

**EXPENSION OF POSITIVE INTEGER OR EXACT DECIMAL NUMBER-II**

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**ABSTRACT**

*In this paper it is shown that a number either positive integer or exact decimal number greater than or equals to one, with different power of a, b, c, d ..... and common power ratio,  $m = \frac{b}{a} = \frac{c}{b} = \frac{d}{c} = \dots\dots$  or common power difference,  $m = b-a = c-b = d-c = \dots\dots$ , can be expanded up to infinity in term of value of m, a & x.*

**Keywords:** Exact decimal number, Positive integer.

**1. INTRODUCTION**

Vide International Journal of Mathematical Archive Ref. –11(9), 2020, 27-30 it has been proved that a number either positive integer or exact decimal number greater than one can presented by an infinite series by different positive integers or by different exact decimal numbers by formula

$$1 + \frac{a-1}{b} + \frac{a-1}{b} \frac{b-1}{c} + \frac{a-1}{b} \frac{b-1}{c} \frac{c-1}{d} + \dots\dots \infty = a$$

Instead of Numbers a, b, c, d.....,  $x_1, x_2, x_3 \dots\dots x_n \dots\dots$ , are taken which is expressed in term of power i.e  $x_1 = x^a, x_2 = x^b, x_3 = x^c \dots\dots$ , these value have been put in above equation and it has been simplify.

**Theorem 1:** If  $x_1, x_2, x_3 \dots\dots x_n \dots\dots$  are positive integer or exact decimal number greater than or equals to one in term of  $x_1 = x^a, x_2 = x^b, x_3 = x^c \dots\dots$ , where ratio of two successive power (m) =  $\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = \dots\dots$  Or difference of two successive power (m) =  $b-a = c-b = d-c = \dots\dots$ , then

**First case:**  $m = \frac{b}{a} = \frac{c}{b} = \frac{d}{c} = \dots\dots$

$$f(m, a, x) = \sum_{i=0}^{\infty} \prod_{n=0}^i x^{-a(m-1)m^n} (1 - x^{-a m^n})$$

**Second case:**  $m = b-a = c-b = d-c = \dots\dots$ ,

$$f(m, a, x) = \sum_{i=0}^{\infty} \prod_{n=0}^i x^{-m} [1 - x^{-(a+mn)}]$$

For both case -  $f(m, a, x) = x^a - 1$  where {m, a > zero (positive) &  $x \geq 1$ }

**Proof:** Vide International Journal of Mathematical Archive Ref. – IJMA-11(9), 2020, 27-30 it has been proved that if a, b, c, d,..... are positive integer or exact decimal numbers greater than one then,

$$1 + \frac{a-1}{b} + \frac{a-1}{b} \frac{b-1}{c} + \frac{a-1}{b} \frac{b-1}{c} \frac{c-1}{d} + \dots\dots \infty = a$$

According to theory If  $x_1, x_2, x_3 \dots\dots x_n \dots\dots$  are positive integer or exact decimal number greater than or equals to one in term of  $x_1 = x^a, x_2 = x^b, x_3 = x^c \dots\dots$ , where ratio of two successive power (m) =  $\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = \dots\dots$  Or difference of two successive power (m) =  $b - a = c - b = d - c, \dots\dots$ , then

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For first case

$$x_1 = x^a, x_2 = x^b, x_3 = x^c \dots \text{and } m = \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

$$b = am, c = am^2, d = am^3, \dots$$

Put the value of  $x_1, x_2, x_3, \dots$  in above equation

$$1 + \frac{x^a-1}{x^b} + \frac{x^a-1}{x^b} \frac{x^b-1}{x^c} + \frac{x^a-1}{x^b} \frac{x^b-1}{x^c} \frac{x^c-1}{x^d} + \dots \infty = x^a$$

Put the value of b, c, d....

$$\frac{x^a-1}{x^{am}} + \frac{x^a-1}{x^{am}} \frac{x^{am}-1}{x^{am^2}} + \frac{x^a-1}{x^{am}} \frac{x^{am}-1}{x^{am^2}} \frac{x^{am^2}-1}{x^{am^3}} + \dots \infty = x^a - 1$$

Above series can be written as

$$\sum_{i=0}^{\infty} \prod_{n=0}^i \frac{x^{am^n} - 1}{x^{am^{n+1}}} = x^a - 1$$

$$\sum_{i=0}^{\infty} \prod_{n=0}^i [x^{(am^n - am^{n+1})} - x^{-am^{n+1}}] = x^a - 1$$

$$\sum_{i=0}^{\infty} \prod_{n=0}^i \left[ x^{-am^n(m-1)} - x^{-am^n} \frac{x^{am^n}}{x^{am^n}} \right] = x^a - 1$$

$$\sum_{i=0}^{\infty} \prod_{n=0}^i \left[ x^{-am^n(m-1)} - \frac{x^{-am^n(m-1)}}{x^{am^n}} \right] = x^a - 1$$

$$\sum_{i=0}^{\infty} \prod_{n=0}^i x^{-am^n(m-1)} [1 - x^{-am^n}] = x^a - 1$$

Since series depend upon value of m, a, x, hence

$$f(m, a, x) = \sum_{i=0}^{\infty} \prod_{n=0}^i x^{-am^n(m-1)} (1 - x^{-am^n}) = x^a - 1$$

For second case

$$x_1 = x^a, x_2 = x^b, x_3 = x^c \dots \text{and } m = b-a = c-b = d-c = \dots$$

$$b = a + m, c = a + 2m, d = a + 3m, \dots$$

Put the value of  $x_1, x_2, x_3, \dots$  in above equation

$$1 + \frac{x^a-1}{x^b} + \frac{x^a-1}{x^b} \frac{x^b-1}{x^c} + \frac{x^a-1}{x^b} \frac{x^b-1}{x^c} \frac{x^c-1}{x^d} + \dots \infty = x^a$$

Put the value of b, c, d....

$$\frac{x^a-1}{x^{a+m}} + \frac{x^a-1}{x^{a+m}} \frac{x^{a+m}-1}{x^{a+2m}} + \frac{x^a-1}{x^{a+m}} \frac{x^{a+m}-1}{x^{a+2m}} \frac{x^{a+2m}-1}{x^{a+3m}} + \dots \infty = x^a - 1$$

Above series can be written as

$$\sum_{i=0}^{\infty} \prod_{n=0}^i \frac{x^{a+mn} - 1}{x^{a+m(n+1)}} = x^a - 1$$

$$\sum_{i=0}^{\infty} \prod_{n=0}^i x^{(a+mn - a - mn - m)} - x^{-(a+mn)} x^{-m} = x^a - 1$$

$$\sum_{i=0}^{\infty} \prod_{n=0}^i x^{-m} - x^{-(a+mn)} x^{-m} = x^a - 1$$

$$\sum_{i=0}^{\infty} \prod_{n=0}^i x^{-m} [1 - x^{-(a+mn)}] = x^a - 1$$

Since series depend upon value of m, a, x, hence

$$f(m, a, x) = \sum_{i=0}^{\infty} \prod_{n=0}^i x^{-m} [1 - x^{-(a+mn)}] = x^a - 1$$

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