuzzy g**p-closed.

Theorem 3.15: Every intuitionistic fuzzy g**p-closed set is IFGPR –closed but converse is not true.

Proof: Let A be intuitionistic fuzzy g**p-closed set. Let A ⊆ U and U is intuitionistic fuzzyregularopen sets in X. Since every intuitionistic fuzzy regular open set is intuitionistic fuzzy g*- open set, U is intuitionistic fuzzy g*- open set such that A ⊆U. By definition of intuitionistic fuzzy g**p-closed sets *IPcl*(A) ⊆ U. We have *IPcl*(A)⊆ U, whenever A ⊆ U and U is intuitionistic fuzzy regular open in X. Therefore, A is IFGPR- closed set.

Example 3.16: Let X = {a, b} and intuitionistic fuzzy set P is defined as P = {< a, (0.6, 0.3) >, < b, (0.7, 0.2) >}. Let ξ= {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X. Q = {< a, (0.7, 0.2) >, < b, (0.7, 0.2) >} is IFGPRCS.
Let P = {< a, (0.7, 0.2) >, < b, (0.8, 0.2) >}. Then P is an IFg*OS. Also, Q⊆P,*IPcl*(Q) =$\tilde{\mathbf{1}}$⊈ P. Hence Q is not IFg**pCS.

Theorem 3.17: Every intuitionistic fuzzy g**p-closed set is intuitionistic fuzzy GSP-closed but converse is not true.

Proof: Let A be intuitionistic fuzzy g**p-closed set. Let A⊆ U and U be intuitionistic fuzzyopen set in X. Since every intuitionistic fuzzy open set is intuitionistic fuzzy g*-open, U is intuitionistic fuzzy g*-open set. Now by definition of intuitionistic fuzzy g**p-closed set, *IPcl*(A) ⊆U. Note that *ISPcl*(A)⊆*IPcl*(A) is always true. We have *ISPcl*(A)⊆U whenever A ⊆ U and U is intuitionisticfuzzy open in X. Therefore, A is intuitionistic fuzzy GSP-closed set.

Example 3.18: Let X = {a, b} and ξ ={$\tilde{\mathbf{0}}$, P,$\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X, where, P = {< a, (0.6, 0.4) >, < b, (0.75, 0.2) >}. Then, Q = {< a, (0.7, 0.2) >, < b, (0.7, 0.2) >} is IFGSP-closed but it is not intuitionistic fuzzy g**p- closed.

Remark 3.19: IFSCS and IFg**pCS are independent

Example 3.20: Let X={a,b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.65) >, < b, (0.2, 0.7) >}. Consider an IFS W= {< a, (0.4, 0.6) >, < b, (0.3, 0.65) >}. Then W is IFSCS but not IFg**pCS.

Example 3.21: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.65) >, < b, (0.2, 0.7) >}. Consider an IFS W= {< a, (0.2, 0.8) >, < b, (0.2, 0.85) >}. Then W is IFg**pCS but not IFSCS.

Remark 3.22: IFGCS and IFg**pCS are independent.

Example 3.23: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.6) >, < b, (0.2, 0.8) >}. Consider an IFS W= {< a, (0.2, 0.8) >, < b, (0.1, 0.9) >}. Then W is IFg**pCS but not IFGCS.

Example 3.24: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.65) >, < b, (0.2, 0.7) >}. Consider an IFS W= {< a, (0.5, 0.5) >, < b, (0.6, 0.4) >}. Then W is IFGCS but not IFg**pCS.

Remark 3.25: IFGαCS and IFg**pCS are independent.

Example 3.26: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P,$\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.2, 0.7) >, < b, (0.3, 0.6) >}. Consider an IFS W= {< a, (0.15, 0.8) >, < b, (0.1, 0.9) >}. Then W is IFg**pCS but *Iαcl* (W)=P^c⊈P. Therefore, W is not IFGαCS.

Example 3.28: Let X= {a, b}. Let ξ = {$\tilde{0}$, P, $\tilde{1}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.7) >, < b, (0.2, 0.8) >}. Consider an IFS W= {< a, (0.5, 0.4) >, < b, (0.7, 0.3) >}. Then W is IFGαCS but not IFg**pCS.

