**International Journal of Mathematical Archive-12(5), 2021, 50-56**
**Available online through www.ijma.info** ISSN 2229 – 5046

# COMMON FIXED POINT THEOREMS FOR FOUR SELF-MAPPINGS SATISFYING (E.A.) PROPERTY VIA A -CLASS FUNCTIONS IN FUZZY METRIC SPACE

**KAMAL WADHWA[1] AND ARVIND KUMAR GUPTA[2]***

[1]**Department of Mathematics Govt. P.G. College Pipariya - (M.P.), India.**
[2]**Department of Mathematics Govt. Polytechnic College Mandla - (M.P.), India.**

***(Received On: 19-04-21; Revised & Accepted On: 10-05-21)***

***ABSTRACT***

*In this paper we prove some common fixed point theorems for weakly compatible mappings in fuzzy metric space of Kromosil and Michelek & George and Veeramani by using a new concept of* ***A*** *– class functions with E.A. property. Some examples are also given in support of the result.*

***Keywords:*** *K.M. & G.V. fuzzy metric space, weakly compatible mappings, E.A. property,* ***A*** *– class function.*

## 1. INTRODUCTION

The notion of fuzzy metric space was given by Zadeh [4] in 1965. George and Veeramani [2] defined continuous t- norms and modified the concept of fuzzy metric space in 1994, which was introduced by Kramosil and Michelek [5] in 1975.Fuzzy set theory comes under A.I. (Artificial Intelligence) or soft computing methods.The concept of fuzzy set is very useful in deferent fields likecomputer science ,linear programming, communication, gaming and control engineering etc. Amri and E.I. Moutawakil [1] introduced the property E.A. in 2002. Objective of this paper is to define a new concept of **A –** class of functions and to prove some common fixed point theorems in fuzzy metric space with the help of this class.

## 2. PRELIMINARIES

**Definition 2.1:** (W. Sintunavarat, P. kumam [7]). A binary operation $*$: $[0, 1] \times [0, 1] \to [0, 1]$ is continuous t-norm if for all p, q, r, s $\in [0, 1]$ the following conditions are satisfied.
Nt-1. $(p* q) = (q * p)$ commutative.
Nt-2. $(p*q)* r = p* (q*r)$ associative.
Nt-3. $p*1 = p$ identity element.
Nt-4.$*$is continuous.
Nt-5 $p*q \le r*s$ whenever $p \le r$ and $q \le s$ for all p, q, r, s $\in [0, 1]$.

**Example 2.2:** [7] There are some examples of t- norms.

1. ( The product t- norm) T.P.: A mapping Tp: $[0, 1] \times [0, 1] \to [0,1]$ is defined by $Tp( p, q) = pq$
2. (The minimum t-norm) T.M.: A mapping Tm: $[0, 1] \times [0, 1] \to [0,1]$ is defined by $Tm( p, q) = \min\{ p, q\}$

**Definition 2.3:** (Kramosil and Michelek (1975) [5]): Let X be an arbitrary set, $*$ is a continuous t-norm, then the 3- tuple $(X,M,*)$ is called a KM fuzzy metric space if M is a fuzzy set on $X \times X \times [0,\infty)$ such that the following axioms hold.
For all x, y, z $\in$X and s, t > 0,
(FM-1) $M(x, y, 0) = 0$ for all x, y $\in$X
(FM-2) $M(x, y, t) = 1 \Leftrightarrow x = y$ for all x, y $\in$X where t > 0,
(FM-3) $M(x, y, t) = M(y, x, t)$ for all x, y $\in$X
(FM-4) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$
(FM-5) $M(x, y,.)$: $[0, \infty) \to [0, 1]$ is left continuous .
(FM-6) $\lim_{t\to\infty} M(x,y,t) = 1$ for all x, y $\in$ X

***Corresponding Author: Arvind Kumar Gupta[2]****
[2]***Department of Mathematics Govt. Polytechnic College Mandla - (M.P.), India.***

