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i- Regular Generalized Closed sets in Isotonic Spaces

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ABSTRACT

The purpose of this paper is to define and study i-regular generalized closed sets in isotonic spaces. We also introduce the concept of i-regular generalized - continuous functions and investigate their properties.

Key Words: *i- regular generalized closed sets, i- regular generalized continuous maps, i regular generalized closed maps,* T_{irg} *space.*

AMS Classification: *54A05.*

1. INTRODUCTION:

Levine [7] initiated the study of g-closed sets, that is, a subset A of a topological space (X, τ) is g-closed if the closure of A is included in every open superset of A.

A function μ from the power set *P(X)* of a nonempty set *X* into itself is called a Generalized closure operator (briefly GCO) on *X* and the pair *(X,μ)* is said to be Generalized closure space (briefly GCS).

In this paper, we introduce and study the notion of irg - closed sets in isotonic spaces. We define a new class of space namely T_{irg} -space and their properties are studied. Further, we introduce a class of irg- continuous maps and irg- closed maps and their characterizations are obtained.

PRELIMINARIES:

An operator μ: *P(X)* →*P(X)* is called grounded if μφ = φ, isotonic if A⊆ B⊆ X implies μA ⊆ μB, expansive if A⊆ μA for every A⊆ X, idempotent if μ μA = μA for every A⊆X and additive if μ (A U B) ⊆ μA U μB for all subsets A and B of X.

Definitions: 1.1
(i) The space (X, μ) is said to be isotonic if μ is grounded and isotonic.
(ii) The space (X, μ) is said to be a neighborhood space if μ is grounded, expansive and isotonic.
(iii) The space (X, μ) is said to be a closure space if μ is grounded, expansive, and isotonic and idempotent.
(iv) The space (X, μ) is said to be a Cech closure space if μ is grounded, expansive, isotonic and additive.
(v) A subset A of X is said to be closed if μA = A. It is open if its complement is closed.
(vi) The empty set and the whole space are both open and closed.

Definition: 1.2 An isotonic space (Y, l) is said to be a subspace of (X, μ) if Y⊆ X and μ (A) = μ (A)∩Y for each subset A ⊆Y. If Y is closed in (X, μ) then the subspace (Y, l) of (X, μ) is said to be closed too.

Definition: 1.3 Let (X, μ) and (Y, l) be isotonic spaces. A map f: (X, μ) →(Y, l) is said to be continuous, if f (μ A) ⊆ μ f(A) for every subset A ⊆ F.

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Definition: 1.4 Let (X, μ) and (Y, l) be isotonic spaces. A map f: (X, μ) →(Y, l) is said to be closed (resp. open) if f (F) is a closed (resp. open) subset of (Y, l) whenever F is a closed (resp. open) subset of (X, μ).

Definition: 1.5 Let Let (X, μ) and (Y, l) be isotonic spaces. A map f: (X, μ) →(Y, l) is said to be closure preserving if μ f(A) ⊆ l f(A) for all A⊆ *P(X)*

Definition: 1.6 The product of a family $\{(X_\alpha, \mu_\alpha) ; \alpha\in I\}$ of isotonic spaces denoted by $\prod_{\alpha\in I}(X_\alpha, \mu_\alpha)$ is the isotonic space $\prod_{\alpha\in I}(X_\alpha, \mu_\alpha)$ where $\prod_{\alpha\in I} X_\alpha$ denotes the Cartesian product of sets X_α , $\alpha\in I$ and μ is isotonic operator generated by the projections $\pi_\alpha : \prod_{\alpha\in I}(X_\alpha, \mu_\alpha) \to (X_\alpha, \mu_\alpha)$, $\alpha\in I$ i.e defined by $\mu(A) = \prod_{\alpha\in I} \mu_\alpha \pi_\alpha(A)$ for each $A\subseteq \prod_{\alpha\in I} X_\alpha$.
Clearly, if $\{(X\alpha, \mu_\alpha): \alpha\in I\}$ is a family of isotonic spaces, then the projection map $\pi_\beta : \prod_{\alpha\in I}(X_\alpha, \mu_\alpha)\to (X_\beta, \mu_\beta)$ is closed and continuous for every $\beta\in I$.