Remark 3.29: From the above discussion. we have the following diagram of implications.

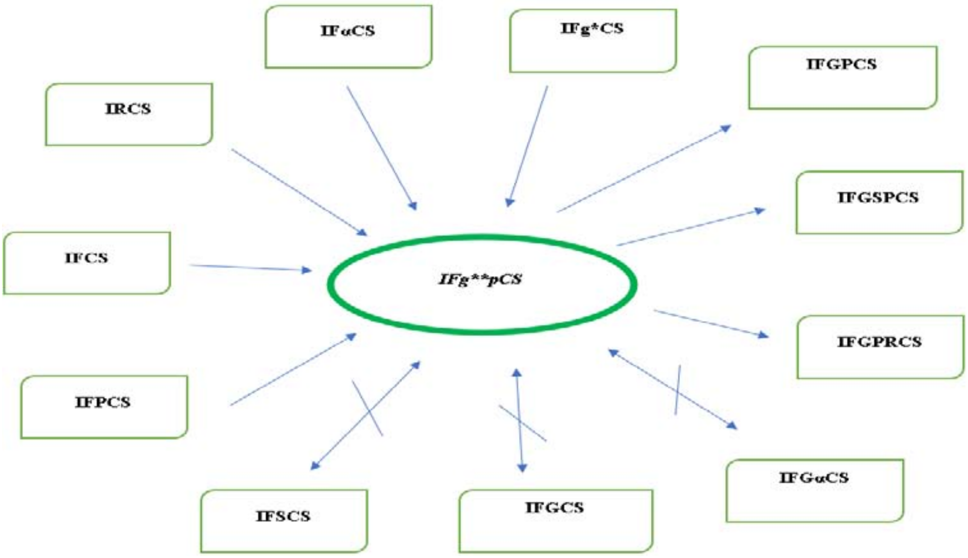

Figure 1: Relationship between intuitionistic fuzzy g**p closed set and other existing intuitionistic fuzzy closed sets.

Where A ⟶ B represents A implies B and A↮ B represents A and B are independent.

Theorem 3.30: Union of two IFg**p- closed sets is again an IFg**p- closed set.

Proof: Let A and B be two IFg**p closed sets in IFTS (X, ξ).And let Q be an IFg* OS in X, Such that A U B⊆ Q. Since A and B are IFg**pCSs we have, *IPcl*(A) ⊆ Q &*IPcl*(B) ⊆ Q. Therefore *IPcl*(A) U *IPcl*(B) ⊆*IPcl*(A U B) ⊆ Q. Hence A U B is an IFg**p closed set.

Remark 3.31: Intersection of two IFg**p closed sets need not be an IFg**p closed set.

Example 3.32: Let X= {a, b}. Let ξ = {$\tilde{0}$, P, $\tilde{1}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.7) >, < b, (0.2, 0.8) >}. Consider two IFg**pCSs W= {< a, (0.5, 0.4) >, < b, (0.9, 0.1) >}, V= {< a, (0.7, 0.3) >, < b, (0.8, 0.2) >}. Then, W ∩ V = {< a, (0.5, 0.4) >, < b, (0.8, 0.2) >} is not an IFg**pCS.

Lemma 3.33: If A is IFg*-open and IFg**p-closed set in X, then A is Intuitionistic Fuzzy pre-closed set.

Proof: Since A is IFg*-open and IFg**p-closed, then *IPcl*(A) ⊆ A, we know that A ⊆*IPcl*(A).Therefore *IPcl*(A)=A. Hence, A is IF pre closed set.

Theorem 3.34: Let A be intuitionistic fuzzy g**p-closed set in an intuitionistic fuzzy topological space (X, τ) and A ⊆ B ⊆*IPcl* (A).Then, B is intuitionistic fuzzy g**p-closed in X.

