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HOMOMORPHISM AND ANTI HOMOMORPHISM FUNCTIONS IN BIPOLAR VALUED VAGUE SUBRINGS OF A RING

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ABSTRACT

In this paper, bipolar valued vague subring of a ring is studied by homomorphism and anti homomorphism and some properties are discussed. These properties are useful to further research.

Key Words: *Fuzzy subset, vague subset, bipolar valued fuzzy subset, bipolar valued vague subset, bipolar valued vague subring, bipolar valued vague normal subring, intersection, image and preimage.*

INTRODUCTION

In 1965, Zadeh [14] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval valued fuzzy sets, vague sets, soft sets etc. Grattan-Guiness [7] discussed about fuzzy membership mapped onto interval and many valued quantities. Vague set is an extension of fuzzy set and it is appeared as a unique case of context dependent fuzzy sets. The vague set was introduced by W.L.Gau and D.J.Buehrer [6]. Lee [8] introduced the notion of bipolar valued fuzzy sets. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [–1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [–1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [8, 9]. Fuzzy subgroup was introduced by Azriel Rosenfeld [2]. RanjitBiswas [11] introduced the vague groups. Cicily Flora. S and Arockiarani.I [4] have introduced a new class of generalized bipolar vague sets. Anitha.M.S., et.al.[1] defined as bipolar valued fuzzy subgroups of a group and Balasubramanian.A et.al[3] have defined the bipolar interval valued fuzzy subgroups of a group. K.Murugalingam and K.Arjunan[10] have discussed about interval valued fuzzy subsemiring of a semiring and Somasundra Moorthy.M.G.,[12] gave a idea about the fuzzy ring. Bipolar valued multi fuzzy subsemirings of a semiring have been introduced by Yasodara.B and KE.Sathappan[13]. Anitha.K., et.al.[5] defined as bipolar valued vague subrings of a ring. Here, the concept of bipolar valued vague subring of a ring is introduced and estaiblished some results. Homomorphism and anti homomorphism are applied in bipolar valued vague subring of a ring.

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1. PRELIMINARIES

Definition 1.1: [14] Let X be any nonempty set. A mapping M: X → [0, 1] is called a fuzzy subset of X.

Definition 1.2: [6] A vague set A in the universe of discourse U is a pair $[t_A, 1-f_A]$, where t_A : U→ [0, 1] and f_A : U→ [0, 1] are mappings, they are called truth membership function and false membership function respectively. Here $t_A(x)$ is a lower bound of the grade of membership of x derived from the evidence for x and $f_A(x)$ is a lower bound on the negation of x derived from the evidence against x and $t_A(x) + f_A(x) \leq 1$, for all x∈U.

Definition 1.3: [6] The interval $[t_A(x), 1-f_A(x)]$ is called the vague value of x in A and it is denoted by $V_A(x)$, i.e., $V_A(x) = [t_A(x), 1-f_A(x)]$.

Example 1.4: A = { < a, [0.2, 0.9] >, < b, [0.3, 0.8] >, < c, [0.5, 0.9] >} is a vague subset of X = {a, b, c }.

Definition 1.5: [8] A bipolar valued fuzzy set (BVFS) A in X is defined as an object of the form A = { < x, $A^+(x)$, $A^-(x)$ >/ x∈X}, where A^+ : X→ [0, 1] and A^- : X→ [−1, 0]. The positive membership degree $A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A.

Example 1.6: A = {< a, 0.8, −0.4 >, < b, 0.6, −0.3 >, < c, 0.2, −0.9 >} is a bipolar valued fuzzy subset of X = {a, b, c}.

Definition 1.7: [4] A bipolar valued vague subset A in X is defined as an object of the form A = {⟨ x, $[t_A^+(x), 1-f_A^+(x)]$, $[-1-f_A^-(x), t_A^-(x)]$ ⟩ / x∈X}, where t_A^+: X→ [0, 1], f_A^+ : X→ [0, 1], t_A^- : X→ [−1, 0] and f_A^- : X→ [−1, 0] are mapping such that $t_A(x) + f_A(x) \leq 1$ and $-1 \leq t_A^- + f_A^-$. The positive interval membership degree $[t_A^+(x), 1-f_A^+(x)]$ denotes the satisfaction region of an element x to the property corresponding to a bipolar valued vague subset A and the negative interval membership degree $[-1-f_A^-(x), t_A^-(x)]$ denotes the satisfaction region of an element x to some implicit counter-property corresponding to a bipolar valued vague subset A. Bipolar valued vague subset A is denoted as A = { ⟨ x, $V_A^+(x)$, $V_A^-(x)$ ⟩ / x∈X }, where $V_A^+(x)$, = $[t_A^+(x), 1-f_A^+(x)]$ and $V_A^-(x) = [-1-f_A^-(x), t_A^-(x)]$.

