International Journal of Mathematical Archive-12(6), 2021, 1-15
IJMA Available online through www.ijma.info ISSN 2229 – 5046

ANALYSIS OF EQUILIBRIUM STRATEGIES IN MARKOVIAN QUEUE WITH PARTIAL BREAKDOWN, TOTAL BREAKDOWN AND DELAYED REPAIR

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(Received On: 06-05-21; Revised & Accepted On: 31-05-21)

ABSTRACT

We consider equilibrium Strategic behaviour in single server markovian queueing system with partial breakdown, total breakdown and delayed repair. In a case of partial breakdown, the system may become defective at any time whether server is busy or idle. When server breaks down then srevice rate decreases without stopping the service completely, service continues at a slower rate. After total breakdown, system goes to repair process. Repair process doesn't start immediately due to non availability of repair facility. Then system consider a delayed repair time .Based on natural reward cost structure and the information of system, arriving customers can decide whether to enter or balk the system. We derive the equilibrium balking strategies and expected social benefits per unit time for the customer in the fully observable, almost observable, fully unobservable and almost unobservable case. Here we investigate Steady-state distribution and the mean sojourn time of the arriving customer by using the probability generating function.

Keyword: *Equilibrium Strategy, Delayed Repair, Markovian Queue, Partially Breakdown, Queueing Theory, Unobservable Queue.*

I. INTRODUCTION

In the field of queueing theory study of customer behavior is a very active research area in many different directions. There are many possibilities and parameters for determining strategic behavior between customers and service providers. Naor[24] is the first who described the effect of information on customer behavior strategy in M/M/1 queueing model. Naor[24] studied the markovian queueing model in which newly arriving customer observes the number of customers and makes his decision whether to join or balk the system in observable case. Edelson and Hildebrand [19] considered a corresponding unobservable case in Naor's model by assuming that arriving customers have no information about number of customers. Hassin and Haviv[26] studied with equilibrium threshold strategies in queueing model with priorities. Hassin and Haviv[26] studied the equilibrium behaviour of customers of queueing system. We studied the fundamental results about various systems after server break down but continue to provide a service of customer at a slower rate than normal working rate. Partial breakdown can happen at any state either server busy or idle. Second state is total breakdown state in which server stops customer service completely. Third state is delayed repair state in which after total breakdown system goes to repair state which causes delay in service.

The aim of our paper is to investigate the equilibrium threshold strategy in four cases with regard to the level of information available to arriving customer.

1. Fully observable case : An arriving customer observe both queue length and state of server
2. Almost observable case: An arriving customer only observe the number of customer and does not know to the state of server.
3. Almost unobservable case: An arriving customer can only observe the state of server and is not allowed to observe the queue length.
4. Fully unobservable case: An arriving customer is not allowed to observe either number of customer or state of server.

Here we discuss the customer's equilibrium strategy for all customer and investigated the stationary behaviour of corresponding situation.

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II. BRIEF REVIEW

Burnets and Economou[1] studied a markoviansingle server queueing system with setup time and analyzed the customer's equilibrium strategy under various level of information. In the queueing literature most of the paper assume that the server is always available but in real service system, a perfectly reliable server do not actually exist. There are some papers in the literature working on analysis of strategic behaviour of customers in such kind of queueing system. Cheng [7] first studied independent breakdown in an unreliable markovian single server queueing system and analyzed optimal internal pricing and backup capacity of computer system. Chaper and Jondral[6] studied independent breakdown in observable m/m/1 queue in radio network. Economou and Kanta[4] studied analysis of equilibrium balking strategies for customer in the observable M/M/1 queue with unreliable server and repair. After that there were many authors who reconsidered the model of Economou and Kanta[4] and extended their corresponding result. Wang and Zhang[13] extended this model to the delayed repair situation . The repair process may not be start immediately due to non-availability of repair facility and therefore system may consider a delayed repair time. The equilibrium threshold balking and equilibrium social benefits are analyzed in fully and partially observable system for all customer. There is an increasing number of research papers that deal with the analysis of balking behaviour of customer in M/M/1 queue Such as Sun Guo & Tian [33], Economou & manou[3], Li & Han[18], Jagannathan[14], Do [8] and Yang[39] among others.some works have incorporated the vacation, different kind of server vacation such as Guo & Hassin[23] , Economou[2] and Li[33]. Similarly Many works have been implemented in working vacation of queue system such as Zhang [43], Sun & Li [35], Sun [36], sun [37]. It is common in above research studied to assume that server will stop service completely in the case of server breakdown due to various factors. However in the modern operational system, there are many situations in a real-world where the server may not stop customer's service completely due to the server breakdown such kind of server breakdown was called partial breakdown (failure). Gnedenko& Kovalenko [12] who first introduced the case of partial breakdown (failure) in m/G/1 system with unreliable server after that Sridharm & Jayshree[31] considered a finite m/m/1 queueing model with normal partial & total failure. Kalidas & Kasturi [16] introduced the concept of working breakdown. They considered markovian queueing model with working breakdown. In real life situations such a problem arises from computer based technology, telecommunication, e-commerce and manufacturing areas. For example, computer is infected by the virus. In that case processing speed of computer degrades but computer keeps working. Here it should be noted that there are difference between partial breakdown and working breakdown. In the partial breakdown situation, server doesn't stop customer's service completely .but provide a service of customers at slower rate than normal processing rate. Partially breakdown can happen at any state either server busy or idel. But in the working breakdown situation. when server is in the working or operational state then it may breakdown only.

In real life there are many reason for server failure/breakdown such that a negative customer or disaster. upon arriving of negative customer , the service of existing customer will be killed . Kim and Li [5] considered and M/G /1queue with disaster and working breakdown in which the system has main server and redundant server when main server breakdown then redundant or substitute server provide the service to arriving customer at a slower rate. Doo Lee[9] studied equilibrium strategies of customer and optimal pricing strategies of server in unobservable queue system with negative customer and repair. Li [17] considered the equilibrium behaviour of customer in M/M/1 queue with partial breakdown and immediately repair. In this paper they described the equilibrium strategy only two fully observable case and fully unobservable case. After that Seline & zaiming [30] studied the strategic behaviour of customer in partially observable markovian queue with partial breakdown. In this paper they also described strategic behaviour of customer only two cases almost unobservable and almost observable case. Chen & Zhou [21] analysed the equilibrium behaviour of customer in m/m/1 queue with setup time, breakdown and immediate repair. After that Xu & Xu [39] studied the equilibrium is strategic behaviour of customer in m/m/1 queue with partial failure and repair. In such a queueing system, during the partial failure period, arriving customer is not allowed to enter the system and server provides service at slower rate instead of stopped serving. In this paper we studied fully observable fully and observable in partial failure server.

After all related research work, Tian & Wang [27] studied the equilibrium strategy in markovian with negative customer and working breakdown. In this paper they described the customer beings serve have to leave the system upon arriving the negative customer. Then server breakdown and server provide a service with slower rate than normal working rate. Here we study strategic behaviour of customer under different level of system and also study equilibrium strategy of customer in four cases such as fully observable, almost observable, fully unobservable, almost unobservable.

The rest of this paper is organized as follows. In Section III, we describe the model description and the reward- cost structure. In Section IV and V, we consider the equilibrium threshold strategiesin the fully observable and almost observable case respectively. Section VI and VII, we determine the equilibrium threshold strategies in the almost unobservable and fully unobservable case respectively. Conclusion and future research are given in the section VIII.

III. MODAL DESCRIPTION

In this paper, we consider the M / M /1 queueing system with an infinite waiting queue where customers arrive according to a Poisson process with intensity λ. During normal working rate, the service times are exponentially distributed with rate μ. When partial breakdown occurs in the system then service times are exponentially distributed with rateμ_0The service alternates between four state with state space I={0,1,2,3}. The server has an exponential lifetime with failure rate or breakdown rate ξ when he is normal working state (1). In the partial breakdown situation (2), server doesn't stop customer's service completely but provide a service of customers at slower rate than normal processing rate and$\mu_0 < \mu$. Partially breakdown can happen at any state either server busy or idle. In total breakdown state (3), system doesn't provide any service to customer. The server has an exponential lifetime with failure rate ξ_0 .The delayed time is exponentially distributed with parameter α.In the delay state, the server doesn't provide any type of service to arriving customers and to begin the repair process, server waits for repair facility. The repair time is exponentially distributed with repair rate θ. In other words, when the server fails, then the repair process doesn't start immediately due to non-availability of the repair facility. The repair delayed time is define as the time interval between the period of server breakdown and the beginning of repair process. We realize that the repair delayed time has two stages and hence it is not memory less

We describe the state of the system at time t by a pair (L (t), I (t)), where L (t) records the number of customers in the system and I (t) denotes the state of the server (1: working state, 2: partial breakdown state, 3: delayed period (total breakdown); 0: under repair). The stochastic process {(L (t), I (t)),≥t 0 } is a two -dimensional continuous-time Markov chain

$$q_{(n,i)(n+1,i)} = \lambda, \qquad n \geq 0;\ i = 0, 1, 2, 3;-$$
$$q_{(n,1)(n-1,1)} = \mu, \qquad n \geq 1;$$
$$q_{(n,2)(n-1,2)} = \mu_0, \qquad n \geq 0;$$
$$q_{(n,1)(n,2)} = \xi, \qquad n \geq 0;$$
$$q_{(n,2)(n,3)} = \xi_0, \qquad n \geq 0;$$
$$q_{(n,3)(n,0)} = \alpha, \qquad n \geq 0;$$
$$q_{(n,0)(n,1)} = \theta, \qquad n \geq 0;$$

The corresponding transition rate diagram is shown in figure1.

We are interested to analyse the behavior of customers when they are allowed to decide whether to join or balk the system at their arrival instants. Every customer receives a reward of R units in the system after completing their service. This may reflect his satisfaction or the added value of being served. On the other hand, customers have a waiting cost of C units per time when they remain in the system including the waiting time in queue and being served. Every Customer wants to maximize their expected net benefit of service. Their decisions are unchangeable that retrials of balking customers and reneging of entering customers are not allowed. Each arriving customer can observe the number of customers.

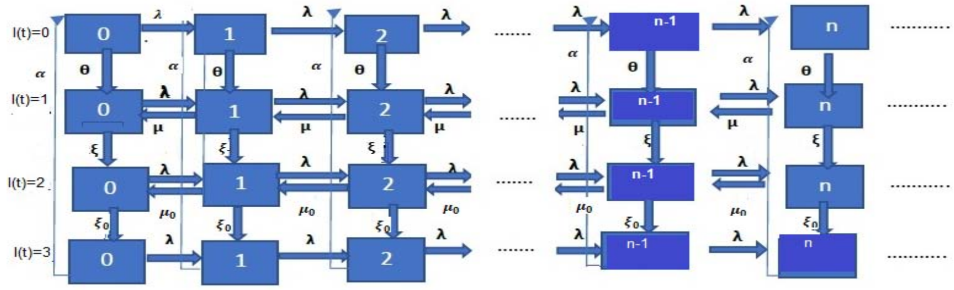

Figure-1: transition rate diagram for the equilibrium strategy with partially breakdown, total breakdown and delayed repair

In fact, the model under consideration can be seen as an M / M /1 queueing system in a random environment. More specifically, the external environment is an irreducible continuous-time Markov chain on a finite state space {3, 2, 1, 0}. That is, when the external environment I (t) is in state I, I = 0, 1, 2 ,3, the system behaves as an M (λ)/ M (μ)/1 queue with arrival intensity λ and service rate μ_i, where $\mu_0 = \mu_3 = 0$ and $\mu_1 =$ µ, $\mu_2 = \mu_0$. The infinitesimal generator (i.e., q -matrix) of the external environment I(t) is given by

$$Q=\begin{pmatrix} -\alpha & 0 & 0 & \alpha \\ \xi_0 & -\xi_0 & 0 & 0 \\ 0 & \xi & -\xi & 0 \\ 0 & 0 & \theta & -\theta \end{pmatrix}$$

Let ($\pi_0, \pi_1, \pi_2, \pi_3$) be the stationary distribution of the external environment, by solving ($\pi_0, \pi_1, \pi_2, \pi_3$) Q = 0,

We have $\pi_0 = \frac{\xi\xi_0\theta}{\xi\xi_0\theta+\xi\alpha\theta+\alpha\xi_0\theta+\xi\xi_0\alpha}$, $\pi_1 = \frac{\xi\alpha\theta}{\xi\xi_0\theta+\xi\alpha\theta+\alpha\xi_0\theta+\xi\xi_0\alpha}$,

$\pi_2 = \frac{\alpha\xi_0\theta}{\xi\xi_0\theta+\xi\alpha\theta+\alpha\xi_0\theta+\xi\xi_0\alpha}$, $\pi_3 = \frac{\xi\xi_0\alpha}{\xi\xi_0\theta+\xi\alpha\theta+\alpha\xi_0\theta+\xi\xi_0\alpha}$

The system is said to be in state (n, I) if there are n customers in the system and the server is found at state I. Let p (n, I) be the limiting probability of the system in state (n, I). That is, p (n, I) = lime $\leftrightarrow\infty$ P (L (t) = n, I(t) = I), n ≥ 0, if I = 0, 1, 2,3.

