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APPLICATION OF FIXED POINTS IN ECONOMICS AND GAME THEORY

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ABSTRACT

In recent years, fixed point theory played a significance role in different branches of Mathematics for example Topology, Complex analysis, Integral and Transformation theory functional analysis, ordinary and partial differential equations and serves as a useful tool in the study of nonlinear phenomena. This research article is the review of foundation of fixed point theory and its possible applications in Equilibrium price determination in Economics, Robustness to marginal changes and equilibrium stability and Nash Equilibrium in Game Theory.

Keywords: *Fixed Point, contractive conditions, Equilibrium price determination in Economics, Game Theory.*

INTRODUCTION

Fixed point theory comprises of several areas like mathematical analysis, calculus topology, Functional and Complex analysis and also is an important tool in mathematical analysis. This theory has enormous applications in countless areas of mathematics such as differential equations, geometry, and algebraic topology.

The derivation of fixed point lies in the theory of consecutive approximations. Joseph Liouville [18] in 1837 and in 1890 Charles Emile Picard [23] proved existence of solutions of differential equations nevertheless properly its roots go back to the start of twentieth century as a significant part of nonlinear analysis.

To evaluate the fixed points of mapping, the famous Polish mathematician Stefan Banach [4] gives in the year 1922 constructive method. Banach contraction principle is classical theory in functional analysis, provides the existence of fixed points as well as their unique property, which are solutions of a lot of problems in several areas of life. Throughout the past years, Banach theorem has go through several generalizations either one by reducing the condition on contractivity or extracting the constraint of completeness or both.

Moreover, in 1912 Brouwer [5], gave the key established theorem for fixed point analysis, which proves that "a continuous map on a closed unit ball has a fixed point". The fixed point theorem by Brouwer is premier aspects of fixed point theory. It is one of the foundation stone in the development in the numerical analysis of differential equations. By using this theorem, lot of fixed point theorems can be proved that are raised in certain areas of economic theory

In 1930, Schauder [24] extended this result and proved a theorem on fixed point which proved that "a continuous map on a convex compact subspace of a Banach space has a fixed point". The Schauder fixed point theorem [24] has huge applications in approximation theory, economical phenomena, game theory, optimization analysis and other scientific area like engineering etc.

Kakutani [17] derived modification of the Brouwer fixed point theorem which applied on multivalued maps in 1941. Jungck [9] generalizes the Banach's fixed point theorem and provides us the pioneering work which was published in the year 1976 which provides a to find the common fixed points of commuting maps. Then, in metric spaces for non-commuting mappings different researchers generalized more results about common fixed point theorem by using different kinds of contractive conditions [1, 6, 13, 14]. Where as Abbas and Jungck [19] established common fixed point theorems for non-commuting mappings in cone metric spaces.

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Popa [27, 28] presented implicit functions which are showing productive in line for their amalgamating influence in addition to stating different contraction conditions. Existence of fixed points is an inherent property of a self-map. Though, there are many criteria for the presence of fixed points which comprise a combination of algebraically, theoretically and topologically properties of functions.

These historic theorems have been used, generalized and prolonged in numerous techniques by a number of researchers, mathematicians, and scientists, economists for an enormous growth in the study of fixed point analysis of single / multivalued and hybrid functions in different contractive conditions in several spaces.

Fixed point results have continuously been sensational in itself and its uses in new areas. At present, it has established as a novel and sizzling zone of research in mathematics, statistics, engineering and economics. Several existence problems such as Equilibrium price determination in Economics, Robustness to marginal changes and equilibrium stability and also in the theory of Game Nash Equilibrium can be expressed as a part of fixed point analysis. On account of this, above results providing sufficient and necessary conditions as well for presence and uniqueness of fixed points. These fixed points have significant impact on game theory and economics. In this article, we are trying to review the literature of fixed points and see the sights them for application persistence.

LITERATURE REVIEW

There are enormous works on this area, which has influenced me in several means. A lot of authors in Game Theory and Economics study unconventional areas in mathematics as a result of their research. Moreover, in the field of economics and game theory fixed point theory is also used to verify that a model has at least one equilibrium model and also it is regularly used in the practice of evaluating the common value of supply and demand functions

On historical fact, John von Neumann verified for two agent games that “minimax” solution exists. John von Neumann generalized theorem by Brouwer’s in the year 1937 and he proved fixed points for economic analysis of a balanced growth equilibrium.[29]

In 1941 Kakutani [17] generalized the fixed point theorem by Brouwer for multivalued mappings and proved an important theorem in the area of game theory by providing a modest methodology to the John Von Neumann‘s minimax theorem. Thus this in turn enriches the applications of fixed point theorems to various areas of economic analysis. Further applications of multi valued mapping and fixed point theorems are lie in the area of Programming, control analysis and in the domain of differential and integral equations. In Kakutani’s theorem after World War II, required generalizations are established by assuming space and corresponding values both are convex This gave geometric conditions which are out of character and overall too robust.