Proposition: 1.7 Let $\{(X_\alpha, \mu_\alpha): \alpha\in I\}$ be a family of isotonic spaces, let $\beta\in I$ and $F\subseteq X_\beta$. Then F is a closed subset of (X_β, μ_β) if and only if $F \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$ is a closed subset of $\prod_{\alpha\in I}(X_\alpha, \mu_\alpha)$.

Proof: Let $\beta\in I$ and let F be a closed subset of (X_β, μ_β).Since π_β is closure preserving, $\pi_\beta^{-1}(F)$ is a closed subset of $\prod_{\alpha\in I}(X_\alpha, \mu_\alpha)$.But $\pi_\beta^{-1}(F) = F \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$

Hence $F \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$ is a closed subset of $\prod_{\alpha\in I}(X_\alpha, \mu_\alpha)$.

Conversely, let $F \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$ is a closed subset of $\prod_{\alpha\in I}(X_\alpha, \mu_\alpha)$. Since π_β is closed,

$\pi_\beta\left(F \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha\right) = F$ is a closed subset of $\prod_{\alpha\in I}(X_\beta, \mu_\beta)$.

Proposition: 1.8 Let $\{(X_\alpha, \mu_\alpha): \alpha\in I\}$ be a family of isotonic spaces, let $\beta\in I$ and $G\subseteq X_\beta$.Then G is a open subset of (X_β, μ_β) if and only if $G \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$ is an open subset of $\prod_{\alpha\in I}(X_\alpha, \mu_\alpha)$.

2. i-REGULAR GENERALIZED - CLOSED SETS:

Definition: 2.1 Let (X, μ) be an isotonic space. A subset A ⊆ X is called a i-regular generalized closed (briefly irg – closed) set, if μA ⊆ G whenever G is a regular open subset of (X, μ) with A ⊆ G. A subset A of X is called a i-regular generalized open set (briefly irg –open) if its complement is a irg- closed subset.

Proposition: 2.2 Every closed set is irg - closed.

Proof: Let G be a regular -open subset of (X, μ) such that A ⊆ G. Since A is a closed set, we have μA = A ⊆ G. Therefore A is irg-closed.

The converse need not true as seen in the following example:

Example: 2.3 Let X = {a, b} and define an isotonic operator μ on X by μϕ = ϕ , μ {a} = μ {b} = μX = X. Then {a} is irg - closed but it is not closed.

Proposition: 2.4 Let (X, μ) be an isotonic and let μ be additive. If A and B are irg - closed subsets of (X, μ), then A ∪ B is also irg - closed.

Proof: Let G be a regular open subset of (X, μ) such that A∪B ⊆ G, then A ⊆ G and B ⊆ G. Since A and B are irg - closed, we have μ (A) ⊆G, and μ (B) ⊆G, Consequently, μ (A∪B) = μ(A) ∪ μ(B) ⊆ G. Therefore A ∪ B is irg - closed.

Remark: The intersection of two irg- closed sets need not be irg - closed as can be seen by the following example.

Example: 2.5 Let X = {1,2,3} and define an isotonic operator μ on X by μϕ = ϕ , μ{1} ={ 1,2}; μ{2}= μ {3 }= μ {2,3} = {2,3}; μ {1,2} = μ {1,3}= μ X = X. If A = {1,2} and B = {1,3}are irg closed, then {1,2}∩{1,3}={1} which is not irg - closed.

Proposition: 2.6 Let (X, μ) be an isotonic space. If A is irg - closed and F is regular closed in (X, μ), then A ∩ F is irg- closed.

Proof: Let G be a regular open subset of (X, μ) such that A ∩ F ⊆ G, Then A ⊆ G ∪ (X-F).and so, Since A is irg- closed , μ (A) ⊆ G ∪ (X-F), Then μ (A) ∩ F ⊆ G , μ (A ∩F) ⊆ G. Therefore A ∩ F is irg - closed.

Proposition: 2.7 Let (Y, l) be a closed subspace of (X, μ). If F is a irg - closed subset of (Y, l), then F is a irg - closed subset of (X, μ).

Proof: Let G be regular open set of (X, μ) such that F ⊆ G. Since F is irg - closed and G ∩ F is regular open μ (F) ∩ Y ⊆ G, But Y is closed subset of (X, μ) and μ (F) ⊆ G, where G is a regular open set. Therefore F is a irg- closed set of (X, μ).