In the following sections, we will analyse the queues with Partial breakdowns, and consider four information cases in this paper:

(1) the fully observable case where arriving customers can observe the system state ($N(t)$; $I(t)$);
(2) the almost observable case where arriving customers can only observe the length of the system $N(t)$;
(3) the almost unobservable case where arriving customers can only observe the server's state I(t);
(4) the fully unobservable case where customers can not observe the system

IV FULLY OBSERVABLE CASE

In this section, we show that there exist equilibrium strategies of threshold type in the fully observable case arriving customers can observe both the number of customers in the system and the server's state at arrival. A pure threshold strategies are specified by (n_e(0), n_e(1), n_e(2), n_e(3)) such that an arriving customer decides to join the system if the number of customers upon arrival does not exceed the specified thresholds. and the balking strategy has the form 'while arriving at time t, observe (N(t), I(t)); enter if N(t) ≤ n_e(I(t)) and balk otherwise'

Let T(n; i) be the expected sojourn time of the arriving customer who finds the system at the system state (n; i) (i = 0, 1, 2, 3) and decides to enter the queue.

We thus conclude the following results.

Theorem 1: In the fully observable M/M/1 queue with partial breakdowns, there exist a pair of thresholds (n_e(0), n_e(1), n_e(2), n_e(3)), such thatthe strategy 'observe (N(t), I(t)) upon arrival, enter if N(t) ≤ n_e(I(t)) and balk otherwise', and (n_e(0), n_e(1), n_e(2), n_e(3)) =($[x_0]$,$[x_1]$, $[x_2]$, $[x_3]$)

where x_i is the unique root of equation

$$\text{ax} + b_i c^{x+1} + d_i = 0, \text{ i} = 1, 2, 3 \tag{1}$$

where

$$\text{a} = -\text{C}\left(\frac{\mu_0+\xi_0+\xi+\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)}{\mu\mu_0+\mu\xi_0+\xi\mu_0} + \frac{\mu_0\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\mu_0+\mu\xi_0+\xi\mu_0)(\mu\xi_0+\xi\mu_0)}\right) \tag{2}$$

$$b_1 = -\text{C}\frac{\mu_0\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\xi_0+\xi\mu_0)^2} \tag{3}$$

$$b_2 = \text{C}\frac{\xi_0\mu\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\xi_0+\xi\mu_0)^2} \tag{4}$$

$$b_0 = b_3 = b_1 \tag{5}$$

$$\text{c} = \frac{\mu\mu_0}{\mu\mu_0+\mu\xi_0+\xi\mu_0} \tag{6}$$

$$d_1 = \text{R} - \text{C}\frac{1}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\left(\mu_0 + \xi_0 + \xi + \xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right) - \frac{\mu\mu_0{}^2\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\xi_0+\xi\mu_0)^2}\right) \tag{7}$$

$$d_2 = d_1 - C\frac{\mu-\mu_0+\mu\xi_0(\frac{1}{\theta}+\frac{1}{\alpha})}{\mu\xi_0+\xi\mu_0} \tag{8}$$

$$d_0 = d_1 - \frac{C}{\theta} \tag{9}$$

$$d_3 = d_1 - \frac{C}{\theta} - \frac{C}{\alpha} \tag{10}$$

and S(n, i) = an $+b_i c^{n+1}$ $+d_i$ is a monotone decreasing function of n.

Proof: It is obvious that for an arriving customer based on the reward-cost structure, his expected net reward if he enters is S(n, i) = R - CT(n, i)
Where T(n, i) denotes his expected mean sojourn time given that he finds the system at state ((N(t), I(t)) upon his arrival.

We have the following equations:

$$T(n,0) = \frac{1}{\theta} + T(n,1) \qquad n = 0, 1, 2, 3\ldots\ldots \tag{11}$$

$$T(0,1) = \frac{\mu}{\mu+\xi}\frac{1}{\mu} + \frac{\xi}{\mu+\xi}T(0,2) \tag{12}$$

$$T(n,1) = \frac{1}{\mu+\xi} + \frac{\mu}{\mu+\xi}T(n-1,1) + \frac{\xi}{\mu+\xi}T(n,2) \qquad n = 1,2,3\ldots\ldots \tag{13}$$

$$T(0,2) = \frac{1}{\mu_0+\xi_0} + \frac{\xi_0}{\mu_0+\xi_0}T(0,3) \tag{14}$$

$$T(n,2) = \frac{1}{\mu_0+\xi_0} + \frac{\mu_0}{\mu_0+\xi_0}T(n-1,2) + \frac{\xi_0}{\mu_0+\xi_0}T(n,3) \qquad n = 1,2,3\ldots\ldots \tag{15}$$

$$T(n,3) = \frac{1}{\alpha} + T(n,0) \qquad n = 0, 1, 2, 3\ldots\ldots \tag{16}$$

Solving the system of (13) and (14) for n = 0 along with (12) we obtain

$$T(0,1) = \frac{\mu_0+\xi_0+\xi+\xi\xi_0(\frac{1}{\theta}+\frac{1}{\alpha})}{\mu\mu_0+\mu\xi_0+\xi\mu_0} \tag{17}$$

$$T(0,2) = \frac{1+\xi_0(\frac{1}{\theta}+\frac{1}{\alpha})}{\mu_0+\xi_0} + \frac{\xi_0}{\mu_0+\xi_0}\left[\frac{\mu_0+\xi_0+\xi+\xi\xi_0(\frac{1}{\theta}+\frac{1}{\alpha})}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\right] \tag{18}$$

Now,

$$T(0,2) - T(0,1) = \frac{\mu-\mu_0+\mu\xi_0(\frac{1}{\theta}+\frac{1}{\alpha})}{\mu\mu_0+\mu\xi_0+\xi\mu_0} \tag{19}$$

From equation (13),

$$T(n,1) = \frac{\mu_0+\xi_0+\xi+\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)+\mu(\mu_0+\xi_0)T(n-1,1)+\xi\mu_0 T(n-1,2)}{\mu\mu_0+\mu\xi_0+\xi\mu_0} \tag{20}$$

And combining equation (15), (16) and (11) in equation (20), then

$$T(n,2) - T(n,1) = \frac{\mu-\mu_0+\mu\xi_0(\frac{1}{\theta}+\frac{1}{\alpha})}{\mu\mu_0+\mu\xi_0+\xi\mu_0} + \frac{\mu\mu_0}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\{T(n-1,2) - T(n-1,1)\} \tag{21}$$

Through iterating (21) and taking account into (19), we get

$$T(n,2) - T(n,1) = \frac{\mu-\mu_0+\mu\xi_0(\frac{1}{\theta}+\frac{1}{\alpha})}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\left[1-\left(\frac{\mu\mu_0}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\right)^{n+1}\right] \tag{22}$$

By plugging (22) in (20), we have

$$T(n,1) = \frac{\mu_0+\xi_0+\xi+\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)}{\mu\mu_0+\mu\xi_0+\xi\mu_0} + T(n-1,1) + \frac{\mu_0\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\mu_0+\mu\xi_0+\xi\mu_0)(\mu\xi_0+\xi\mu_0)}\left[1-\left(\frac{\mu\mu_0}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\right)^{n+1}\right] \tag{23}$$

Through iterating (23) and taking account into (17), we get

$$T(n,1) = n\left[\frac{\mu_0+\xi_0+\xi+\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)}{\mu\mu_0+\mu\xi_0+\xi\mu_0} + \frac{\mu_0\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\mu_0+\mu\xi_0+\xi\mu_0)(\mu\xi_0+\xi\mu_0)}\right] + \frac{\mu_0\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\xi_0+\xi\mu_0)^2}\left(\frac{\mu\mu_0}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\right)^{n+1}$$
$$+\frac{1}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\left(\mu_0+\xi_0+\xi+\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right) - \frac{\mu\mu_0{}^2\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\xi_0+\xi\mu_0)^2}\right) \tag{24}$$

$$T(n,2) = n\left[\frac{\mu_0+\xi_0+\xi+\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)}{\mu\mu_0+\mu\xi_0+\xi\mu_0} + \frac{\mu_0\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\mu_0+\mu\xi_0+\xi\mu_0)(\mu\xi_0+\xi\mu_0)}\right] - \frac{\xi_0\mu\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\xi_0+\xi\mu_0)^2}\left(\frac{\mu\mu_0}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\right)^{n+1}$$
$$+\frac{1}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\left(\mu_0+\xi_0+\xi+\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right) - \frac{\mu\mu_0{}^2\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\xi_0+\xi\mu_0)^2}\right) + \frac{\mu-\mu_0+\mu\xi_0(\frac{1}{\theta}+\frac{1}{\alpha})}{\mu\xi_0+\xi\mu_0} \tag{25}$$

T(n,0) and T(n,3) are define by putting this value in equation (11) and (16).

$$T(n,0) = \frac{1}{\theta} + n\left[\frac{\mu_0+\xi_0+\xi+\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)}{\mu\mu_0+\mu\xi_0+\xi\mu_0} + \frac{\mu_0\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\mu_0+\mu\xi_0+\xi\mu_0)(\mu\xi_0+\xi\mu_0)}\right] + \frac{\mu_0\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\xi_0+\xi\mu_0)^2}\left(\frac{\mu\mu_0}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\right)^{n+1}$$
$$+\frac{1}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\left(\mu_0+\xi_0+\xi+\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right) - \frac{\mu\mu_0{}^2\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\xi_0+\xi\mu_0)^2}\right) \tag{26}$$

$$T(n,3) = \frac{1}{\theta} + \frac{1}{\alpha} + n\left[\frac{\mu_0+\xi_0+\xi+\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)}{\mu\mu_0+\mu\xi_0+\xi\mu_0} + \frac{\mu_0\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\mu_0+\mu\xi_0+\xi\mu_0)(\mu\xi_0+\xi\mu_0)}\right] + \frac{\mu_0\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\xi_0+\xi\mu_0)^2}\left(\frac{\mu\mu_0}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\right)^{n+1}$$
$$+\frac{1}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\left(\mu_0+\xi_0+\xi+\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right) - \frac{\mu\mu_0{}^2\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\xi_0+\xi\mu_0)^2}\right) \tag{27}$$

The expected net reward of such a customer is

$$S(n,i) = R - C\,T(n,i), \qquad i = 0, 1, 2, 3 \tag{28}$$

A customer does not enter the system if S(n, i) < 0, otherwise, he enters the queue. We can use (16), (24) and (25) to obtain that the customer arriving at time t decides to enter if and only if n ≤ ne(i), where ($n_e(0)$, $n_e(1)$, $n_e(2)$, $n_e(3)$) are obtained by using the unique solution $x_i$of equation (1).

$$S(n,1) – S(n-1,1) = - C(T(n,1) – T(n-1,1))$$
$$= - C\frac{1}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\left(\mu_0+\xi_0+\xi+\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)+\frac{\mu_0\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\xi_0+\xi\mu_0)}\left[1-\left(\frac{\mu\mu_0}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\right)^n\right]\right)$$

Since $0 < \frac{\mu\mu_0}{\mu\mu_0+\mu\xi_0+\xi\mu_0} < 1$, S(n, 1) < S(n - 1, 1) is decreasing in n.