Proof: Let V be an intuitionistic fuzzy g*-open set in X such that B ⊆ V. Then A ⊆V and since A is intuitionistic fuzzy g**p-closed, *IPcl*(A) ⊆ V. By hypothesis B ⊆*IPcl* (A) then *IPcl* (B) ⊆*IPcl*(*IPcl* (A)).Thus *IPcl* (B) ⊆*IPcl*(A). This implies *IPcl* (B) ⊆ V. Hence B is intuitionistic fuzzy g**p-closed set.

Theorem 3.35: Let (X, τ) be an IFTS. Then IFC(X) = IF g**pC(X) if every IFS in (X, τ) is an IFg*OS in X, where IFC(X)denotes the collection of IFCSs of an IFTS (X, τ).

Proof: Suppose that every IFS in (X, τ) is an IFg*OS in X. Let A ∈ IF g**pC(X). Then *IPcl*(A) ⊆ U whenever A ⊆ U and U is an IFg*OS in X. Since every IFS is an IFg*OS, A is also an IFg*OS andA ⊆ A. Therefore *IPcl*(A) ⊆ A. Hence *IPcl*(A) = A. Therefore A ∈ IFC(X). Hence IF g**pC(X) ⊆ IFC(X) → (1) Let A ∈ IFC(X). Then by Theorem 3.5, A ∈ IF g**pC(X). Hence IFC(X) ⊆IFg**pC(X) → (2). From (1) and (2), we have IFC(X) = IFg**pC(X).

Theorem 3.36: In an intuitionistic fuzzy topological space (X, τ), every IFg*-open set is Intuitionistic Fuzzy pre closed iff every subset of X is IFg**p- closed set.

Proof:
Necessity: Suppose that every IFg*-open set is pre closed. Let A be a subset of X such that A⊆ U whenever U is IFg*-open. But *IPcl*(A) ⊆*IPcl*(U)=U. Therefore, A is IFg**p closed set.

Sufficiency: Suppose that every subset of X is IFg**p closed. Let U be IFg*-open. Since U is g**p-closed, we have *IPcl*(U) ⊆ U. Therefore, *IPcl*(U)=U. Hence the proof.

IV. INTUITIONISTIC FUZZY g**p -OPEN SET

In this section we introduce and studied Intuitionistic fuzzy g**p- open set and its properties.

Definition 4.1: An IFS A is said to be an Intuitionistic fuzzy g**p open set (IFg**pOS in short) in (X, ξ) if the complement A^c is an IFg**pCS in X. The family of all IFg**pOSs of IFTS (X, ξ) is denoted by IFg**pO(X)

Example 4.2: Let X = {a, b} and intuitionistic fuzzy set M is defined as M = {< a, (0.6, 0.3) >, < b, (0.7, 0.2) >}. Let ξ = {$\tilde{\mathbf{0}}$, M, $\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X.
Then P = {< a, (0.6, 0.4) >, < b, (0.4, 0.5) >} is intuitionistic fuzzy g**p- open set.

Theorem 4.3: For any IFTS (X, ξ), we have the following:

- Every IFOS is an IFg**pOS.
- Every IFPOS is an IFg**pOS.
- Every IFαOS is an IFg**pOS.
- Every IFROS is an IFg**pOS.
- Every IFg*OS is an IFg**pOS.

Proof: Straight forward.

Corollary 4.4: Every IFg**pOS need not be an IFPOS in (X,ξ). It is shown in the following example.

Example 4.5: Let X = {a, b} and intuitionistic fuzzy set P is defined as P = {< a, (0.3, 0.7) >, < b, (0.2, 0.8) >}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X.Then the intuitionistic fuzzy set Q = {< a, (0.4,0.5) >, < b, (0.1,0.9) >} is intuitionistic fuzzy g**p open set but not an intuitionistic fuzzy pre- open set.

Corollary 4.6: Every IFg**pOS need not be an IFOS in (X,ξ). It is shown in the following example.

Example 4.7 Let X = {a, b} and ξ = {$\tilde{\mathbf{0}}$, P,$\tilde{\mathbf{1}}$} be an IFTS on X, where P = {< a, (0.3, 0.7) >, < b, (0.2, 0.8) >}. Then the IFS Q = {< a, (0.8, 0.2) >, < b, (0.85, 0.1) >} is an IF g**p-open set. But *Iint*(Q) ≠ Q, therefore, it is not intuitionistic fuzzy open set.