The following statement is obvious

Proposition: 2.8 Let (X, μ) be an isotonic space and let A ⊆ X. If A is both regular open and irg - closed then A is closed.

Proposition: 2.9 Let (X, μ) be an isotonic space and let k be idempotent. If A is a irg- closed subset of (X, μ) such that A ⊆ B ⊆ μ (A), then B is a irg - closed subset of (X, μ).

Proof: Let G be a regular open subset of (X, μ) such that B ⊆ G. Then A ⊆ G, Since A is irg - closed, μ (A) ⊆ G. As G is idempotent, μ (B) ⊆ μ (μ (A)) = μ (A) ⊆ G, Hence B is irg - closed. Since (X, μ) is grounded, expansive, isotonic and idempotent. It now becomes a closure space.

Proposition: 2.10 Let (X, μ) be an isotonic space and let A ⊆ X. If A is irg - closed, then μ (A) – A has no non empty regular closed subset.

Proof: Suppose that A is irg- closed. Let F be a regular - closed set of μ (A) – A. Then F⊆ μ (A) ∩ (X– A), so A ⊆ X-F. Consequently, since A is irg – closed F ⊆ X- μ (A), Since F ⊆ μ (A) , F ⊆ (X- μ (A)) ∩ μ (A) = ϕ,

thus F = ϕ.Therefore μ (A) – A contains no non empty regular closed subset.

Proposition: 2.11. Let (X, μ) be an isotonic space. A set A ⊆ X is irg- open if and only if F ⊆ X - μ (X-A) whenever F is regular closed subset of (X, μ) with F ⊆ A.

Proof: Suppose that A is irg- open and F be a regular closed subset of (X, μ) such that F⊆ A. Then X-A ⊆ X –F. But X- A is irg- closed and X-F is regular open. It follows that μ (X-A) ⊆ X –F. (i.e) F ⊆ X- μ (X-A).

Conversely, Let G be a regular open subset of (X, μ) such that X-A ⊆ G. Then X-G ⊆ A. Therefore X-U ⊆ μ (X-A). Consequently, μ (X-A) ⊆ G. Hence X- A is irg- closed and so A is irg- open.

Remark: 2.12 The union of two irg- open sets need not be irg- open.

Proposition: 2.13 Let (X, μ) be an isotonic space. If A and B are irg- open of (X, μ), then A ∩ B is irg – open.

Proof: Let F be a regular closed subset of (X, μ) such that F ⊆ A ∩ B . Then X – (A ∩ B) ⊆ X – F.

Consequently, (X-A) ∪ (X-B) ⊆ X – F. By proposition 2.4, (X-A) ∪ (X-B) is irg – closed. Thus, μ [(X-A) ∪ (X-B)] ⊆ X – F. Hence F ⊆ X – μ [(X-A) ∪ (X-B)] ⊆ X - μ (X- (A ∩ B)), By proposition 2.11, A ∩ B is irg – open.

Proposition: 2.14 Let (X, μ) be an isotonic space. If A is irg- open subsets of (X, μ), then $X = G$ whenever G is regular open and $(X- \mu (X- A)) \cup (X- A) \subseteq G$.

Proof: Suppose that A is irg – open. Let G be a regular open subset of (X, μ) such that $(X- \mu (X- A)) \cup (X- A) \subseteq G$. Then $X – G \subseteq X-[(X- \mu (X- A)) \cup (X- A)]$. Therefore $X – G \subseteq \mu (X- A) \cap A$ or equivalently, $X- G \subseteq \mu (X- A) – (X- A)$. But X – G is regular closed and X – A is irg – closed. Then by proposition 2.10, $X – G = \phi$ .Consequently X = G.

Proposition: 2.15 Let (X, μ) be an isotonic space and let $A \subseteq X$. If A is irg - closed, then $\mu (A) – A$ is irg- open.

Proof: Suppose that A is irg - open. Let F be a regular - closed set of (X, μ) such that $F \subseteq \mu (A) – A$ .By proposition 2.10 $F = \phi$, and hence $F \subseteq X - \mu [(X- \mu (A- A))$. By proposition 2.11 $\mu (A) – A$ is irg – open.