When n = 1, S(1, 1) < S(0, 1) < 0, so S(n, 1) < S(n -1, 1) < 0 for n = 1, 2, 3. . ..

Similarly we can show that S(n, 2) - S(n - 1, 2) < 0,

so that S(n, i) - S(n - 1, i) < 0, so S(n, i), i = 0, 1, 2, 3 is a monotone decreasing function.

V. ALMOST OBSERVABLE CASE

In this section, we focus on the almost observable case, where a customer can only observe the number of customers present and cannot know the state of the server at their arrival instant. We seek equilibrium threshold for the customers. Thus, the customers follow the same threshold strategy ne.

Taking ne(0) = ne(1) = ne(2) = ne(3) = ne, we get stationary distribution of the corresponding Markov chain with process{(N(t), J(t)), t ≥ 0 }.

We have the following result

Theorem 2: Consider the M/M/1 queue with partial breakdowns where all customers use the same threshold for entrance ne, the stationary distribution (P(n, i): (n, i) ∈{0, 1, . . . , ne + 1} × {0, 1}) is

$$p(n, 0) =\sum_{i=0}^{5} w_i\rho_i^n,\ \ n=0, 1, 2, 3,\dots n_e \tag{29}$$
$$p(n, 1) =\sum_{i=0}^{5} v_i\rho_i^n,\ \ n=0,1, 2, 3\dots\ n_e \tag{30}$$
$$p(n, 2) = \sum_{i=0}^{5} c_i\rho_i^n,\ \ n=0, 1, 2\dots \dots n_e \tag{31}$$
$$p(n, 3) = \sum_{i=0}^{5} u_i\rho_i^n,\ \ n=0, 1, 2, 3\dots\ n_e \tag{32}$$
$$p(n_e+1,0) = \frac{\lambda\mu_0(\mu+\xi)\sum_{i=0}^{5}(w_i+u_i)\rho_i^{ne}+\lambda\mu\xi_0\sum_{i=0}^{5}(c_i+w_i+u_i)\rho_i^{ne}+\lambda\xi\xi_0\sum_{i=0}^{5}(c_i+v_i+w_i+u_i)\rho_i^{ne}}{\theta(\mu\mu_0+\mu\xi_0+\mu_0\xi)} \tag{33}$$
$$p(n_e+1,1) = \frac{\lambda\mu_0\sum_{i=0}^{5}(v_i+w_i+u_i)\rho_i^{ne}+\lambda\xi_0\sum_{i=0}^{5}(c_i+v_i+w_i+u_i)\rho_i^{ne}}{\mu\mu_0+\mu\xi_0+\mu_0\xi} \tag{34}$$
$$p(n_e+1,2) = \frac{\lambda\mu\sum_{i=0}^{5}c_i\rho_i^{ne}+\lambda\xi\sum_{i=0}^{5}(c_i+v_i+w_i+u_i)\rho_i^{ne}}{\mu\mu_0+\mu\xi_0+\mu_0\xi} \tag{35}$$
$$p(n_e+1,3) = \frac{\lambda\mu_0(\mu+\xi)\sum_{i=0}^{5}u_i\rho_i^{ne}+\lambda\mu\xi_0\sum_{i=0}^{5}(c_i+u_i)\rho_i^{ne}+\lambda\xi\xi_0\sum_{i=0}^{5}(c_i+v_i+w_i+u_i)\rho_i^{ne}}{\alpha(\mu\mu_0+\mu\xi_0+\mu_0\xi)} \tag{36}$$

Where

$$v_i = \frac{c_i}{\xi}\ (\lambda+\xi_0+\mu_0-\mu_0\rho_i-\frac{\lambda}{\rho_i})\quad i = 1, 2...5 \tag{37}$$
$$w_i = \frac{v_i}{\theta}\ (\lambda+\xi+\mu-\mu\rho_i-\frac{\lambda}{\rho_i})\quad i = 1, 2\dots5 \tag{38}$$
$$u_i = w_i\frac{\{(\lambda+\theta)\rho_i-\lambda\}}{\alpha\rho_i}\quad i = 1, 2\dots5 \tag{39}$$
$$a = -\frac{[\mu_0\{\lambda(\lambda+\theta+\alpha)(\lambda+\mu+\xi)+\lambda\theta\alpha+\lambda^2\mu\}+\mu\mu_0(\lambda^2+\lambda\alpha+\lambda\theta+\theta\alpha)(\lambda+\mu_0+\xi_0)+\mu_0\alpha\theta\xi]}{\mu\mu_0(\lambda^2+\lambda\alpha+\lambda\theta+\theta\alpha)} \tag{40}$$
$$b = \frac{[\{\lambda(\lambda+\theta+\alpha)(\lambda+\mu+\xi)+\lambda\theta\alpha+\lambda^2\mu\}(\lambda+\mu_0+\xi_0)+\mu\lambda(\lambda^2+\lambda\alpha+\lambda\theta+\theta\alpha)+\lambda^2\mu_0(2\lambda+\mu+\xi+\theta+\alpha)]}{\mu\mu_0(\lambda^2+\lambda\alpha+\lambda\theta+\theta\alpha)} \tag{41}$$
$$c = -\frac{[\lambda\{\lambda(\lambda+\theta+\alpha)(\lambda+\mu+\xi)+\lambda\theta\alpha+\lambda^2\mu\}+\lambda^2(2\lambda+\mu+\xi+\theta+\alpha)(\lambda+\mu_0+\xi_0)+\lambda^3\mu_0]}{\mu\mu_0(\lambda^2+\lambda\alpha+\lambda\theta+\theta\alpha)} \tag{42}$$
$$d = \frac{\lambda^3(3\lambda+\mu+\xi+\theta+\alpha+\mu_0+\xi_0)}{\mu\mu_0(\lambda^2+\lambda\alpha+\lambda\theta+\theta\alpha)} \tag{43}$$
$$e = -\frac{\lambda^4}{\mu\mu_0(\lambda^2+\lambda\alpha+\lambda\theta+\theta\alpha)} \tag{44}$$

Proof: The steady-state balance equations are given below,

$$(\lambda+\theta)\ p(0, 0) = \alpha p(0, 3), \tag{45}$$
$$(\lambda+\theta)\ p(n, 0) = \lambda\ p(n-1, 0) + \alpha\ p(n, 3),\ n =1, 2,\dots n_e \tag{46}$$
$$\theta\ p(n_e+1, 0) = \lambda\ p(n_e, 0) + \alpha\ p(n_e+1, 3), \tag{47}$$
$$(\lambda+\xi)\ p(0, 1) = \mu\ p(1, 1) + \theta p(0, 0), \tag{48}$$
$$(\lambda+\mu+\xi)\ p(n, 1) = \mu p(n+1, 1) + \lambda\ p(n-1, 1) + \theta\ p(n, 0),\ n =1,2,\dots n_e\ , \tag{49}$$

$$(\mu + \xi)\, p(n_e+1, 1) = \lambda\, p(n_e, 1) + \theta\, p(n_e+1, 0) \quad (50)$$
$$(\lambda + \xi_0)\, p(0, 2) = \mu_0\, p(1, 2) + \xi p(0, 1), \quad (51)$$
$$(\lambda + \mu_0 + \xi_0)\, p(n, 2) = \lambda\, p(n-1, 2) + \mu_0\, p(n+1, 2) + \xi p(n, 1), \ n = 1,2,\ldots n_e \quad (52)$$
$$(\mu_0 + \xi_0)\, p(n_e+1, 2) = \lambda\, p(n_e, 2) + \xi p(n_e+1, 1) \quad (53)$$
$$(\lambda + \alpha)\, p(0, 3) = \xi_0\, p(0, 2), \quad (54)$$
$$(\lambda + \alpha)\, p(n, 3) = \lambda\, p(n-1, 3) + \xi_0\, p(n, 2), \ n = 1,2,\ldots n_e \quad (55)$$
$$\alpha p(n_e+1, 3) = \lambda\, p(n_e, 3) + \xi_0\, p(n_e+1, 2), \quad (56)$$

Starting with n = 0 and summing each of these balance equations over i, i = 0, 1, 2, 3, Then

$$\mu\, p(n+1, 1) + \mu_0\, p(n+1, 2) = \lambda\, p(n, 3) + \lambda\, p(n, 2) + \lambda\, p(n, 1) + \lambda\, p(n, 0), \ 0 \le n \le n_e. \quad (57)$$

Find the value of p (n, 0), p (n, 1), p (n, 3) and p (n+1, 1) in terms of p (n, 2) then put into equation (57), we have

$$\mu\mu_0(\lambda^2 + \lambda\alpha + \lambda\theta + \theta\alpha)\, p(n+2, 2) - \begin{bmatrix} \mu_0\{\lambda(\lambda+\theta+\alpha)(\lambda+\mu+\xi) + \lambda\theta\alpha + \lambda^2\mu\} \\ +\mu\mu_0(\lambda^2 + \lambda\alpha + \lambda\theta + \theta\alpha)(\lambda + \mu_0 + \xi_0) + \mu_0\alpha\theta\xi \end{bmatrix} p(n+1,2)$$
$$+ \begin{bmatrix} \{\lambda(\lambda+\theta+\alpha)(\lambda+\mu+\xi) + \lambda\theta\alpha + \lambda^2\mu\}(\lambda + \mu_0 + \xi_0) + \mu\lambda(\lambda^2 + \lambda\alpha + \lambda\theta + \theta\alpha) \\ +\lambda^2\mu_0(2\lambda + \mu + \xi + \theta + \alpha) \end{bmatrix} p(n, 2)$$
$$-[\lambda\{\lambda(\lambda+\theta+\alpha)(\lambda+\mu+\xi) + \lambda\theta\alpha + \lambda^2\mu\} + \lambda^2(2\lambda + \mu + \xi + \theta + \alpha)(\lambda + \mu_0 + \xi_0) + \lambda^3\mu_0]\, p(n-1,2)$$
$$+\lambda^3(3\lambda + \mu + \xi + \theta + \alpha + \mu_0 + \xi_0)\, p(n-2,2) - \lambda^4 p(n-3,2) = 0$$
$$x^5 + ax^4 + bx^3 + cx^2 + dx + e = 0$$

which is a fifth -order difference equation with solution ρ_i(i = 1, 2, 3, 4, 5) and a, b, c, d, e are define in equations (40)-(44) Therefore, we can set:

$$p(n, 2) = \sum_{i=0}^{5} c_i \rho_i^n \qquad n = 0,1,2\ldots\ldots n_e$$

where c_i, i = 1, 2, 3, 4 ,5 are constants to be determined. By plugging (31) in (52),

$$p(n, 1) = \sum_{i=0}^{5} v_i \rho_i^n \qquad n = 0,1,2,3\ldots\ldots n_e$$

Again, by plugging (30) in (49), we get:

$$p(n, 0) = \sum_{i=0}^{5} w_i \rho_i^n \quad n = 0,2,3\ldots\ldots n_e$$

Again, by plugging (29)in (46), we get:

$$P(n,3) = \sum_{i=0}^{5} u_i \rho_i^n \quad n = 0,1,2,\ldots\ldots n_e$$

Where v_i, w_i and u_i are described in equations (37)-(39)