Note that:. [0] = [0, 0], [1] = [1, 1] and [−1] = [−1, −1].

Example 1.8: [A] = {< a, [0.2, 0, 4], [−0.6, −0.2] >, < b, [0.3, 0.4], [−0.5, −0.3] >, < c, [0.4, 0.6], [−0.7, −0.2] > } is a bipolar valued vague subset of X = {a, b, c }.

Definition 1.9: [4] Let A = $\langle V_A^+, V_A^- \rangle$ and B = $\langle V_B^+, V_B^- \rangle$ be two bipolar valued vague subsets of a set X. We define the following relations and operations:

(i) [A] ⊂ [B] if and only if $V_A^+(u) \leq V_B^+(u)$ and $V_A^-(u) \geq V_B^-(u)$, ∀ u∈X.

(ii) [A] = [B] if and only if $V_A^+(u) = V_B^+(u)$ and $V_A^-(u) = V_B^-(u)$, ∀ u∈X.

(iii) [A]∩[B] = { ⟨ u, rmin ($V_A^+(u)$, $V_B^+(u)$), rmax ($V_A^-(u)$, $V_B^-(u)$) ⟩ / u∈X }.

(iv) [A]∪[B] = { ⟨ u, rmax ($V_A^+(u)$, $V_B^+(u)$), rmin ($V_A^-(u)$, $V_B^-(u)$) ⟩ / u∈X }. Here rmin ($V_A^+(u)$, $V_B^+(u)$) = [min $\{t_A^+(x), t_B^+(x)\}$, min $\{1-f_A^+(x), 1-f_B^+(x)\}$], rmax ($V_A^+(u)$, $V_B^+(u)$) = [max $\{t_A^+(x), t_B^+(x)\}$, max $\{1-f_A^+(x), 1-f_B^+(x)\}$], rmin ($V_A^-(u)$, $V_B^-(u)$) = [min $\{-1-f_A^-(x), -1-f_B^-(x)\}$, min $\{t_A^-(x), t_B^-(x)\}$], rmax ($V_A^-(u)$, $V_B^-(u)$) = [max $\{-1-f_A^-(x), -1-f_B^-(x)\}$, max $\{t_A^-(x), t_B^-(x)\}$].

Definition 1.10: [5] Let R be a ring. A bipolar valued vague subset A of R is said to be a bipolar valued vague subring of R (BVVSR) if the following conditions are satisfied,

(i) $V_A^+(x-y) \geq$ rmin{ $V_A^+(x)$, $V_A^+(y)$ }

(ii) $V_A^+(xy) \geq$ rmin{ $V_A^+(x)$, $V_A^+(y)$ }

(iii) $V_A^-(x-y) \leq$ rmax{ $V_A^-(x)$, $V_A^-(y)$ }

(iv) $V_A^-(xy) \leq$ rmax{ $V_A^-(x)$, $V_A^-(y)$ } for all x and y in R.

Example 1.11: Let R = Z_3 = {0, 1, 2} be a ring with respect to the ordinary addition and multiplication. Then A = {< 0, [0.6, 0.9], [– 0.8, – 0.5] >, < 1, [0.3, 0.6], [– 0.7, – 0.4] >, < 2, [0.3, 0.6], [– 0.7, – 0.4] >} is a bipolar valued vague subring of R.

Definition 1.12: Let R be a ring. A bipolar valued vague subring A = $\langle V_A^+, V_A^- \rangle$ of R is said to be a bipolar valued vague normal subring of R if V_A^+ (xy) = V_A^+ (yx) and V_A^- (xy) = V_A^- (yx) for all x and y in R.

Definition 1.13: [12] Let R and R^l be any two rings. Then the function f: R → R^l is said to be an antihomomorphism if f(x+y) = f(y) + f(x) and f(xy) = f(y)f(x) for all x and y in R.