Corollary 4.8: Every IFg**pOS need not be an IFαOS in (X,ξ). It is shown in the following example.

Example 4.9: Let X = {a, b} and ξ = {$\tilde{\mathbf{0}}$, P,$\tilde{\mathbf{1}}$} be an IFTS on X, where P = {< a, (0.3, 0.6) >, < b, (0.2, 0.7) >}. Then, the IFS S = {< a, (0.2, 0.8) >, < b, (0.1, 0.9) >} is an IF g**p- closed set. But S ⊄ *Iint(Icl(Iint(S)))* = A. Therefore, S is not IFα-open.

Corollary 4.10: Every IFg**pOS need not be an IFROS in (X, ξ). It is shown in the following example.

Example 4.11: Let X = {a, b} and intuitionistic fuzzy set P is defined as P = {< a, (0.3, 0.7) >, < b, (0.2, 0.7) >}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X.

Then the intuitionistic fuzzy set A = {< a, (0.15, 0.85) >, < b, (0.2, 0.8) >} is intuitionistic fuzzy g**p- open set. But *Iint*(*Icl*(A)) ≠ A, therefore it is not intuitionistic fuzzy regular-open set.

Corollary 4.12: Every IFg**pOS need not be an IFg*OS in (X,ξ). It is shown in the following example.

Example 4.13: Let X = {a, b} and intuitionistic fuzzy set G is defined as G = {< a, (0.5, 0.3) >, < b, (0.7, 0.2) >}. Let ξ = {$\tilde{\mathbf{0}}$, G, $\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X. Then the IFS S = {< a, (0.7, 0.3) >, < b, (0.6, 0.4) >} is intuitionistic fuzzy g**p – open set but not IF g*-open set.

Theorem 4.14: For any IFTS (X, ξ), we have the following:

- Every IF g**p-open set IF GP-open.
- Every IF g**p-open set IFGPR- open.
- Every IF g**p-open set IFGSP- open.

Corollary 4.15: Every IFGPOS need not be an IFg**pOS in (X, ξ). It is shown in the following example.

Example 4.16: Let X = {a, b} and intuitionistic fuzzy sets P is defined as P = {< a, (0.3, 0.7) >, < b, (0.2, 0.7) >}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set M = {< a, (0.4, 0.5) >, < b, (0.6, 0.4) >} is intuitionistic fuzzy GP-open but it is not intuitionistic fuzzy g**p-open.

Corollary 4.17: Every IFGPROS need not be an IFg**pOS in (X, ξ). It is shown in the following example.

Example 4.18: Let X = {a, b} and intuitionistic fuzzy sets P is defined as P = {< a, (0.6, 0.3) >, < b, (0.7, 0.2) >}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X. Q = {< a, (0.2, 0.7) >, < b, (0.2, 0.7) >} is IFGPROS but not IFg**pOS.

Corollary 4.19: Every IFGSPOS need not be an IFg**pOS in (X,ξ). It is shown in the following example.

Example 4.20: Let X = {a, b} and intuitionistic fuzzy sets P is defined as P = {< a, (0.6, 0.4) >, < b, (0.75, 0.2) >}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X. Q = {< a, (0.2, 0.7) >, < b, (0.2, 0.7) >} is IFGSP-open but it is not intuitionistic fuzzy g**p- open.

Remark 4.21: IFSOS and IFg**pOS are independent.

Example 4.22: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.65) >, < b, (0.2, 0.7) >}. Consider an IFS W= {< a, (0.6, 0.4) >, < b, (0.65, 0.3) >}. Then W is IFSOS but not IFg**pOS.

Example 4.23: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.65) >, < b, (0.2, 0.7) >}. Consider an IFS W= {< a, (0.8, 0.2) >, < b, (0.85, 0.15) >}. Then W is IFg**pOS but not IFSOS.

Remark 4.24: IFGOS and IFg**pOS are independent.