$p(n_e+1, 0)$, $p(n_e+1, 1)$, $p(n_e+1, 2)$ and $p(n_e+1, 3)$ are described in equations (33)-(36)

From equation (45), (48), (51) and (54)

$$\sum_{i=0}^{5}\left(\frac{\lambda}{\rho_i} - \mu_0\right)c_i = 0 \quad (58)$$
$$\sum_{i=0}^{5}\left(\frac{\lambda}{\rho_i} - \mu\right)\left(\lambda + \xi_0 + \mu_0 - \mu_0\rho_i - \frac{\lambda}{\rho_i}\right)c_i = 0 \quad (59)$$
$$\sum_{i=0}^{5}\frac{\lambda}{\rho_i}\left(\lambda + \xi + \mu - \mu\rho_i - \frac{\lambda}{\rho_i}\right)\left(\lambda + \xi_0 + \mu_0 - \mu_0\rho_i - \frac{\lambda}{\rho_i}\right)c_i = 0 \quad (60)$$
$$\sum_{i=0}^{5}\left\{(\lambda+\alpha)\left(\lambda + \theta - \frac{\lambda}{\rho_i}\right)\left(\lambda + \xi + \mu - \mu\rho_i - \frac{\lambda}{\rho_i}\right)\left(\lambda + \xi_0 + \mu_0 - \mu_0\rho_i - \frac{\lambda}{\rho_i}\right) - \xi_0\right\}c_i = 0 \quad (61)$$

Lemma 1: Consider the almost observable case where all customers use the same threshold for entrance n_e, the net reward of a joining customer that finds n customers is

$$S(n) = R - C\left[T(n,1) + \frac{(T(n,2) - T(n,1))\sum_{i=0}^{5} c_i\rho_i^n + \frac{1}{\theta}\sum_{i=0}^{5} w_i\rho_i^n + (\frac{1}{\theta}+\frac{1}{\alpha})\sum_{i=0}^{5} u_i\rho_i^n}{\sum_{i=0}^{5}(c_i+v_i+w_i+u_i)\rho_i^n}\right], n = 0, 1, 2, \ldots n_e \quad (62)$$

$$S(n_e+1) = R - C\left[T(n_e+1,1) + \frac{\begin{matrix}(T(n_e+1,2) - T(n_e+1,1))(\lambda\mu\sum_{i=0}^{5} c_i\rho_i^{n_e} + \lambda\xi\sum_{i=0}^{5}(c_i+v_i+w_i+u_i)\rho_i^{n_e})\theta\alpha + \\ \frac{\alpha}{\theta}[\lambda(\mu\mu_0+\xi\mu_0+\mu\xi_0)\sum_{i=0}^{5}(w_i+u_i)\rho_i^{n_e}] + \lambda\mu\xi_0(\frac{\alpha^3+\alpha\theta+\theta^2}{\theta\alpha^2})\sum_{i=0}^{5} c_i\rho_i^{n_e} \\ \frac{\theta}{\alpha}(\frac{1}{\theta}+\frac{1}{\alpha})[\lambda(\mu\mu_0+\xi\mu_0+\mu\xi_0)\sum_{i=0}^{5} u_i\rho_i^{n_e}] + \lambda\xi\xi_0(\frac{\alpha^3+\alpha\theta+\theta^2}{\theta\alpha^2})\sum_{i=0}^{5}(c_i+v_i+w_i+u_i)\rho_i^{n_e}\end{matrix}}{\begin{matrix}[\alpha\lambda(\mu\mu_0+\xi\mu_0+\mu\xi_0+\alpha\mu_0)\sum_{i=0}^{5}(w_i+u_i)\rho_i^{n_e} + \lambda(\xi\xi_0\alpha+\xi_0\alpha\theta+\xi\alpha\theta+\xi\xi_0\theta)\sum_{i=0}^{5}(c_i+v_i+w_i+u_i)\rho_i^{n_e}] + \\ +\lambda\mu_0\theta\alpha\sum_{i=0}^{5} v_i\rho_i^{n_e} + \lambda\mu(\alpha\xi_0+\theta\xi_0+\alpha\theta)\sum_{i=0}^{5} c_i\rho_i^{n_e} + [\lambda\theta(\mu\mu_0+\xi\mu_0+\mu\xi_0)\sum_{i=0}^{5} u_i\rho_i^{n_e}]\end{matrix}}\right] \quad (63)$$

Proof: The expected utility of a joining customer that finds n customers is S(n) = R− CT(n), where T(n) denotes his mean sojourn time given that he finds n customers in the system.

Let p(I = i|L = n) be the probability that the server at state i when he observes n customers upon his arrival. Conditioning on the state of the server,

$$T(n) = T(n, 0)\, p(I = 0|L = n) + T(n, 1)\, p(I = 1|L = n) + T(n, 2)\, p(I = 2|L = n) + T(n, 3)\, p(I = 3|L = n)$$
$$T(n) = T(n, 1) + (T(n, 2) - T(n, 1))p(I = 2|L = n) + \frac{1}{\theta}p(I = 0|L = n) + (\frac{1}{\theta} + \frac{1}{\alpha})p(I = 3|L = n).$$

T(n, 2) and T(n, 1) are given in equation (24) and (25).

By using PASTA property, the probability that there are n customers in the system given that the server is found at state i is

$$p(I= i|L = n) = \frac{\lambda P(n,i)}{\lambda P(n,0)+\lambda P(n,1)+\lambda P(n,2)+\lambda P(n,3)} \quad I = 0, 2, 3$$

$$T(n) = T(n,1) + \frac{(T(n,2)-T(n,1))\sum_{i=0}^{5} c_i\rho_i^n + \frac{1}{\theta}\sum_{i=0}^{5} w_i\rho_i^n + (\frac{1}{\theta}+\frac{1}{\alpha})\sum_{i=0}^{5} u_i\rho_i^n}{\sum_{i=0}^{5}(c_i+v_i+w_i+u_i)\rho_i^n} \qquad (64)$$

$$T(n_e+1) = T(n_e+1,1) + \frac{\begin{matrix}(T(n_e+1,2)-T(n_e+1,1))(\lambda\mu\sum_{i=0}^{5} c_i\rho_i^{n_e} + \lambda\xi\sum_{i=0}^{5}(c_i+v_i+w_i+u_i)\rho_i^{n_e})\theta\alpha + \\ \frac{\alpha}{\theta}[\lambda(\mu\mu_0+\xi\mu_0+\mu\xi_0)\sum_{i=0}^{5}(w_i+u_i)\rho_i^{n_e}] + \lambda\mu\xi_0(\frac{\alpha^3+\alpha\theta+\theta^2}{\theta\alpha^2})\sum_{i=0}^{5} c_i\rho_i^{n_e} \\ \frac{\theta}{\alpha}(\frac{1}{\theta}+\frac{1}{\alpha})[\lambda(\mu\mu_0+\xi\mu_0+\mu\xi_0)\sum_{i=0}^{5} u_i\rho_i^{n_e}] + \lambda\xi\xi_0(\frac{\alpha^3+\alpha\theta+\theta^2}{\theta\alpha^2})\sum_{i=0}^{5}(c_i+v_i+w_i+u_i)\rho_i^{n_e}\end{matrix}}{\begin{matrix}[\alpha\lambda(\mu\mu_0+\xi\mu_0+\mu\xi_0+\alpha\mu_0)\sum_{i=0}^{5}(w_i+u_i)\rho_i^{n_e} + \lambda(\xi\xi_0\alpha+\xi_0\alpha\theta+\xi\alpha\theta+\xi\xi_0\theta)\sum_{i=0}^{5}(c_i+v_i+w_i+u_i)\rho_i^{n_e}] + \\ +\lambda\mu_0\theta\alpha\sum_{i=0}^{5} v_i\rho_i^{n_e} + \lambda\mu(\alpha\xi_0+\theta\xi_0+\alpha\theta)\sum_{i=0}^{5} c_i\rho_i^{n_e} + [\lambda\theta(\mu\mu_0+\xi\mu_0+\mu\xi_0)\sum_{i=0}^{5} u_i\rho_i^{n_e}]\end{matrix}} \qquad (65)$$

It should be noted that a customer does not enter the system even if he finds no customers in front of him if S(0) < 0, otherwise, he enters the queue.

Next, we describe the equilibrium balking pure threshold strategies in the partially observable case by assuming S(0) > 0. We have the following result.
Let

$$f_1(n) = R - C\left[T(n,1) + \frac{(T(n,2)-T(n,1))\sum_{i=0}^{5} c_i\rho_i^n + \frac{1}{\theta}\sum_{i=0}^{5} w_i\rho_i^n + (\frac{1}{\theta}+\frac{1}{\alpha})\sum_{i=0}^{5} u_i\rho_i^n}{\sum_{i=0}^{5}(c_i+v_i+w_i+u_i)\rho_i^n}\right], n = 0,1,2,\ldots \qquad (66)$$

$$f_2(n) = R - C\left[T(n,1) + \frac{\begin{matrix}(T(n,2)-T(n,1))(\lambda\mu\sum_{i=0}^{5} c_i\rho_i^{n-1} + \lambda\xi\sum_{i=0}^{5}(c_i+v_i+w_i+u_i)\rho_i^{n-1})\theta\alpha + \\ \frac{\alpha}{\theta}[\lambda(\mu\mu_0+\xi\mu_0+\mu\xi_0)\sum_{i=0}^{5}(w_i+u_i)\rho_i^{n-1}] + \lambda\mu\xi_0(\frac{\alpha^3+\alpha\theta+\theta^2}{\theta\alpha^2})\sum_{i=0}^{5} c_i\rho_i^{n-1} \\ \frac{\theta}{\alpha}(\frac{1}{\theta}+\frac{1}{\alpha})[\lambda(\mu\mu_0+\xi\mu_0+\mu\xi_0)\sum_{i=0}^{5} u_i\rho_i^{n-1}] + \lambda\xi\xi_0(\frac{\alpha^3+\alpha\theta+\theta^2}{\theta\alpha^2})\sum_{i=0}^{5}(c_i+v_i+w_i+u_i)\rho_i^{n-1}\end{matrix}}{\begin{matrix}[\alpha\lambda(\mu\mu_0+\xi\mu_0+\mu\xi_0+\alpha\mu_0)\sum_{i=0}^{5}(w_i+u_i)\rho_i^{n-1} + \lambda(\xi\xi_0\alpha+\xi_0\alpha\theta+\xi\alpha\theta+\xi\xi_0\theta)\sum_{i=0}^{5}(c_i+v_i+w_i+u_i)\rho_i^{n-1}] + \\ +\lambda\mu_0\theta\alpha\sum_{i=0}^{5} v_i\rho_i^{n-1} + \lambda\mu(\alpha\xi_0+\theta\xi_0+\alpha\theta)\sum_{i=0}^{5} c_i\rho_i^{n-1} + [\lambda\theta(\mu\mu_0+\xi\mu_0+\mu\xi_0)\sum_{i=0}^{5} u_i\rho_i^{n-1}]\end{matrix}}\right], n = 0,1\ldots.. \qquad (67)$$

By definition $f_1(n) = S(n)$, n = 0, 1, 2….n_e and $f_2(n_e+1) = S(n_e+1)$

Moreover, $f_1(n) > f_2(n)$, n = 0, 1,…..