**Definition 2.4:** (George and Veeramani (1994) [2] ): A 3- tuple (X, M, $*$) is called a GV fuzzy metric space where X is nonempty set, $*$ is a continuous t-norm, and if M is a fuzzy set on X × X × [0,∞) is satisfy the following conditions.
For all x, y, z ∈X and s, t > 0,
(FM'-1) M(x, y, t) > 0 for all x, y ∈X
(FM'-2) M(x, y, t) = 1 ⇔ x = y for all x, y ∈X where t > 0,
(FM'-3) M(x, y, t) = M(y, x, t) for all x, y ∈X
(FM'-4) M(x, y, t) $*$ M(y, z, s) ≤ M(x, z, t + s)
(FM-5) M(x, y, .): [0, ∞) → [0, 1] is continuous for all x, y, z ∈ X and s, t > 0
Where M(x, y, t) is a degree of nearness between x and y with respect to t. and axiom (FM'-1) and (FM'-2) we see that if x ≠ y and ∀ t > 0

$$0 < M(x, y, t) < 1$$

**Example 2.5:** [7] Let (X, d) be a metric space, p$*$q = Tm(p, q) and for all x, y ∈X where t > 0,

$$M(x, y, t) = \frac{t}{t+d(x,y)}$$

Then (X, M, $*$) is a GV- fuzzy metric space.

**Definition 2.6:** [6] Let (X, M, $*$) be fuzzy metric space. Then a sequence $\{x_n\}$ in X is called convergent to x ∈X if

$$\lim_{x\to\infty} M(x_n, x, t) = 1$$

**Definition 2.7:** [6] Let (X, M, $*$) be fuzzy metric space. Then a sequence $\{x_n\}$ in X is called Cauchy sequence if $\lim_{x\to\infty} M(x_n, x_{n+m}, t) = 1$ ∀ t > 0 and m∈N

**Definition 2.8:** [6] A fuzzy metric space (X, M, $*$) is called complete if every Cauchy sequence convergent to a point in X.

**Definition 2.9:** Jungck and Rhoades [7] Suppose that f and g are two self-mappings which are defined on nonempty set X. i.e. f, g: X→X, Then they are said to be weakly compatible

If they are commute at their coincidence point.

$$fx = gx \Rightarrow fgx = gfx.$$

**Definition 2.10:** Aamri and EI Moutawakil [1] Let f and g be two self -mappings of a metric space (X, d).Then we say that f and g satisfy E.A. property if there exist a sequence $\{x_n\}$ in X. such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t \text{ for some } t \in X$$

Let X = [0, 4] and we define d: X ×X→ [0, ∞) as d(x, y) = $|x - y|$

**Example 2.11:** Let f, g : X→ X be two function defined by

$$f(x) = \begin{cases} 2 & \text{for } x \in [0, 2] \\ \frac{2x+3}{2} & \text{for } x \in (0, 4] \end{cases} \quad \text{and} \quad g(x) = \begin{cases} \frac{3-x}{2} & \text{for } x \in [0, 2] \\ \frac{3x+8}{16} & \text{for } x \in (0, 4] \end{cases}$$

for a sequence $\{x_n\}$ in X such that

$$x_n = 2 + \frac{1}{n+3},\ n = 0, 1, 2, 3\ldots$$

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = \frac{7}{8}$$

So f and g are satisfying E.A. property. But f and g are non-compatible.

**Definition 2.12:** Let $\varphi$ be a class of all mappings $\emptyset$ : [0, 1] → [0 , 1] satisfying the condition
($\emptyset_1$) $\emptyset$is continuous and non-decreasing on [0, 1].
($\emptyset_2$) $\emptyset$(x) >x for all x ∈ (0, 1).
($\emptyset_3$) $\emptyset$(0) = 0 and $\emptyset$(1) = 1.

**Definition 2.13:** Let $\psi$ : [0, 1] → [0, 1] is monotonic increasing function with $\psi$(t) > t and $\psi$(0) = 0 and $\psi$(1) = 1.

We now introduce the concept of A-class of function.

**Definition 2.14:** A mapping $\mathcal{F}$: (0, 1] × (0, 1] → $R^+$ is called A-class function, if it is continuous and satisfies following conditions.