Example 4.25: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.6) >, < b, (0.2, 0.8) >}. Consider an IFS W= {< a, (0.8, 0.2) >, < b, (0.9, 0.1) >}. Then W is IFg**pOS but not IFGOS.

Example 4.26: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.65) >, < b, (0.2, 0.7) >}. Consider an IFS W= {< a, (0.5, 0.5) >, < b, (0.4, 0.6) >}. Then W is IFGOS but not IFg**pOS.

Remark 4.27: IFGαOS and IFg**pOS are independent.

Example 4.28: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.2, 0.7) >, < b, (0.3, 0.6) >}. Consider an IFS W= {< a, (0.8, 0.15) >, < b, (0.9, 0.1) >}. Then W is IFg**pOS but not IFGαOS.

Example 4.29: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.3, 0.7) >, < b, (0.2, 0.8) >}. Consider an IFS W= {< a, (0.4, 0.5) >, < b, (0.3, 0.7) >}. Then W is IFGαOS but not IFg**pOS.
.

Theorem 4.30: If A and B are IFg**p-open sets in an IFTS (X, ξ), then (A∩B) is an IFg**p-open sets in (X, ξ).

Proof: Let A and B be IFg**p-open sets in IFTS (X, ξ). Therefore, A^c and B^c are IFg**p-closed sets in (X, ξ).
By Theorem 3.30, ($A^c \cup B^c$) is IFg**p-closed sets in (X, ξ).Since ($A^c \cup B^c$) = $(A\cap B)^c$, (A∩B) is IFg**p open sets in (X, ξ).

Remark 4.31: Union of two IFg**p-open sets need not be an IFg**p-open set.

Example 4.32: Let X= {a, b}. Let ξ = {$\tilde{\mathbf{0}}$, P, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where P = {< a, (0.7, 0.3) >, < b, (0.8, 0.2) >}. Consider two IFg**pOSs W= {< a, (0.4, 0.5) >, < b, (0.1, 0.9) >}, V= {< a, (0.3, 0.7) >, < b, (0.2, 0.8) >}. Then, W ∪ V = {< a, (0.4, 0.5) >, < b, (0.2, 0.9) >} is not an IFg**pOS.

Theorem 4.33: An IFsubset B of a IFTS (X,ξ) is IFg**p- open if and only if K⊆*IPint*(B) whenever K is IFg*-closed and K⊆B.

Proof: Suppose that B is IFg**p - open in X, K is IFg*-closed and K⊆B. Then K^c is IFg*-open and $B^c \subseteq K^c$. Since, B^c is IFg**p- closed, then *IPcl*(B^c) ⊆K^c. But, *IPcl* (B^c) = [*IPint*(B)]^c⊆K^c. Hence K⊆*IPint*(B).

Conversely, suppose that K⊆*IPint*(B) whenever K⊆B and K is IFg*-closed. If H is an IF g*-open set in X containing B^c, then H^c is a IFg*-closed setcontained in B. Hence by hypothesis, H^c⊆*IPint*(B), then by taking the complements, we have, *IPcl*(B^c) ⊆H. Therefore B^c is IFg**p − closed in X and hence B is IFg**p-open in X.

Corollary 4.34: If B is IFg**p -open in IFTS (X,ξ), then H=X, whenever H is IFg*-open and *IPint*(B) U (B^c) ⊆H.

Proof: Assume that H is IFg*-open and *IPint*(B) ∪(B^c) ⊆H. Hence H^c⊆*IPcl*(B^c) ∩B = *IPcl*(B^c) - (B^c). Since, H^c is IFg*-closed and B^c is IFg**p −closed, then by Theorem 3.34, H^c= ∅ and hence, H=X.

Theorem 4.35: Let (X, ξ) be an IFTS. Then IFO(X) = IFg**pO(X) if every IFS in (X, ξ) is an IFg*-open in X, where IFO(X) denotes the collection of IFOSs of an IFTS(X, ξ).

Proof: Suppose that every IFS in (X, ξ) is an IFg*-open in X. Then by theorem 3.35, we have IFC(X) = IFg**pC(X). Therefore IFO(X) = IFg**pO(X).

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