Theorem 3: For the sequences ($f_1(n)$, n ≥ 0) and ($f_2(n)$, n ≥ 0), Then there exist finite non-negative integers $n_L \le n_U$ such that

$$f_1(0), f_1(1), f_1(2) \ldots\ldots\ldots f_1(n_U) > 0, \quad f_1(n_U+1) \le 0 \qquad (68)$$

and

$$f_2(n_U+1), f_2(n_U), \ldots\ldots f_2(n_L+2)\ f_2(n_L+1) \le 0,\ f_2(n_L) > 0 \qquad (69)$$

Or

$$f_2(n_U+1),\ f_2(n_U),\ f_2(n_U-1), \ldots\ldots f_2(1),\ f_2(0) \le 0 \qquad (70)$$

In the partially observable M/M/1 queue with redundant server with breakdowns and delayed repairs the pure threshold strategies of the form 'While arriving at time t, observe N(t); enter if N(t) $\le n_e$-1 and balk otherwise'.

For $n_e \epsilon \{n_L, n_L+1, \ldots\ldots. n_U\}$ are equilibrium strategies.

Proof: if we assume that S(0) > 0 then We have $f_1(0) > 0$ and $\lim_{n\to\infty} f_1(n) = -\infty$, so if n_U is the subscript of the first negativeterm of the sequence ($f_1(n)$), we have that for the finite number n_U the condition (68) holds.

On the other hand, $f_1(n) > f_2(n)$, n = 0, 1, ... In particular we conclude that $f_2(n_U) < f_1(n_U) \le 0$. Now, we begin to gobackwards, starting from the subscript n_U, towards 0 and we let $n_L - 1$ be the subscript of the first positive term of the sequence ($f_2(n)$). Then we have (69). If all the terms of ($f_2(n)$) going backwards from n_U to 0 are non-positive we have (70).

Now we can establish the existence of equilibrium threshold policies in the partially observable case.

In this model we consider an arrival customer assume that all other customers follow the same threshold strategy 'while arriving at time t, observe N(t), enter if N(t) $\le n_e$ and balk otherwise'. for some fixed r $n_e \epsilon \{n_L, n_L+1, \ldots\ldots. n_U\}$

If the tagged customer finds n≤ n_e customers in front of him and decides to enter, his expected benefit is equal to $f_1(n) > 0$ because of (66) and (68). So in this case the customer prefers to enter.

If the tagged customer finds n = $n_e + 1$ customers in front of him and decides to enter, his expected benefit is equal to $f_2(n_e) \leq 0$ because of (67), (69) or (70). So in this case the customer prefers to balk.

Therefore we conclude that customer follow the equilibrium threshold strategies $n_e = n_L, n_L + 1, \dots \dots . n_U$.

The social benefits per unit time when all the customers follow the equilibrium threshold strategies n_e given in theorm 3 equals

$SB_{ao} = \lambda R\ (1 - p(n_e)) - C\sum_{n=0}^{n_e} np(n)$

Where $p(n_e) = p(n_e + 1,0) + p(n_e + 1,1) + p(n_e + 1,2) + p(n_e + 1,3)$

$p(n) = p(n, 0) + p(n, 1) + p(n, 2) + p(n, 3)$

VI. THE ALMOST UNOBSERVABLE CASE

We now proceed to the almost unobservable case with partial breakdown where the arriving customers observe the state of the server upon arrival, but not the queue size. From a methodological point of view, the almost unobservable case is interesting. In the almost unobservable case, a mixed strategy is specified by a vector of joining probabilities (q_0, q_1, q_2, q_3), $q_i \in$ [0, 1], where q_i denotes the joining probability of a customer if the server is found at state i upon arrival, i = 0, 1, 2 ,3.

Clearly, the new queue is equivalent to the original queue except that the arrival intensity λ should be replaced by λq_i when the server is found at state i. The mixed strategy has the form 'while arriving at time t, observe I (t), enter with probability q_i when I(t) = i'. Let p(n, i) be the stationary distribution of the corresponding system.

Lemma 2: For the M / M /1 queue with partial breakdowns, total breakdown and delayed repairs for the almost unobservable case, the stability holds if $\mu > \lambda q_1$ and $\mu_0 > \lambda q_2$

Proof: The steady-state balance equations are given below,

$(\lambda q_0 + \theta)\ p(0, 0) = \alpha p(0, 3),$ (71)

$(\lambda q_0 + \theta)\ p(n, 0) = \lambda q_0\ p(n-1, 0) + \alpha\ p(n, 3), \quad n \geq 1,$ (72)

$(\lambda q_1 + \xi)\ p(0, 1) = \mu\ p(1, 1) + \theta p(0, 0),$ (73)

$(\lambda q_1 + \mu + \xi)\ p(n, 1) = \mu p(n+1, 1) + \lambda q_1\ p(n-1, 1) + \theta\ p(n, 0), \quad n \geq 1,$ (74)

$(\lambda q_2 + \xi_0)\ p(0, 2) = \mu_0\ p(1, 2) + \xi p(0, 1),$ (75)

$(\lambda q_2 + \mu_0 + \xi_0)\ p(n, 2) = \lambda q_2\ p(n-1, 2) + \mu_0\ p(n+1, 2) + \xi p(n, 1), \ n \geq 1.$ (76)

$(\lambda q_3 + \alpha)\ p(0, 3) = \xi_0\ p(0, 2),$ (77)

$(\lambda q_3 + \alpha)\ p(n, 3) = \lambda q_3\ p(n-1, 3) + \xi_0\ p(n, 2), \quad n \geq 1.$ (78)

Starting with n = 0 and summing each of these balance equations over i, i = 0, 1, 2, 3, Then

$\mu\ p(n+1, 1) + \mu_0\ p(n+1, 2) = \lambda q_3\ p(n, 3) + \lambda q_2\ p(n, 2) + \lambda q_1\ p(n, 1) + \lambda q_0\ p(n, 0),\ n \geq 0.$ (79)

Clearly, $\sum_{n=0}^{\infty} p(n, i) = \pi_i$, i = 0, 1, 2. By summing (79) overall n, we arrive at

$\mu\ (\pi_1 - p(0, 1)) + \mu_0\ (\pi_2 - p(0, 2)) = \lambda q_3 \pi_3 + \lambda q_2 \pi_2 + \lambda q_1 \pi_1 + \lambda q_0 \pi_0\ ) = \lambda$, that is,

$\mu p(0, 1) + \mu_0\ p(0, 2) + \lambda q_0 \pi_0 + \lambda q_3 \pi_3 = (\mu - \lambda q_1)\pi_1 + (\mu_0 - \lambda q_2)\pi_2$ (80)

Since all states are communicating, from the theory of recurrent events, all the probabilities p (n, i) (n ≥ 0, i = 0, 1, 3) are either all positive or, alternatively, all equal to zero. This property is crucial for our analysis.

If the Markov chain {(L (t), I (t)), t ≥ 0} is ergodic (positive recurrent), then all the probabilities p (n, i) (n ≥0, i = 0, 1, 3) are positive.

Thus p (0, 2) > 0 and p (0, 3) > 0 we have $\mu > \lambda q_1\ and\ (\mu_0 > \lambda q_2)$, all the probabilities P(n, i) are positive (and sum to one) from the ergodicity theory. The system is stable

Define the partial generating functions as $G_i\ (z) = \sum_{n=0}^{\infty} p(n, i)\ z^n$, | z | ≤1, i = 0, 1, 2, 3.

For Equations (71) – (73) and (77), (78), multiplying both sides by z^n and summing overall n for state i, note that λ should be replaced by λq_i for state i, then

$(\lambda q_0(1-z) + \theta)\ G_0\ (z) = \alpha G_3\ (z)$ (81)

$(\lambda q_3(1-z) + \alpha) G_3\ (z) = \xi_0 G_2\ (z)$ (82)

For Equations (74) and (76), multiplying both sides by z^{n+1} and summing overall n for state i, note that λ should be replaced by λq_i for state i, then

$$\{(\lambda q_1 z-\mu)(1-z)+\xi z\}\, G_1(z) - \theta z G_0(z) = (z-1)\,\mu\, p(0,1) \tag{83}$$

$$\{(\lambda q_2 z-\mu_0)(1-z)+\xi_0 z\}\, G_2(z) - \xi z G_1(z) = (z-1)\mu_0 p(0,2) \tag{84}$$

$$\mu\,(G_2(z) - p(0,2)) + \mu\,(G_3(z) - p(0,3)) = \lambda q_3\, z G_3(z) + \lambda q_2\, z G_2(z) + \lambda q_1\, z G_1(z) + \lambda q_0\, z G_0(z). \tag{85}$$

$$(\mu-\lambda q_1 z)G_1(z) + (\mu_0-\lambda q_2 z)G_2(z) = \mu\, p(0,1) + \mu_0\, p(0,2) + \lambda q_0 z G_0(z) + \lambda q_3 z G_3(z) \tag{86}$$

From equations (81) and (82)

$$G_2 = G_0\left\{\frac{\lambda^2(1-z)^2 q_0 q_3+\lambda(1-z)(\alpha q_0+\theta q_3)+\theta\alpha}{\alpha\xi_0}\right\} \tag{87}$$

Put value of G_2 in equation (84), we have

$$\{(\lambda q_2 z-\mu_0)(1-z)+\xi_0 z\}\{\lambda^2(1-z)^2 q_0 q_3+\lambda(1-z)(\alpha q_0+\theta q_3)+\theta\alpha\} - \xi\alpha\xi_0 z G_1(z) = (z-1)\mu_0\alpha\xi_0 p(0,2) \tag{88}$$

Solve the equation (83) and (88), then

$$G_0 = \frac{(z-1)\mu p(0,1) z\alpha\xi\,\xi_0+(z-1)\mu_0\alpha\xi_0 p(0,2)\{(\lambda q_1 z-\mu)(1-z)+\xi z\}}{Q(z)} \tag{89}$$

Where

$$Q(z) = -\,[\{(\lambda q_1 z-\mu)(\lambda q_2 z-\mu_0)(1-z)+(\lambda q_1 z-\mu)z\xi_0+(\lambda q_2 z-\mu_0)z\xi\} * \{\lambda^2(1-z)^2 q_0 q_3 + \lambda(1-z)(\alpha q_0+\theta q_3)+\theta\alpha\} + z^2\xi\xi_0\{\lambda^2(1-z)^2 q_0 q_3+\lambda(1-z)(\alpha q_0+\theta q_3)\}]$$

Lemma 2:1 the function Q(z) has a unique root g in (0, 1).

Proof: If $0 < q_i < 1$, $Q(0) = -\mu\mu_0\{\lambda^2 q_0 q_3+\lambda(\alpha q_0+\theta q_3)+\theta\alpha\} < 0$ and

$$Q(1) = \left\{(\mu-\lambda q_1)\xi_0+(\mu_0-\lambda q_2)\xi-\lambda\xi\xi_0\left(\frac{q_0}{\theta}+\frac{q_3}{\alpha}\right)\right\}\alpha\theta > 0$$

Q(z) has a roots lie (0, 1), clearly $\mu\xi_0+\mu_0\xi > \lambda q_1\xi_0+\lambda q_2\xi$

if $\frac{\mu}{\lambda q_1} \geq \frac{\mu_0}{\lambda q_2}, \frac{\mu}{\lambda q_1} > 1$ and $Q(\frac{\mu}{\lambda q_1}) = \frac{\mu\xi}{\lambda q_1}\left(\mu_0-\frac{\mu q_2}{q_1}\right)\alpha\theta \leq 0$ (if $q_0 = q_3 = 0$)

Similarly, if $\frac{\mu_0}{\lambda q_2} > \frac{\mu}{\lambda q_1}, \frac{\mu_0}{\lambda q_2} > 1$ and $Q(\frac{\mu_0}{\lambda q_2}) < 0$

Then function Q(z) has a unique root g in (0, 1).