1. $\mathcal{F}(x, y) \geq x$
2. $\mathcal{F}(x, y) = x \Rightarrow$ either $x = 1$ or $y = 1$ for all $x, y \in (0, 1]$
3. $\mathcal{F}(1, 1) = 1$

We denote A–class functions by $\mathcal{A}$.

**Example 2.15:** Following functions $\mathcal{F}$: (0, 1] × (0, 1] → $R^+$ are elements of $\mathcal{A}$

(1) $\mathcal{F}(x, y) = \frac{2x}{y+1}$,
(2) $\mathcal{F}(x, y) = x + \frac{1-y}{y}$,
(3) $\mathcal{F}(x, y) = (x+m)^{\frac{1}{y^r}} - m$, $m > 1$, $r \in (0, \infty)$,
(4) $\mathcal{F}(x, y) = \frac{x(y+1)}{2y}$,
(5) $\mathcal{F}(x, y) = x + \left(\frac{1-y}{2+y}\right)\left(\frac{1+x}{2x}\right)$,
(6) $\mathcal{F}(x, y) = x(2-y)^r$ , $r \in (0, \infty)$,
(7) $\mathcal{F}(x, y) = x, log_{(1+y)}^{(1+y^x)}$
(8) $\mathcal{F}(x, y) = x.e^{(1-x)}$
(9) $\mathcal{F}(x, y) = \frac{2x}{(x+1)}$

## 3. MAIN RESULT

We now start the main result of the paper.

**Theorem 3.1:** Let (X, M, ∗) be a fuzzy metric space and P, Q, S, T : X→ X are four self -mappings with P(X) ⊆ T(X) and Q(X) ⊆ S(X) such that.

$$\psi\{M(Px, Qy, t)\} \geq \mathcal{F}\{\psi\{M_a(x, y, t)\}, \emptyset\{M_a(x, y, t)\} \text{for all } x, y \in X \text{ where } t > 0, \tag{3.1.1}$$

where $M_a(x, y, t) \geq \emptyset\left[\min\{M(Sx, Ty, t), M(Px, Sx, t), M(Qy, Ty, t), M(Px, Ty, t), M(Sx, Qy, t)\}\right]$

Suppose that the pair (P, S) or (Q, T) satisfy E.A. property and one of the subspace P(X), Q(X), S(X), T(X), is closed in X. Then the pair (P, S) and (Q, T) have a point of coincidence in X. and if pairs (P, S) and (Q, T) are weakly compatible. Then P, Q, S, T have a unique common fixed point.

**Proof:** Suppose that the pair (P, S) satisfy the E.A. property then there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} S x_n = q \text{ for some } q \in X$$

Since P(X) ⊆ T(X) there exists a sequence $\{y_n\}$ in X, such that $Px_n = Ty_n$.

Hence $\lim_{n\to\infty} Ty_n = q$

Now we show that

$$\lim_{n\to\infty} Qy_n = q$$

We put $x = x_n$ and $y = y_n$ in (3.1.1)

$$\psi\{M(Px_n, Qy_n, t)\} \geq \mathcal{F}\{\psi\{M_a(x_n, y_n, t)\}, \emptyset\{M_a(x_n, y_n, t)\}\} \text{for all } x, y \in X \text{ where } t > 0,$$

Where

$$M_a(x_n, y_n, t) \geq \emptyset\left[\min\left[\begin{matrix} M(Sx_n, Ty_n, t), M(Px_n, Sx_n, t), M(Qy_n, Ty_n, t), \\ M(Px_n, Ty_n, t), M(Sx_n, Qy_n, t) \end{matrix}\right]\right]$$

On taking limit n →∞

$$\lim_{n\to\infty} M_a(x_n, y_n, t) \geq \emptyset\left[\min\left[\begin{matrix} M(q, q, t), M(q, q, t), M(Qy_n, q, t), \\ M(q, q\ t), M(q\ Qy_n, t) \end{matrix}\right]\right]$$

$$= \emptyset\{\min\{1, 1, M(Qy_n, q, t), 1, M(q\ Qy_n, t)\}\}$$
$$= \emptyset\{M(Qy_n, q, t)\}$$
$$> M(Qy_n, q, t)$$