Proposition: 2.16 Let $\{(X_\alpha, \mu_\alpha): \alpha\in I\}$ be a family of isotonic spaces, let $\beta\in I$ and $G \subseteq X_\beta$. Then G is a irg - open subset of (X_β, μ_β) if and only if $G \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$ is a irg- open subset of $\prod_{\alpha\in I} (X_\alpha, \mu_\alpha)$.

Proof: Let F be a regular closed subset of $\prod_{\alpha\in I} (X_\alpha, \mu_\alpha)$. such that $F \subseteq G \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$, Then $\pi_\beta (F) \subseteq G$. Since $\pi_\beta (F)$ is regular closed and G is irg - open in (X_β, μ_β),

$\pi_\beta(F) \subseteq X_\beta - \mu_\beta (X_\beta\text{-}G)$. Therefore $F\subseteq \pi_\beta^{-1}(X_\beta - \mu_\beta (X_\beta\text{-}G)) = \prod_{\alpha\in I} X_\alpha - \prod_{\alpha\in I} \mu_\alpha \pi_\alpha(\prod_{\alpha\in I} X_\alpha - G\times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha)$, hence $G \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$ is a irg- open subset of $\prod_{\alpha\in I} (X_\alpha,\mu_\alpha)$.

Conversely, Let F be a regular closed subset of (X_β,μ_β) such that $F\subseteq G$. Then $F \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha \subseteq G\times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$).

since $F \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$ is regular closed and $G \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$ is irg- open in $\prod_{\alpha\in I} (X_\alpha,\mu_\alpha)$.

$$F \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha \subseteq \prod_{\alpha\in I} X_\alpha - \prod_{\alpha\in I} \mu_\alpha\pi_\alpha(\prod_{\alpha\in I} X_\alpha - G\times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha).$$

Therefore

$$\prod_{\alpha\in I} \mu_\alpha\pi_\alpha(X_\beta - G)\times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha) \subseteq \prod_{\alpha\in I} X_\alpha - F \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha = (X_\beta - F) \prod_{\alpha\in I} X_\alpha$$

Consequently, $\mu_\beta (X_\beta - G) \subseteq X_\beta - F$ implies $F \subseteq X_\beta - \mu_\beta (X_\beta - G)$. Hence G is an irg - open subset of (X_β, μ_β).

Proposition: 2.17 Let $\{(X_\alpha, \mu_\alpha): \alpha\in I\}$ be a family of isotonic spaces, let $\beta\in I$ and $F\subseteq X_\beta$. Then F is a irg- closed subset of (X_β,μ_β) if and only if $F \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$ is a irg - closed subset of $\prod_{\alpha\in I} (X_\alpha, \mu_\alpha)$.

Proof: Let F be a irg- closed subset of (X_β,μ_β).Then X_β -F is an irg - open subset of (X_β,μ_β). By proposition 2.16, $(X_\beta -F) \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha = \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha - F \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$ is an irg - open subset of $\prod_{\alpha\in I} (X_\alpha, \mu_\alpha)$.Hence $F \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$ is an irg - closed subset of $\prod_{\alpha\in I} (X_\alpha, \mu_\alpha)$.

Conversely, let G be a regular open subset of (X_β,μ_β) such that $F \subseteq G$, Then $F \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha \subseteq G \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$. Since $F \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$ is irg closed and $G \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$ is regular open in $\prod_{\alpha\in I} (X_\alpha, \mu_\alpha)$,

$\prod_{\alpha\in I} \mu_\alpha\pi_\alpha(F\times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\beta) \subseteq G \times \prod_{\alpha\in I} X_\alpha$. Consequently, $\mu_\beta(F) \subseteq G$. Therefore, F is a irg - closed subset of (X_β, μ_β).

Proposition: 2.18 Let $\{(X_\alpha, \mu_\alpha): \alpha\in I\}$ be a family of isotonic spaces, For each $\beta\in I$ and let $\pi_\beta : \prod_{\alpha\in I} X_\alpha \to X_\beta$ be a projection map. Then

(i) If F is an irg closed subset of $\prod_{\alpha\in I} (X_\alpha, \mu_\alpha)$, then $\pi_\beta(F)$ is an irg- closed subset of (X_β, μ_β).