Put z = g in Q(z) and $q_0 = q_1 = q_2 = q_3 = 0$ now

$$-\;[\mu\mu_0(1-g)-\mu g\xi_0-\xi g\mu_0]\alpha\theta = 0$$

$$g = \frac{\mu\mu_0}{\mu\mu_0+\mu\xi_0+\xi\mu_0}$$

The numerator of (89) must equal zero when z = g. then

$$\mu p(0,1) = -\frac{\mu_0 p(0,2)\varphi(g)}{g\xi}$$

put this value in equation (80),we have

$$P(0,2) = \frac{g\xi\{(\mu-\lambda q_1)\pi_1+(\mu_0-\lambda q_2)\pi_2-\lambda q_0\pi_0-\lambda q_3\pi_3\}}{g\xi\mu_0-\mu_0\varphi(g)} \tag{90}$$

where $\varphi(g) = \{(\lambda q_1 g-\mu)(1-g)+\xi g\}$

Since $G_i(1) = \pi_i$, by differentiating (81) – (86) with respect to z and setting z = 1, then

$$\theta G_0'(1) = \lambda q_0\pi_0 + \alpha G_3'(1)$$

$$\alpha G_3'(1) = \lambda q_3\pi_3 + \xi_0 G_2'(1)$$

$$\xi G_1'(1) - \theta G_0'(1) = \theta\pi_0 + \pi_1(\lambda q_1-\mu-\xi) - \mu\, p(0,1)$$

$$\xi_0 G_2'(1) - \xi G_1'(1) = \xi\pi_1 + \pi_2(\lambda q_2-\mu_0-\xi_0) - \mu_0\, p(0,2)$$

$$\mu G_1'(1) + \mu_0 G_2'(1)) = \lambda q_0\pi_0+\lambda q_1\pi_1+\lambda q_2\pi_2+\lambda q_3\pi_3+\lambda q_0 G_0'(1)+\lambda q_1 G_1'(1)+\lambda q_2 G_2'(1)+\lambda q_3 G_3'(1)$$

Now solving these equations

$$G_0'(1) = \frac{\mu\xi_0(\xi\pi_1-\mu_0\pi_2-\xi_0\pi_2)-\mu_0\xi_0(\mu-\lambda q_1)p(0,2)+\xi_0(\mu_0+\xi_0)\lambda q_1\pi_2+\lambda\xi_0(\mu-\lambda q_1+\xi)(q_0\pi_0+q_2\pi_2+q_3\pi_3)+\lambda\xi(\mu_0-\lambda q_2)(q_0\pi_0+q_3\pi_3) - \frac{\lambda^2 q_0 q_3\pi_0\xi\xi_0}{\alpha}}{\theta\{(\mu-\lambda q_1)\xi_0+(\mu_0-\lambda q_2)\xi-\lambda\xi\,\xi_0\left(\frac{q_0}{\theta}+\frac{q_3}{\alpha}\right)\}} \tag{91}$$

$$G_1'(1) = \frac{\lambda\xi_0\left[q_0\pi_0+q_1\pi_1+q_3\pi_3+\mu_0\pi_2+\frac{\lambda q_0^2\pi_0}{\theta}+\left(\frac{q_0}{\theta}+\frac{q_3}{\alpha}\right)\{\pi_2(\mu_0+\lambda q_2-\lambda\xi_0)+\xi\pi_1+\lambda q_3\pi_3\}\right] + (\mu_0-\lambda q_2)\{(\mu_0-\lambda q_2)\pi_2-\xi\pi_1\}+\mu_0 p(0,2)[\mu_0-\lambda q_2-\lambda\xi\,\xi_0\left(\frac{q_0}{\theta}+\frac{q_3}{\alpha}\right)\}}{\left\{(\mu-\lambda q_1)\xi_0+(\mu_0-\lambda q_2)\xi-\lambda\xi\,\xi_0\left(\frac{q_0}{\theta}+\frac{q_3}{\alpha}\right)\right\}} \tag{92}$$

$$G_2'(1)=\frac{\mu(\xi\pi_1-\mu_0\pi_2-\xi_0\pi_2)-\mu_0(\mu-\lambda q_1)p(0,2)+(\mu-\lambda q_1)\lambda q_2\pi_2+(\mu_0+\xi_0)\lambda q_1\pi_2+\lambda\xi(q_0\pi_0+q_2\pi_2+q_3\pi_3)+\frac{\lambda^2 q_0^2\pi_0\xi}{\theta}+\frac{\lambda^2 q_3^2\pi_3\xi}{\alpha}+\frac{\lambda^2 q_0 q_3\pi_3\xi}{\theta}}{\left\{(\mu-\lambda q_1)\xi_0+(\mu_0-\lambda q_2)\xi-\lambda\xi\xi_0\left(\frac{q_0}{\theta}+\frac{q_3}{\alpha}\right)\right\}} \quad (93)$$

$$G_3'(1)=\frac{\mu\xi_0(\xi\pi_1-\mu_0\pi_2-\xi_0\pi_2)-\mu_0\xi_0(\mu-\lambda q_1)p(0,2)+\xi_0(\mu_0+\xi_0)\lambda q_1\pi_2+\lambda\xi\xi_0(q_0\pi_0+q_2\pi_2+q_3\pi_3)+(\mu-\lambda q_1)\lambda q_2\pi_2\xi_0+\lambda q_3\pi_3\{\xi_0(\mu-\lambda q_1)+\xi(\mu_0-\lambda q_2)\}+\frac{\lambda^2 q_0^2\pi_0\xi\xi_0}{\theta}}{\alpha\{(\mu-\lambda q_1)\xi_0+(\mu_0-\lambda q_2)\xi-\lambda\xi\xi_0\left(\frac{q_0}{\theta}+\frac{q_3}{\alpha}\right)\}} \quad (94)$$

The quantity G_i (1) can be considered as the contribution of state i to the mean queue length. Intuitively, since all the three states are communicating, the accumulation of customers in one state will influence all the three states.

By using PASTA property, the probability that there are n customers in the system given that the server is found at state i is

$P(n|i)=\frac{p(n,i)}{\sum_{n=0}^{\infty}p(n,i)}=\frac{p(n,i)}{\pi_i}$, $n\geq 0$, i = 0, 1, 2, 3.

Let $L_i(q_0,q_1,q_2,q_3)=\sum_{n=0}^{\infty}np(n|i)=\frac{\sum_{n=0}^{\infty}np(n|i)}{\pi_i}=\frac{G_i'(1)}{\pi_i}$, i = 0, 1, 2,3.

$$L_0(q_0,q_1,q_2,q_3)=\frac{\mu\alpha\theta(\xi^2-\mu_0\xi_0-\xi_0^2)+\xi_0(\mu_0+\xi_0)\lambda q_1\alpha\theta+\lambda\xi_0(\mu-\lambda q_1+\xi)(q_0\xi\theta+q_2\theta\alpha+q_3\xi\alpha)+\lambda\xi^2(\mu_0-\lambda q_2)(q_0\xi\theta+q_3\xi\alpha)-\frac{\lambda^2 q_0 q_3\theta\xi^2\xi_0}{\alpha}}{\xi\theta^2\{(\mu-\lambda q_1)\xi_0+(\mu_0-\lambda q_2)\xi-\lambda\xi\xi_0\left(\frac{q_0}{\theta}+\frac{q_3}{\alpha}\right)\}}-\frac{g(\mu-\lambda q_1)\{(\mu-\lambda q_1)\xi\alpha\theta+(\mu_0-\lambda q_2)\xi_0\alpha\theta-\lambda q_0\xi_0\xi\theta-\lambda q_3\xi\xi_0\alpha\}}{\theta^2\{g\xi-\varphi(g)\}\{(\mu-\lambda q_1)\xi_0+(\mu_0-\lambda q_2)\xi-\lambda\xi\xi_0\left(\frac{q_0}{\theta}+\frac{q_3}{\alpha}\right)\}}$$

$$L_1(q_0,q_1,q_2,q_3)=-\frac{g\xi\{\mu_0-\lambda q_2-\lambda\xi_0\left(\frac{q_0}{\theta}+\frac{q_3}{\alpha}\right)\}\{(\mu-\lambda q_1)\xi\alpha\theta+(\mu_0-\lambda q_2)\xi_0\alpha\theta-\lambda q_0\xi_0\xi\theta-\lambda q_3\xi\xi_0\alpha\}}{\alpha\theta(g\xi-\varphi(g))\{(\mu-\lambda q_1)\xi_0+(\mu_0-\lambda q_2)\xi-\lambda\xi\xi_0\left(\frac{q_0}{\theta}+\frac{q_3}{\alpha}\right)\}}$$

$$L_2(q_0,q_1,q_2,q_3)=\frac{\mu\alpha\theta(\xi^2-\mu_0\xi_0-\xi_0^2)+(\mu_0+\xi_0)\lambda q_1\xi_0\alpha\theta+(\mu-\lambda q_1)\lambda q_2\xi_0\alpha\theta+\lambda\xi\xi_0(q_0\xi\theta+q_2\theta\alpha+q_3\xi\alpha)+\lambda^2\xi^2\xi_0(q_0^2+q_3^2)+\frac{\lambda^2 q_0 q_3\alpha\xi^2\xi_0}{\theta}}{\xi_0\alpha\theta\{(\mu-\lambda q_1)\xi_0+(\mu_0-\lambda q_2)\xi-\lambda\xi\xi_0\left(\frac{q_0}{\theta}+\frac{q_3}{\alpha}\right)\}}-\frac{g\xi(\mu-\lambda q_1)\{(\mu-\lambda q_1)\xi\alpha\theta+(\mu_0-\lambda q_2)\xi_0\alpha\theta-\lambda q_0\xi_0\xi\theta-\lambda q_3\xi\xi_0\alpha\}}{\xi_0\theta\alpha\{g\xi-\varphi(g)\}\{(\mu-\lambda q_1)\xi_0+(\mu_0-\lambda q_2)\xi-\lambda\xi\xi_0\left(\frac{q_0}{\theta}+\frac{q_3}{\alpha}\right)\}}$$

$$L_3(q_0,q_1,q_2,q_3)=\frac{\mu\alpha\theta(\xi^2-\mu_0\xi_0-\xi_0^2)+(\mu_0+\xi_0)\lambda q_1\xi_0\alpha\theta+(\mu-\lambda q_1)\lambda q_2\xi_0\alpha\theta+\lambda^2 {q_0}^2\xi^2\xi_0+\lambda\xi\xi_0(q_0\xi\theta+q_2\theta\alpha+q_3\xi\alpha)+\lambda q_3\xi\alpha\{\xi_0(\mu-\lambda q_1)+\xi(\mu_0-\lambda q_2)\}}{\xi\alpha^2\{(\mu-\lambda q_1)\xi_0+(\mu_0-\lambda q_2)\xi-\lambda\xi\xi_0\left(\frac{q_0}{\theta}+\frac{q_3}{\alpha}\right)\}}-\frac{g(\mu-\lambda q_1)\{(\mu-\lambda q_1)\xi\alpha\theta+(\mu_0-\lambda q_2)\xi_0\alpha\theta-\lambda q_0\xi_0\xi\theta-\lambda q_3\xi\xi_0\alpha\}}{\alpha^2\{g\xi-\varphi(g)\}\{(\mu-\lambda q_1)\xi_0+(\mu_0-\lambda q_2)\xi-\lambda\xi\xi_0\left(\frac{q_0}{\theta}+\frac{q_3}{\alpha}\right)\}}$$