Definition 1.14: Let X and X^l be any two sets. Let f : X→ X^l be any function and let A = $\langle V_A^+, V_A^- \rangle$ be a bipolar valued vague subset in X, V = $\langle V_V^+, V_V^- \rangle$ be a bipolar valued vague subset in f(X) = X^l, defined by V_V^+ (y) = $r\sup_{x\in f^{-1}(y)} V_A^+$ (x) and V_V^- (y) = $r\inf_{x\in f^{-1}(y)} V_A^-$ (x), for all x in X and y in X^l. A is called a preimage of V under f and is defined as V_A^+ (x) = V_V^+ (f(x)), V_A^- (x) = V_V^- (f(x)) for all x in X and is denoted by $f^{-1}(V)$.

2. SOME THEOREMS

Theorem 2.1: Let R and R^l be any two rings. The homomorphic image of a bipolar valued vague subring of R is a bipolar valued vague subring of R^l.

Proof: Let f: R→ R^l be a homomorphism. Let V = f(A) = $\langle V_V^+, V_V^- \rangle$, where A = $\langle V_A^+, V_A^- \rangle$ is a bipolar valued vague subring of R. We have to prove that V is a bipolar valued vague subring of R^l. Now for f(x), f(y) in R^l, V_V^+ (f(x)–f(y)) = V_V^+ (f(x–y)) ≥ V_A^+ (x–y) ≥ rmin{V_A^+ (x), V_A^+ (y)} = rmin{V_V^+ (f(x)), V_V^+ (f(y)) } which implies that V_V^+ (f(x)–f(y)) ≥ rmin{V_V^+ (f(x)), V_V^+ (f(y))}. And V_V^+ (f(x)f(y)) = V_V^+ (f(xy)) ≥ V_A^+ (xy) ≥ rmin{V_A^+ (x), V_A^+ (y)} = rmin { V_V^+ (f(x)), V_V^+ (f(y)) } which implies that V_V^+ (f(x)f(y)) ≥ rmin{ V_V^+ (f(x)), V_V^+ (f(y)) }. Also V_V^- (f(x)–f(y)) = V_V^- (f(x–y)) ≤ V_A^- (x–y) ≤ rmax{ V_A^- (x), V_A^- (y) } = rmax{ V_V^- (f(x)), V_V^- (f(y))} which implies that V_V^- (f(x)–f(y)) ≤ rmax{ V_V^- (f(x)), V_V^- (f(y)) }. And V_V^- (f(x)f(y)) = V_V^- (f(xy)) ≤ V_A^- (xy) ≤ rmax{V_A^- (x), V_A^- (y)} = rmax{V_V^- (f(x)), V_V^- (f(y))} which implies that V_V^- (f(x)f(y)) ≤ rmax{ V_V^- (f(x)), V_V^- (f(y))}. Hence V is a bipolar valued vague subring of R^l.

2.2 Theorem: Let R and R^l be any two rings. The homomorphic preimage of a bipolar valued vague subring of R^l is a bipolar valued vague subring of R.

Proof: Let f : R → R^l be a homomorphism. Let V = f(A) = $\langle V_V^+, V_V^- \rangle$ where V is a bipolar valued vague subring of R^l. We have to prove that A = $\langle V_A^+, V_A^- \rangle$ is a bipolar valued vague subring of R. Let x and y in R. Now V_A^+ (x–y) = V_V^+ (f(x–y))= V_V^+ (f(x)–f(y)) ≥ rmin { V_V^+ (f(x)), V_V^+ (f(y)) }= rmin{ V_A^+ (x), V_A^+ (y)} which implies that V_A^+ (x–y) ≥ rmin{ V_A^+ (x), V_A^+ (y) }. And V_A^+ (xy) = V_V^+ (f(xy)) = V_V^+ (f(x)f(y)) ≥ rmin { V_V^+ (f(x)), V_V^+ (f(y)) }= rmin {V_A^+ (x), V_A^+ (y)} which implies that V_A^+ (xy) ≥ rmin { V_A^+ (x), V_A^+ (y) }. Also V_A^- (x–y) = V_V^- (f(x–y)) = V_V^- (f(x) –f(y)) ≤ rmax{V_V^- (f(x)), V_V^- (f(y))} = rmax{V_A^- (x), V_A^- (y)} which implies that V_A^- (x–y) ≤ rmax{V_A^- (x), V_A^- (y) }. And V_A^- (xy) = V_V^- (f(xy)) = V_V^- (f(x)f(y)) ≤ rmax{V_V^- (f(x)), V_V^- (f(y))} = rmax{ V_A^- (x), V_A^- (y)} which implies that V_A^- (xy) ≤ rmax{ V_A^- (x), V_A^- (y)}. Hence A is a bipolar valued vague subring of R.