(ii) If F is an irg closed subset of (X_β,μ_β),then $\pi_\beta^{-1}(F)$ is an irg closed subset of $\prod_{\alpha\in I} (X_\alpha, \mu_\alpha)$,

Proof: (i) Let F be an irg closed subset of $\prod_{\alpha\in I} (X_\alpha, \mu_\alpha)$ and let G be a regular open subset of (X_β,μ_β) such that $\pi_\beta(F)\subseteq G$. Then $F\subseteq \pi_\beta^{-1}(G) = G\times\prod_{\alpha\in I} X_\alpha$. Since F is an irg closed and $G\times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\beta$ is regular open, $\prod_{\alpha\in I} \mu_\alpha\pi_\alpha(F) \subseteq G \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$. Consequently $\mu_\beta\pi_\beta(F) \subseteq G$. Hence $\pi_\beta(F)$ is an irg closed subset of (X_β,μ_β).

(ii) Let F be an irg closed subset of (X_β,μ_β),Then $\pi_\beta^{-1}(F) = F \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$.

Therefore, we have, $F \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$ is an irg closed subset of $\prod_{\alpha\in I} (X_\alpha, \mu_\alpha)$.Therefore, $\pi_\beta^{-1}(F)$ is an irg closed subset of $\prod_{\alpha\in I} (X_\alpha, \mu_\alpha)$.

Definition: 2.19 An isotonic space (X, μ) is said to be a T_{irg} -space if every irg closed subset of (X, μ) is closed.

Proposition: 2.20 Let (X, μ) be an isotonic space. Then

(i) If (X, μ) is a T_{irg} -space then every singleton subset of X is either regular closed or open.

(ii)If every singleton subset of X is a regular - closed subset of (X, μ), then (X, μ) is a T_{irg} -space.

Proof: (i) Suppose that $(X. \mu)$ is a T_{irg} -space. Let $x\in X$ and assume that $\{x\}$ is not regular - closed. Then $X-\{x\}$ is not regular - open. Since X is the only regular - open set which contains $X-\{x\}$ this implies $X-\{x\}$ is irg - closed.

Since (X, μ) is a T_{irg}-space, $X-\{x\}$ is closed or equivalently $\{x\}$ is open.

(ii)Let A be an irg - closed subset of (X, μ).To prove: A is closed. Suppose that $x\notin A$. Then $\{x\} \subseteq X-\{x\}$. Since A is irg - closed and $X-\{x\}$ is regular – open, $\mu(A) \subseteq X-\{x\}$,(i.e) $\{x\} \subseteq X- \mu(A)$. Hence $x\notin \mu(A)$ and thus $\mu(A) \subseteq A$.

Therefore A is closed subset of (X,μ). Hence (X,μ) is a T_{irg}-space.

3. i REGULAR GENERALIZED - CONTINUOUS MAPS:

Definition: 3.1 Let (X, μ) and (Y, l) be an isotonic space. A mapping $f : (X, \mu) \to (Y, l)$ is said to be irg - continuous, if $f^{-1}(F)$ is irg - closed set of (X, μ) for every closed set F in (Y, l).

Proposition: 3.2 Every continuous map is irg - continuous.

Proof: Let f : (X, μ) → (Y, l) be continuous, Let F be a closed set of (Y, l).Since f is continuous, then $f^{-1}(F)$ is closed set of (X, μ).Since every closed set is irg - closed of (X, μ),we have $f^{-1}(F)$ is a irg - closed set of (X, μ).Therefore f is a irg – continuous map.

Proposition: 3.3 Let (X, μ) be a T_{irg} space and let (Y, l) be an isotonic space.
If f : (X, μ) → (Y, l) is said to be regular - continuous, then f is irg- continuous,

Proof: Let F be a closed subset of (Y, l).Since F is regular - continuous, then $f^{-1}(F)$ is regular-closed set of (X, μ). Since (X, μ) is a T_{irg} space, $f^{-1}(F)$ is an irg- closed set of (X, μ). Hence, f is irg - continuous,

The following statement is obvious.

Proposition: 3.4 Let (X, μ), (Y, l) and (Z, m) be an isotonic spaces. If f: (X, μ) →(Y, l) is irg - continuous and g: (Y, l) →(Z, m) is continuous then g ∘ f : (X, μ) →(Z, m) is irg - continuous.