If a joining customer finds the server at state i, his mean sojourn time is T($L_i(q_0,q_1,q_2,q_3)$, i), T(n, 0) and T(n, 1), T(n,2) and T(n,3) are given in equation (24)-(27), then

$$T(L_1(q_0,q_1,q_2,q_3),1)=L_1(q_0,q_1,q_2,q_3)\left[\frac{\mu_0+\xi_0+\xi+\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)}{\mu\mu_0+\mu\xi_0+\xi\mu_0}+\frac{\mu_0\xi\left\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\right\}}{(\mu\mu_0+\mu\xi_0+\xi\mu_0)(\mu\xi_0+\xi\mu_0)}\right]+\frac{\mu_0\xi\left\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\right\}}{(\mu\xi_0+\xi\mu_0)^2}\left(\frac{\mu\mu_0}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\right)^{L_1(q_0,q_1,q_2,q_3),+1}+\frac{1}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\left(\mu_0+\xi_0+\xi+\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\frac{\mu{\mu_0}^2\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\xi_0+\xi\mu_0)^2}\right)$$

$$T(L_2(q_0,q_1,q_2,q_3),2)=L_2(q_0,q_1,q_2,q_3)\left[\frac{\mu_0+\xi_0+\xi+\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)}{\mu\mu_0+\mu\xi_0+\xi\mu_0}+\frac{\mu_0\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\mu_0+\mu\xi_0+\xi\mu_0)(\mu\xi_0+\xi\mu_0)}\right]-\frac{\xi_0\mu\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\xi_0+\xi\mu_0)^2}\left(\frac{\mu\mu_0}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\right)^{L_2(q_0,q_1,q_2,q_3)+1}+\frac{1}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\left(\mu_0+\xi_0+\xi+\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)-\frac{\mu{\mu_0}^2\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\xi_0+\xi\mu_0)^2}\right)+\frac{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)}{\mu\xi_0+\xi\mu_0}$$

$$T(L_0(q_0,q_1,q_2,q_3),0)=L_0(q_0,q_1,q_2,q_3)\left[\frac{\mu_0+\xi_0+\xi+\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)}{\mu\mu_0+\mu\xi_0+\xi\mu_0}+\frac{\mu_0\xi\left\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\right\}}{(\mu\mu_0+\mu\xi_0+\xi\mu_0)(\mu\xi_0+\xi\mu_0)}\right]+\frac{\mu_0\xi\left\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\right\}}{(\mu\xi_0+\xi\mu_0)^2}\left(\frac{\mu\mu_0}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\right)^{L_0(q_0,q_1,q_2,q_3),+1}+\frac{1}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\left(\mu_0+\xi_0+\xi+\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\frac{\mu{\mu_0}^2\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\xi_0+\xi\mu_0)^2}\right)+\frac{1}{\theta}$$

$T(L_3(q_0,q_1,q_2,q_3),3) =$

$$L_3(q_0,q_1,q_2,q_3)\left[\frac{\mu_0+\xi_0+\xi+\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)}{\mu\mu_0+\mu\xi_0+\xi\mu_0}+\frac{\mu_0\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\mu_0+\mu\xi_0+\xi\mu_0)(\mu\xi_0+\xi\mu_0)}\right]-\frac{\xi_0\mu\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\xi_0+\xi\mu_0)^2}\left(\frac{\mu\mu_0}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\right)^{L_3(q_0,q_1,q_2,q_3)+1}$$
$$+\frac{1}{\mu\mu_0+\mu\xi_0+\xi\mu_0}\left(\mu_0+\xi_0+\xi+\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)-\frac{\mu\mu_0{}^2\xi\{\mu-\mu_0+\mu\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{(\mu\xi_0+\xi\mu_0)^2}\right)+\frac{1}{\theta}+\frac{1}{\alpha}$$

The expected net reward of such a customer is

$$S_i\ (q_0,q_1,q_2,q_3) = R - C\ T(L_i(q_0,q_1,q_2,q_3),\ i),\ i = 0, 1, 2, 3$$

VII. THE FULLY UNOBSERVABLE CASE

In this section, arriving customers can observe neither the state of the server at their arrival instant nor the number of customers present. A strategy can be described by a fraction q ($0 < q < 1$), which is the probability of joining, and the effective arrival rate is q.

To identify the equilibrium strategies of the customers, we should first investigate the stationary distribution of the system when all customers follow a given strategy q, which can be obtained by lemma 2 By taking $q_0 = q_1 = q_2 = q_3 = q$, we can get the expressions of $G_i'(1)$ in a way similar to that exhibited in Section VI, Then

$$G_0'(1) = \frac{\mu\xi_0(\xi\pi_1-\mu_0\pi_2-\xi_0\pi_2)-\mu_0\xi_0(\mu-\lambda q)p(0,2)+\xi_0(\mu_0+\xi_0)\lambda q\pi_2+\lambda q\xi_0(\mu-\lambda q+\xi)(\pi_0+\pi_2+\pi_3)+\lambda q\xi(\mu_0-\lambda q)(\pi_0+\pi_3)-\frac{\lambda^2q^2\pi_0\xi\xi_0}{\alpha}}{\theta\left\{(\mu-\lambda q)\xi_0+(\mu_0-\lambda q)\xi-\lambda q\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\right\}}$$

$$G_1'(1) = \frac{\lambda\xi_0\left[q\pi_0+q\pi_1+q\pi_3+\mu_0\pi_2+\frac{\lambda q^2\pi_0}{\theta}+q\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\{\pi_2(\mu_0+\lambda q-\lambda\xi_0)+\xi\pi_1+\lambda q\pi_3\}\right]+(\mu_0-\lambda q)\{(\mu_0-\lambda q)\pi_2-\xi\pi_1\}+\mu_0p(0,2)\{\mu_0-\lambda q-\lambda q\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\}}{\left\{(\mu-\lambda q)\xi_0+(\mu_0-\lambda q)\xi-\lambda q\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\right\}}$$

$$G_2'(1) = \frac{\mu(\xi\pi_1-\mu_0\pi_2-\xi_0\pi_2)-\mu_0(\mu-\lambda q)p(0,2)+(\mu-\lambda q)\lambda q\pi_2+(\mu_0+\xi_0)\lambda q\pi_2+\lambda q\xi(\pi_0+\pi_2+\pi_3)+\frac{\lambda^2q^2\xi(\pi_0+\pi_3)}{\theta}+\frac{\lambda^2q^2\pi_3\xi}{\alpha}}{\left\{(\mu-\lambda q)\xi_0+(\mu_0-\lambda q)\xi-\lambda q\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\right\}}$$

$G_3'(1) =$

$$\frac{\mu\xi_0(\xi\pi_1-\mu_0\pi_2-\xi_0\pi_2)-\mu_0\xi_0(\mu-\lambda q)p(0,2)+\xi_0(\mu_0+\xi_0)\lambda q\pi_2+\lambda q\xi\xi_0(\pi_0+\pi_2+\pi_3)+(\mu-\lambda q)\lambda q\pi_2\xi_0+\lambda q\pi_3\{\xi_0(\mu-\lambda q)+\xi(\mu_0-\lambda q)\}+\frac{\lambda^2q^2\pi_0\xi\xi_0}{\theta}}{\alpha\left\{(\mu-\lambda q)\xi_0+(\mu_0-\lambda q)\xi-\lambda q\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\right\}}$$

Thus, the mean queue length is given by

$E(L) = G_0'(1) + G_1'(1) + G_2'(1) + G_3'(1) =$

$$\frac{\begin{gathered}\mu\xi^2\alpha\theta(\xi_0\theta+\xi_0\alpha+\alpha\theta)-(\mu_0+\xi_0)(\mu-\lambda q)(\xi_0\theta+\xi_0\alpha+\alpha\theta)\alpha\theta\xi_0+\lambda q(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta)\\(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\\+(\mu-\lambda q)\lambda q\xi_0\alpha\{(\xi_0\theta+\xi_0\alpha+\alpha\theta)\theta+2\xi\xi_0\theta+\xi\xi_0\alpha\}+\alpha\theta(\mu_0-\lambda q)\{\lambda q\xi\xi_0(2\xi+\alpha)+\xi_0\alpha\theta(\mu_0-\lambda q)+\alpha\theta\xi^2\}\\+\mu_0(\xi_0\theta\alpha)^2+\\\lambda^2q^2\xi\xi_0{}^2\theta\alpha+\lambda^2q^2\xi^2\xi_0(\alpha^2+\alpha\theta+\theta)+\lambda\xi_0q(\theta+\alpha)\{(\mu_0+\lambda q-\lambda\xi_0)\xi_0\alpha\theta+\xi^2\alpha\theta+\lambda q\xi\xi_0\alpha\}\end{gathered}}{\alpha\theta(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\left\{(\mu-\lambda q)\xi_0+(\mu_0-\lambda q)\xi-\lambda q\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\right\}}$$
$$-\frac{g\xi(\mu-\lambda q)\{(\xi_0\theta+\xi_0\alpha+\alpha\theta)+(\mu_0-\lambda q)\alpha\theta-\lambda q\xi\xi_0(\theta+\alpha)\}\{(\mu-\lambda q)\xi\alpha\theta+(\mu_0-\lambda q)\xi_0\alpha\theta-\lambda q\xi_0\xi(\theta+\alpha)\}}{\alpha\theta(g\xi-\varphi(g))(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\left\{(\mu-\lambda q)\xi_0+(\mu_0-\lambda q)\xi-\lambda q\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\right\}}$$

Therefore, the mean sojourn time of a customer who joins the system can be derived by using Little's law, then

$$E(W) = \frac{E(L)}{\lambda q}$$

$$E(W) = \frac{\begin{gathered}\mu\xi^2\alpha\theta(\xi_0\theta+\xi_0\alpha+\alpha\theta)-(\mu_0+\xi_0)(\mu-\lambda q)(\xi_0\theta+\xi_0\alpha+\alpha\theta)\alpha\theta\xi_0+\lambda q(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta)(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\\+(\mu-\lambda q)\lambda q\xi_0\alpha\{(\xi_0\theta+\xi_0\alpha+\alpha\theta)\theta+2\xi\xi_0\theta+\xi\xi_0\alpha\}+\alpha\theta(\mu_0-\lambda q)\{\lambda q\xi\xi_0(2\xi+\alpha)+\xi_0\alpha\theta(\mu_0-\lambda q)+\alpha\theta\xi^2\}+\mu_0(\xi_0\theta\alpha)^2+\\\lambda^2q^2\xi\xi_0{}^2\theta\alpha+\lambda^2q^2\xi^2\xi_0(\alpha^2+\alpha\theta+\theta)+\lambda\xi_0q(\theta+\alpha)\{(\mu_0+\lambda q-\lambda\xi_0)\xi_0\alpha\theta+\xi^2\alpha\theta+\lambda q\xi\xi_0\alpha\}\end{gathered}}{\lambda q\alpha\theta(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\left\{(\mu-\lambda q)\xi_0+(\mu_0-\lambda q)\xi-\lambda q\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\right\}} - \frac{g\xi(\mu-\lambda q)\{(\xi_0\theta+\xi_0\alpha+\alpha\theta)+(\mu_0-\lambda q)\alpha\theta-\lambda q\xi\xi_0(\theta+\alpha)\}\{(\mu-\lambda q)\xi\alpha\theta+(\mu_0-\lambda q)\xi_0\alpha\theta-\lambda q\xi_0\xi(\theta+\alpha)\}}{\lambda q\alpha\theta(g\xi-\varphi(g))(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\left\{(\mu-\lambda q)\xi_0+(\mu_0-\lambda q)\xi-\lambda q\xi\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\right\}}$$

Clearly, E (W) is strictly increasing for q, $q \in [0, 1]$. This property is crucial for our analysis. A general balking strategy in the fully unobservable case is specified by a single joining probability q. The case q = 0 corresponds to the pure strategy 'to balk' whereas the case q = 1 corresponds to the pure strategy 'to join'. Any value of $q \in (0, 1)$ corresponds to a mixed strategy 'to join with probability q'. We can describe the equilibrium behavior of the customers in the following theorem. The equilibrium strategies depend on the value of the ratio R /C. Customers have a greater incentive to enter the system if the value of R/ C is higher.