2.3 Theorem: Let R and R^{l} be any two rings. The antihomomorphic image of a bipolar valued vague subring of R is a bipolar valued vague subring of R^{l}.

Proof: Let f : R → R^{l} be an antihomomorphism. Let V = f(A) = $\langle V_V^+ , V_V^- \rangle$ where A = $\langle V_A^+ , V_A^- \rangle$ is a bipolar valued vague subring of R. We have to prove that V is a bipolar valued vague subring of R^{l}. Now for f(x), f(y) in R^{l}, V_V^+(f(x) –f(y)) = V_V^+(f(y–x)) ≥ V_A^+(y–x) ≥ rmin {V_A^+(x), V_A^+(y) } = rmin {V_V^+(f(x)), V_V^+(f(y)) } which implies that V_V^+(f(x) –f(y))≥ rmin{ V_V^+(f(x)), V_V^+(f(y)) }. And V_V^+(f(x)f(y)) = V_V^+(f(yx)) ≥ V_A^+(yx)≥ rmin {V_A^+(x), V_A^+(y)} = rmin {V_V^+(f(x)), V_V^+(f(y)) } which implies that V_V^+(f(x)f(y))≥ rmin { V_V^+(f(x)), V_V^+(f(y)) }. Also V_V^-(f(x) –f(y)) = V_V^-(f(y–x)) ≤ V_A^-(y–x) ≤ rmax{ V_A^-(x), V_A^-(y)} = rmax{ V_V^-(f(x)), V_V^-(f(y)) } which implies that V_V^-(f(x) –f(y)) ≤ rmax{V_V^-(f(x)), V_V^-(f(y))}. And V_V^-(f(x)f(y)) = V_V^-(f(yx)) ≤ V_A^-(yx) ≤ rmax{ V_A^-(x), V_A^-(y)} = rmax{ V_V^-(f(x)), V_V^-(f(y)) } which implies that V_V^-(f(x)f(y)) ≤ rmax{ V_V^-(f(x)), V_V^-(f(y)) }. Hence V is a bipolar valued vague subring of R^{l}.

2.4 Theorem: Let R and R^{l} be any two rings. The antihomomorphic preimage of a bipolar valued vague subring of R^{l} is a bipolar valued vague subring of R.

Proof: Let f : R → R^{l} be an antihomomorphism. Let V = f(A) = $\langle V_V^+ , V_V^- \rangle$ where V is a bipolar valued vague subring of R^{l}. We have to prove that A = $\langle V_A^+ , V_A^- \rangle$ is a bipolar valued vague subring of R. Let x and y in R. Now V_A^+(x–y) = V_V^+(f(x–y)) = V_V^+(f(y) –f(x)) ≥ rmin{ V_V^+(f(x)), V_V^+(f(y)) } = rmin{ V_A^+(x), V_A^+(y) } which implies that V_A^+(x–y) ≥ rmin{ V_A^+(x), V_A^+(y)}. And V_A^+(xy) = V_V^+(f(xy)) = V_V^+(f(y)f(x)) ≥ rmin { V_V^+(f(x)), V_V^+(f(y)) } = rmin{ V_A^+(x), V_A^+(y) } which implies that V_A^+(xy) ≥ rmin { V_A^+(x), V_A^+(y) }. Also V_A^-(x–y) = V_V^-(f(x–y)) = V_V^-(f(y) –f(x)) ≤ rmax{ V_V^-(f(x)), V_V^-(f(y)) } = rmax{V_A^-(x), V_A^-(y)} which implies that V_A^-(x–y) ≤ rmax{V_A^-(x), V_A^-(y)}. And V_A^-(xy) = V_V^-(f(xy)) = V_V^-(f(y)f(x)) ≤ rmax{ V_V^-(f(x)), V_V^-(f(y))} = rmax { V_A^-(x), V_A^-(y) } which implies that V_A^-(xy) ≤ rmax{ V_A^-(x), V_A^-(y) }. Hence A is a bipolar valued vague subring of R.

2.5 Theorem: Let R and R^{l} be any two rings. The homomorphic image of a bipolar valued vague normal subring of R is a bipolar valued vague normal subring of R^{l}.