Proposition: 3.5 Let (X, μ), (Z, m) be isotonic spaces and let (Y, l) be a T_{irg} - space. If f: (X, μ) →(Y, l) and g: (Y, l) →(Z, m) are irg - continuous, then g ∘ f: (X, μ) → (Z,m) is irg - continuous.

Proof: Let F be a closed subset of (Z, w).Since g is irg - continuous, then $g^{-1}(F)$ is irg - closed set of (Y, l).Since (Y, l) is a T_{irg} space, $g^{-1}(F)$ is a closed set of (Y, l) which implies that $(g \circ f)^{-1}(F)$ is a irg – closed subset of (X, μ). Hence, g ∘ f is irg – continuous.

Proposition: 3.6 Let $\{(X_\alpha, \mu_\alpha): \alpha\in I\}$ and $\{(Y_\alpha, l_\alpha): \alpha\in I\}$ be families of isotonic spaces. For each α∈I, let $f_\alpha: X_\alpha \to Y_\alpha$ be a map and f: $\prod_{\alpha\in I} X_\alpha \to \prod_{\alpha\in I} Y_\alpha$ be a map defined by $f((x_\alpha)_{\alpha\in I}) = (f_\alpha(x_\alpha)_{\alpha\in I})$.If f: $\prod_{\alpha\in I}(X_\alpha, \mu_\alpha) \to \prod_{\alpha\in I}(Y_\alpha, l_\alpha)$ is irg - continuous, then $f_\alpha: (X_\alpha, \mu_\alpha) \to (Y_\alpha, l_\alpha)$ is irg - continuous for each α∈I.

Proof: Let β∈I and F be a closed subset of (Y_β, l_β).Then $F \times \prod_{\alpha\neq\beta, \alpha\in I} Y_\alpha$ is a closed subset of $\prod_{\alpha\in I}(Y_\alpha, l_\alpha)$, Since f is irg - continuous, $f^{-1}(F \times \prod_{\alpha\neq\beta, \alpha\in I} Y_\alpha) = f_\beta^{-1}(F) \times \prod_{\alpha\neq\beta, \alpha\in I} X_\alpha$ is an irg - closed subset of $\prod_{\alpha\in I}(X_\alpha, \mu_\alpha)$.By proposition 2.17, $f_\beta^{-1}(F)$ is a irg- closed subset of (X_β, μ_β), Hence f_β is irg - continuous.

Definition: 3.7 Let (X, μ) and (Y, l) be an isotonic spaces. A map f : (X, μ) →(Y, l) is called irg – irresolute if $f^{-1}(F)$ is irg - closed set in (X, μ) for every irg - closed set F in (Y, l).

Definition: 3.8 Let (X, μ) and (Y, l) be isotonic spaces. A map f : (X, μ) →(Y, l) is called irg–closed if f(F) is a irg - closed subset of (Y, l) for every closed set F of (X, μ).

Proposition: 3.9 Let (X, μ) and (Y, l) be isotonic spaces and let μ be additive. Let A and B be closed subsets of (X, μ) such that X = A U B. Let f : (A, μ_A) →(Y, l) and g: (B, μ_B) → (Y, l) be irg – continuous maps such that f(x) = g(x) for every x ε A and h(x) = g(x) if x ε B. Then h: (X, μ) →(Y, l) is irg – continuous.

Proof: Let F be a closed subset of (Y, l).Clearly $h^{-1}(F) = f^{-1}(F) \cup g^{-1}(F)$ since f : (A, μ_A) →(Y, l) and g: (B, μ_B)→(Y, l) are irg –continuous, $f^{-1}(F)$ and $g^{-1}(F)$ are irg – closed subset of (A, μ_A) and (B, μ_B) respectively. As A is a closed subset of (X, μ), $f^{-1}(F)$ is a irg – closed subset of (X, μ) by proposition 2.7 Similarly $g^{-1}(F)$ is a irg – closed subset of (X, μ).By proposition 2.4, $f^{-1}(F) \cup g^{-1}(F)$ is a irg – closed subset of (X, μ). Therefore $h^{-1}(F)$ is an irg-closed subset of (X,).

Hence h is irg-continuous.

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