Again taking limit $n \to \infty$ in (3.1.1)

$$\psi\{\lim_{n\to\infty} M(Px_n, Qy_n, t)\} \geq \mathcal{F}\{\ \psi\{\lim_{n\to\infty} M_a(x_n, y_n, t)\},\ \emptyset\{\lim_{n\to\infty} M_a(x_n, y_n, t)\}\}$$

$$\psi\{M(q, Qy_n, t)\} > \mathcal{F}\{\ \psi\{M(Qy_n, q, t)\},\ \emptyset\{M(Qy_n, q, t)\}\} \geq \psi\{M(q, Qy_n, t)\}$$

Thus $\mathcal{F}\{\ \psi\{M(Qy_n, q, t)\}, \emptyset\{M(Qy_n, q, t)\}\}= \psi\{M(q, Qy_n, t)\}$

implies that
either $\psi\{M(q, Qy_n, t)\}=1$ or $\emptyset\{M(Qy_n, q, t)\} = 1$

$$\Rightarrow M(q, Qy_n, t) = 1$$

Hence $\lim_{n\to\infty} Qy_n = q$

If S(X) is closed subspace of X, then there exists a $x \in X$ such that $Sx = q$
Put $x = x$ and $y = y_n$ in (3.1.1)

$$\psi\{M(Px\ Qy_n, t)\} \geq \mathcal{F}\{\ \psi\{M_a(x, y_n, t)\},\ \emptyset\{M_a(xy_n, t)\}\} \text{ for all } x, y \in X \text{ where } t > 0, \qquad (3.1.2)$$

Where

$$M_a(x, y_n, t) \geq \emptyset\ \{\min\{M(Sx,Ty_n, t), M(Px, Sx, t), M(Qy_n, Ty_n, t), M(Px, Ty_n, t), M(Sx, Qy_n,t)\}\}$$

On taking limit $n \to\infty$

$$\lim_{n\to\infty} M_a\ (x, y_n, t) \geq \emptyset\left[\ \min\left[M(q, q, t), M(Px,q,t), M(q ,q, t), M(Px,q,t), M(q, q, t),\right]\right]$$
$$= \emptyset\{\min\{1, M(Px, q, t),1, M(Px, q, t), 1\}\}$$
$$= \emptyset\{M(Px, q, t)\}$$
$$> M(Px, q, t)$$

On taking limit $n\to\infty$ in (3.1.2)

$$\psi\{M(Px, q, t)\} \geq \mathcal{F}\{\psi\{M(Px, q, t)\},\ \emptyset\{M(Px, q, t)\}\} \geq \psi\{M(Px, q, t)\}$$

Thus $\mathcal{F}\{\psi\{M(Px, q, t)\},\ \emptyset\{M(Px, q, t)\}\}= \psi\{M(Px, q, t)\}$
implies that
either $\psi\{M(Px, q\ t)\}= 1$ or $\emptyset\{M(Px\ q, t)\} = 1$
$\Rightarrow M(Px, q\ t) = 1$

So $Px = q$

Therefore $Sx = Px = q$

So x is a coincidence point of (P, S). Again $P(X) \subseteq T(X)$ there exists a point $y \in X$ such that $Ty = q$

Now we prove that $Qy = Ty = q$ from equation (3.1.1)

$$\psi\{M(Px, Qy, t)\} \geq \mathcal{F}\{\psi\{M_a(x, y, t)\},\ \emptyset\{M_a(x, y, t)\}\} \text{for all } x, y \in X$$

Where $M_a(x, y, t) \geq \emptyset\{\min\{M(Sx, Ty, t), M(Px, Sx, t), M(Qy, Ty, t), M(Px,Ty,t), M(Sx, Qy, t)\}\}$
$$= \emptyset\{\min\{M(q, q, t), M(q, q, t), M(Qy, q, t), M(q, q, t), M(q, Qy, t)\}\}$$
$$= \emptyset\{\min\{1, 1, M(Qy, q, t), 1, M(q, Qy, t)\}\}$$
$$= \emptyset\{M(Qy, q, t)\}$$
$$> M(Qy, q, t)$$