Proof: Let f : R → R^{l} be a homomorphism. Let V = f(A) = $\langle V_V^+ , V_V^- \rangle$ where A = $\langle V_A^+ , V_A^- \rangle$ is a bipolar valued vague normal subring of R. We have to prove that V is a bipolar valued vague normal subring of R^{l}. By Theorem 2.1, V is a bipolar valued vague subring of R^{l}. Now for f(x), f(y) in R^{l}, V_V^+(f(x)f(y)) = V_V^+(f(xy)) ≥ V_A^+(xy) = V_A^+(yx) ≤ V_V^+(f(yx)) = V_V^+(f(y)f(x)) which implies that V_V^+(f(x)f(y)) = V_V^+(f(y)f(x)). Also V_V^-(f(x)f(y)) = V_V^-(f(xy)) ≥ V_A^-(xy) = V_A^-(yx) ≤ V_V^-(f(yx)) = V_V^-(f(y)f(x)) which implies that V_V^-(f(x)f(y)) = V_V^-(f(y)f(x)). Hence V is a bipolar valued vague normal subring of R^{l}.

2.6 Theorem: Let R and R^{l} be any two rings. The homomorphic preimage of a bipolar valued vague normal subring of R^{l} is a bipolar valued vague normal subring of R.

Proof: Let f : R → R^{l} be a homomorphism. Let V = f(A) = $\langle V_V^+ , V_V^- \rangle$ where V is a bipolar valued vague normal subring of R^{l}. We have to prove that A = $\langle V_A^+ , V_A^- \rangle$ is a bipolar valued vague normal subring of R. By Theorem 2.2, A = $\langle V_A^+ , V_A^- \rangle$ is a bipolar valued vague subring of R. Let x and y in R. Now V_A^+(xy) = V_V^+(f(xy)) = V_V^+(f(x)f(y)) = V_V^+(f(y)f(x)) = V_V^+(f(yx)) = V_A^+(yx) which implies that V_A^+(xy) = V_A^+(yx). Also V_A^-(xy) = V_V^-(f(xy)) = V_V^-(f(x)f(y)) = V_V^-(f(y)f(x)) = V_V^-(f(yx)) = V_A^-(yx) which implies that V_A^-(xy) = V_A^-(yx). Hence A is a bipolar valued vague normal subring of R.

2.7 Theorem: Let R and R^{l} be any two rings. The antihomomorphic image of a bipolar valued vague normal subring of R is a bipolar valued vague normal subring of R^{l}.

Proof: Let f : R $\rightarrow$ R^{l} be an antihomomorphism. Let V = f(A) = $\langle V_V^+ , V_V^- \rangle$ where A = $\langle V_A^+ , V_A^- \rangle$ is a bipolar valued vague normal subring of R. We have to prove that V is a bipolar valued vague normal subring of R^{l}. By Theorem 2.3, V is a bipolar valued vague subring of R^{l}. Now for f(x), f(y) in R^{l}, V_V^+(f(x)f(y)) = V_V^+(f(yx)) $\geq$ V_A^+(yx) = V_A^+(xy) $\leq$ V_V^+(f(xy)) = V_V^+(f(y)f(x)) which implies that V_V^+(f(x)f(y)) = V_V^+(f(y)f(x)). Also V_V^-(f(x)f(y)) = V_V^-(f(yx))$\leq$ V_A^-(yx) = V_A^-(xy) $\geq$ V_V^-(f(xy)) = V_V^-(f(y)f(x)) which implies that V_V^-(f(x)f(y)) = V_V^-(f(y)f(x)). Hence V is a bipolar valued vague normal subring of R^{l}.

2.8 Theorem: Let R and R^{l} be any two rings. The antihomomorphic preimage of a bipolar valued vague normal subring of R^{l} is a bipolar valued vague normal subring of R.

Proof: Let f : R $\rightarrow$ R^{l} be an antihomomorphism. Let V = f(A) = $\langle V_V^+ , V_V^- \rangle$ where V is a bipolar valued vague normal subring of R^{l}. We have to prove that A = $\langle V_A^+ , V_A^- \rangle$ is a bipolar valued vague normal subring of R. By Theorem 2.4, A = $\langle V_A^+ , V_A^- \rangle$ is a bipolar valued vague subring of R. Let x and y in R. Now V_A^+(xy) = V_V^+(f(xy)) = V_V^+(f(y)f(x)) = V_V^+(f(x)f(y)) = V_V^+(f(yx)) = V_A^+(yx) which implies that V_A^+(xy) = V_A^+(yx). Also V_A^-(xy) = V_V^-(f(xy)) = V_V^-(f(y)f(x)) = V_V^-(f(x)f(y)) = V_V^-(f(yx)) = V_A^-(yx) which implies that V_A^-(xy) = V_A^-(yx). Hence A is a bipolar valued vague normal subring of R.

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