Theorem 4: In the fully unobservable M / M /1 queue with partial breakdowns, total breakdowns and delayed repairs, there exists a unique equilibrium strategy 'enter with probability q_e ', where q_e is specified

$$q_e = \begin{cases} 0, & \frac{R}{C} < \mathrm{E(W)}|_{q=0}, \\ q_e^*, & \mathrm{E(W)}|_{q=0} \le \frac{R}{C} \le \mathrm{E(W)}|_{q=1} \\ 1, & \frac{R}{C} > E(W)|_{q=1} \end{cases}$$

Where q_e^* is the unique root of the equation R-CE(W) = 0 and

$$\mathrm{E(W)}|_{q=0} > \frac{(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta)(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)-(\alpha\theta)^2\{\mu_0\xi_0+\xi^2\} \quad \mu\xi_0\alpha\{(\xi_0\theta+\xi_0\alpha+\alpha\theta)\theta+2\xi\xi_0\theta+\xi\xi_0\alpha\}+\xi_0\alpha\theta(\theta+\alpha)\{(\mu_0-\lambda\xi_0)\xi_0+\xi^2\}}{\alpha\theta(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\{\mu\xi_0+\xi\mu_0\}} + \frac{g\xi\{(\xi_0\theta+\xi_0\alpha+\alpha\theta)+\mu_0\alpha\theta\}\{\mu\xi+\mu_0\xi_0\}}{(g-1)(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\{\mu\xi_0+\xi\mu_0\}}$$

$$\mathrm{E(W)}|_{q=1} = \frac{\begin{gathered}\mu\xi^2\alpha\theta(\xi_0\theta+\xi_0\alpha+\alpha\theta)-(\mu_0+\xi_0)(\mu-\lambda)(\xi_0\theta+\xi_0\alpha+\alpha\theta)\alpha\theta\xi_0+\lambda(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta)(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\\ +(\mu-\lambda q)\lambda\xi_0\alpha\{(\xi_0\theta+\xi_0\alpha+\alpha\theta)\theta+2\xi\xi_0\theta+\xi\xi_0\alpha\}+\alpha\theta(\mu_0-\lambda)\{\lambda q\xi\,\xi_0(2\xi+\alpha)+\xi_0\alpha\theta(\mu_0-\lambda)+\alpha\theta\xi^2\}+\mu_0(\xi_0\theta\alpha)^2+\\ \lambda^2\xi\xi_0{}^2\theta\alpha+\lambda^2\xi^2\xi_0(\alpha^2+\alpha\theta+\theta)+\lambda\xi_0(\theta+\alpha)\{(\mu_0+\lambda-\lambda\xi_0)\xi_0\alpha\theta+\xi^2\alpha\theta+\lambda\xi\,\xi_0\alpha\}\end{gathered}}{\lambda\alpha\theta(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\left\{(\mu-\lambda)\xi_0+(\mu_0-\lambda)\xi-\lambda\xi\,\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\right\}} - \frac{g\xi(\mu-\lambda)\{(\xi_0\theta+\xi_0\alpha+\alpha\theta)+(\mu_0-\lambda)\alpha\theta-\lambda\xi\xi_0(\theta+\alpha)\}\{(\mu-\lambda)\xi\alpha\theta+(\mu_0-\lambda)\xi_0\alpha\theta-\lambda\xi_0\xi(\theta+\alpha)\}}{\lambda\alpha\theta(g\xi-\varphi(g))(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\left\{(\mu-\lambda)\xi_0+(\mu_0-\lambda)\xi-\lambda\xi\,\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\right\}}$$

Proof: Based on the reward-cost structure, if a tagged customer decides to enter the system at his arrival instant, his expected net reward is

$$S(q) = R- C\frac{\begin{gathered}\mu\xi^2\alpha\theta(\xi_0\theta+\xi_0\alpha+\alpha\theta)-(\mu_0+\xi_0)(\mu-\lambda q)(\xi_0\theta+\xi_0\alpha+\alpha\theta)\alpha\theta\xi_0+\lambda q(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta)(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\\ +(\mu-\lambda q)\lambda q\xi_0\alpha\{(\xi_0\theta+\xi_0\alpha+\alpha\theta)\theta+2\xi\xi_0\theta+\xi\xi_0\alpha\}+\alpha\theta(\mu_0-\lambda q)\{\lambda q\xi\xi_0(2\xi+\alpha)+\xi_0\alpha\theta(\mu_0-\lambda q)+\alpha\theta\xi^2\}+\mu_0(\xi_0\theta\alpha)^2+\\ \lambda^2q^2\xi\xi_0{}^2\theta\alpha+\lambda^2q^2\xi^2\xi_0(\alpha^2+\alpha\theta+\theta)+\lambda\xi_0q(\theta+\alpha)\{(\mu_0+\lambda q-\lambda\xi_0)\xi_0\alpha\theta+\xi^2\alpha\theta+\lambda q\xi\xi_0\alpha\}\end{gathered}}{\lambda q\alpha\theta(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\left\{(\mu-\lambda q)\xi_0+(\mu_0-\lambda q)\xi-\lambda q\xi\,\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\right\}} - \frac{g\xi(\mu-\lambda q)\{(\xi_0\theta+\xi_0\alpha+\alpha\theta)+(\mu_0-\lambda q)\alpha\theta-\lambda q\xi\xi_0(\theta+\alpha)\}\{(\mu-\lambda q)\xi\alpha\theta+(\mu_0-\lambda q)\xi_0\alpha\theta-\lambda q\xi_0\xi(\theta+\alpha)\}}{\lambda q\alpha\theta(g\xi-\varphi(g))(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\left\{(\mu-\lambda q)\xi_0+(\mu_0-\lambda q)\xi-\lambda q\xi\,\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\right\}} \qquad (95)$$

Clearly, S (q) is strictly decreasing for q, q ∈ [0, 1]. In addition,

$$\mathrm{E(W)}|_{q=0} > \frac{(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta)(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)-(\alpha\theta)^2\{\mu_0\xi_0+\xi^2\} \quad \mu\xi_0\alpha\{(\xi_0\theta+\xi_0\alpha+\alpha\theta)\theta+2\xi\xi_0\theta+\xi\xi_0\alpha\}+\xi_0\alpha\theta(\theta+\alpha)\{(\mu_0-\lambda\xi_0)\xi_0+\xi^2\}}{\alpha\theta(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\{\mu\xi_0+\xi\mu_0\}} + \frac{g\xi\{(\xi_0\theta+\xi_0\alpha+\alpha\theta)+\mu_0\alpha\theta\}\{\mu\xi+\mu_0\xi_0\}}{(g-1)(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\{\mu\xi_0+\xi\mu_0\}}$$

$$E(W)|_{q=1} = \frac{\begin{gathered}\mu\xi^2\alpha\theta(\xi_0\theta+\xi_0\alpha+\alpha\theta)-(\mu_0+\xi_0)(\mu-\lambda)(\xi_0\theta+\xi_0\alpha+\alpha\theta)\alpha\theta\xi_0+\lambda(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta)(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\\ +(\mu-\lambda q)\lambda\xi_0\alpha\{(\xi_0\theta+\xi_0\alpha+\alpha\theta)\theta+2\xi\xi_0\theta+\xi\xi_0\alpha\}+\alpha\theta(\mu_0-\lambda)\{\lambda q\xi\,\xi_0(2\xi+\alpha)+\xi_0\alpha\theta(\mu_0-\lambda)+\alpha\theta\xi^2\}+\mu_0(\xi_0\theta\alpha)^2+\\ \lambda^2\xi\xi_0{}^2\theta\alpha+\lambda^2\xi^2\xi_0(\alpha^2+\alpha\theta+\theta)+\lambda\xi_0(\theta+\alpha)\{(\mu_0+\lambda-\lambda\xi_0)\xi_0\alpha\theta+\xi^2\alpha\theta+\lambda\xi\,\xi_0\alpha\}\end{gathered}}{\lambda\alpha\theta(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\left\{(\mu-\lambda)\xi_0+(\mu_0-\lambda)\xi-\lambda\xi\,\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\right\}} - \frac{g\xi(\mu-\lambda)\{(\xi_0\theta+\xi_0\alpha+\alpha\theta)+(\mu_0-\lambda)\alpha\theta-\lambda\xi\xi_0(\theta+\alpha)\}\{(\mu-\lambda)\xi\alpha\theta+(\mu_0-\lambda)\xi_0\alpha\theta-\lambda\xi_0\xi(\theta+\alpha)\}}{\lambda\alpha\theta(g\xi-\varphi(g))(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\left\{(\mu-\lambda)\xi_0+(\mu_0-\lambda)\xi-\lambda\xi\,\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\right\}}$$

When $\frac{R}{C} < \mathrm{E(W)}|_{q=0}$, S (q) is negative for every q. Therefore, the best response of tagged customer is balking.

If $\frac{R}{C} > \mathrm{E(W)}|_{q=1}$, then S (q)≥ S (1) > 0, the expected net benefit of the tagged customer is positive, thus he joins the system with probability 1.

When $\mathrm{E(W)}|_{q=0} \le \frac{R}{C} \le \mathrm{E(W)}|_{q=1}$, there exists a unique root q_e^* of the equation S (q) = 0 in the interval [0, 1].

The social benefit per unit time can now be easily calculated as

$$SB_{fu} = \lambda qR - C\frac{\begin{gathered}\mu\xi^2\alpha\theta(\xi_0\theta+\xi_0\alpha+\alpha\theta)-(\mu_0+\xi_0)(\mu-\lambda q)(\xi_0\theta+\xi_0\alpha+\alpha\theta)\alpha\theta\xi_0+\lambda q(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta)(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\\ +(\mu-\lambda q)\lambda q\xi_0\alpha\{(\xi_0\theta+\xi_0\alpha+\alpha\theta)\theta+2\xi\xi_0\theta+\xi\xi_0\alpha\}+\alpha\theta(\mu_0-\lambda q)\{\lambda q\xi\xi_0(2\xi+\alpha)+\xi_0\alpha\theta(\mu_0-\lambda q)+\alpha\theta\xi^2\}+\mu_0(\xi_0\theta\alpha)^2+\\ \lambda^2q^2\xi\xi_0{}^2\theta\alpha+\lambda^2q^2\xi^2\xi_0(\alpha^2+\alpha\theta+\theta)+\lambda\xi_0q(\theta+\alpha)\{(\mu_0+\lambda q-\lambda\xi_0)\xi_0\alpha\theta+\xi^2\alpha\theta+\lambda q\xi\xi_0\alpha\}\end{gathered}}{\alpha\theta(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\left\{(\mu-\lambda q)\xi_0+(\mu_0-\lambda q)\xi-\lambda q\xi\,\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\right\}} - C\frac{g\xi(\mu-\lambda q)\{(\xi_0\theta+\xi_0\alpha+\alpha\theta)+(\mu_0-\lambda q)\alpha\theta-\lambda q\xi\xi_0(\theta+\alpha)\}\{(\mu-\lambda q)\xi\alpha\theta+(\mu_0-\lambda q)\xi_0\alpha\theta-\lambda q\xi_0\xi(\theta+\alpha)\}}{\alpha\theta(g\xi-\varphi(g))(\xi\xi_0\theta+\xi\xi_0\alpha+\xi\alpha\theta+\xi_0\alpha\theta)\left\{(\mu-\lambda q)\xi_0+(\mu_0-\lambda q)\xi-\lambda q\xi\,\xi_0\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\right\}} \qquad (96)$$

VIII CONCLUSION AND FUTURE WORK

In this paper we considered equilibrium Strategic behaviour in single server markovian queueing system with partial breakdown,total breakdown and delayed repair. We studied the equilibrium balking strategies and expected social benefits per unit time for the customer in four different case with respect to the level of information provided to arriving customers . We also discussed the sensitivity of the equilibrium threshold with various parameters in fully and almost observable queue and sensitivity of the expected sojourn time of customer in almost and fully unobservable queues. This work can be extended in several different directions. One direction is to study the equilibrium Strategic in multi-server system. The direct generalization is to concern the optimal price of social planner due to the increased operating costs caused by switching service rates.

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Source of support: Nil, Conflict of interest: None Declared.