However, thereafter, several generalizations and extensions of the Kakutani’s theorem are given by many researchers. In 1950, Nash [16] generalized this theorem and gave a significant application to prove the presence and uniqueness of an equilibrium model in the field of non-cooperative finite Game Theory.

The most fundamental model in the domain of Economics is the proof on general equilibrium theory given by Arrow and Debreu (1954) [3].The analysis of refinements of Nash equilibrium (e.g., Selten (1975); Myerson (1978); Kreps andWilson (1982); Kohlberg and Mertens (1986); Mertens (1989, 1991); Govindan and Wilson (2008)) have several notions which extent a wilting of the concept of essential set. Essential sets with particular types of perturbations of the mapping are robust. In particular, Jiang (1963) established the application to the domain of game theory; He defined what essential Nash equilibrium is and what is an essential set of the same with robustness property wrt perturbations of the finestretort correspondence persuaded by perturbations of the payoffs.

Problems based upon comparative statistics, robustness under perturbations, stability of equilibrium with respect to dynamic adjustment processes, and the algorithmic and complexity of equilibrium computation are addressed by Fixed Point Analysis.

Game theory has been intensely predisposed by refinement notions defined mainly in terms of robustness with respect to definite kinds of perturbations in 1970s.

Finite games to non-cooperative games with nonlinear payoff functions are extended by Debreu[7]. Also Ky Fan generalized the theorem on Nash equilibrium for Hausdorff topological vector spaces [8].

Kakutani’s theorem on Fixed point generalized Arrow and Debreu [3] based upon general equilibrium theory as well as the established the case of non-cooperative games. [13, 19, 26] used the Brouwer’s theorem to prove many theorems.

[32] Established that for all excess demand mapping and functions as well which are created by the Exchange Economy. [25] proved the existence and uniqueness of fixed points in closed simplex possesses appropriately defined equilibrium then and there all continuous mapping from closed simplex.

Vesna Rajić, Dragan Azdejković, Dragan Lončar [30] proved some basic results on fixed point theory and apply these results to find out possible applications in economics. This results are the start of applications such as competitive relationship equilibrium in diverse arcades of the domain of economic

More over to evaluate potential uses [30] presents Fixed point theorem by Brouwer's and Kakutani's in the field of economic exploration. [30] proved generalization of this result where as Kakutani in the year 1941 simplified the result. Gergely Kõhegyi1 [11] studied the methodology by John von Neumann and gave a discussion on ordinary' historical explanations. He concerned about the work by John von Neumann and established a correspondence of fixed point theorem by Brouwer's with existence of equilibrium in a simple exchange model with monotonic consumers.

To establish an infinite-dimensional generalization of the fixed point theorem by Brouwer, Jian Yu, Neng-Fa Wang and Zhe Yang [15] used the Fan extension of the Nash equilibrium theorem.

They also generalized the theorems by Berge and Kakutani and proved some interesting concepts for the Nash equilibrium. [5] explores the structure of macroeconomic models using major concepts and algorithms of the graph theory. [30] Proved other economics applications on fixed point theorem such as Robustness to marginal changes and equilibrium stability, Measuring market concentration and competitive dynamics within industry. Recently [6] proved a convergence theorem on J fixed points and constructed several algorithms for approaching roots of inverse strongly monotone functions. Also [12] defined a new φ-weakly contraction of the rational type for separable Banach Spaces and [31] established some ions of applications of fixed point theorems in Integral Equations.

Fixed Point theory can be considered in several problems in Economics analysis such as presence of competitive equilibrium in common theory of equilibrium. Theorems based upon the Nash Equilibrium in game theory gives us necessary and sufficient approach for uniqueness and existence of fixed points which are playing premium role in the area of Economics. Also various algorithms of Graph theory are used in the structures of macroeconomic models

CONCLUSION

It is an effort to review the theorems on fixed point which are applicable on different types of contractive conditions and their applications. I believe that there is a vast scope of research to cultivate theory of fixed points in economics and game theory. The main object of this literature review article is to analysis the significance of fixed point's theory in several applications for researchers that they can use it for further analysis and research in economics analysis.

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