So $\psi\{M(q, Qy, t)\}> \mathcal{F}\{\ \psi\{M(Qy, q, t)\},\ \emptyset\{M(Qy, q, t)\}\} \geq \psi\{M(q, Qy, t)\}$

Thus $\mathcal{F}\{\ \psi\{M(Qy, q, t)\},\ \emptyset\{M(Qy, q, t)\}\} = \psi\{M(q, Qy, t)\}$

$\Rightarrow \psi\{M(Qy, q, t)\} = 1$ or $\emptyset\{M(Qy, q, t)\} = 1$

$\Rightarrow M(Qy, q, t) = 1$
Thus $Qy = q$
$\Rightarrow Qy =Ty = q$

Hence y is a coincidence point of pair (Q, T).

So $Px = Sx = Qy = Ty = q$

By compatibility of pairs (P, S) and (Q, T)

$$Pq = Sq = Qq = Tq$$

Now we will prove that q is common fixed point of P, Q, S and T

We have

$$\psi\{M(Tq, q, t)\} = \psi\{M(Tq, Sx, t)\} = \psi\{M(Sx, Tq, t)\}$$
$$\geq \mathcal{F}\{\psi\{M_a(x, q, t)\}, \emptyset\{M_a(x, q, t)\}\} \text{ for all } x, y \in X$$

Where

$$\begin{aligned} M_a(x, q, t) &\geq \emptyset\{\min\{M(Sx, Tq, t), M(Px, Sx, t), M(Qq, Tq, t), M(Px,Tq, t), M(Sx, Qq, t)\}\} \\ &= \emptyset\{\min\{M(q, Tq, t), M(q, q, t), M(Tq, Tq, t), M(q, Tq,t), M(q, Qq, t)\}\} \\ &= \emptyset\{\min\{M(q, Tq, t), 1, 1, M(q, Tq,t), M(q, Qq, t)\}\} \\ &= \emptyset\{M(q, Tq, t)\} \\ &> M(q, Tq, t) \end{aligned}$$

So $\psi\{M(Tq, q, t)\} \geq \mathcal{F}\{\psi\{M(q, Tq, t)\}, \emptyset\{M(q,Tq, t)\}\} \geq \psi\{M(q, Tq, t)\}$

Thus $\mathcal{F}\{\psi\{M(q, Tq, t)\}, \emptyset\{M(q, Tq, t)\}\} = \psi\{M(q, Tq, t)\}$

$\Rightarrow \psi\{M(q, Tq, t)\}= 1$ or $\emptyset\{M(q, Tq, t)\} = 1$

$\Rightarrow M(q,Tq, t) = 1$

Thus $Tq = q \Rightarrow Qq = Tq = q$

Similarly we can prove that $Pq = Sq = q$

So $Pq = Sq = Qq = Tq = q$

Hence q is common fixed point of P, Q, S, and T

To prove uniqueness, suppose that p is another fixed point of P, Q, S, and T. Then by (3.1.1)

$$\psi\{M(q, p, t)\} = \psi\{M(Pq, Qp, t)\} \geq \mathcal{F}\{\psi\{M_a(q, p, t)\}, \emptyset\{M_a(q, p, t)\}\}$$

and

$$\begin{aligned} M_a(q, p, t) &\geq \emptyset\{\min\{M(Sq ,Tp, t), M(Pq, Sq, t), M(Qp, Tp, t), M(Pq, Tp, t), M(Sq, Qp, t)\}\} \\ &= \emptyset\{\min\{M(q, p, t), M(q, q, t), M(p, p, t), M(q, p, t), M(q, p, t)\}\} \\ &= \emptyset\{\min\{M(q, p, t), 1, 1, M(q, p, t), M(q, p, t)\}\} \\ &= \emptyset\{M(q, p, t)\} \\ &> M(q, p, t) \end{aligned}$$

Therefore $\psi\{M(q, p, t)\} = \psi\{M(Pq, Qp, t)\} > \mathcal{F}\{\psi\{M(q, p, t)\}, \emptyset\{M(q, p, t)\}\} \geq \psi\{M(q, p, t)\}$

Thus $\mathcal{F}\{\psi\{M(q, p, t)\}, \emptyset\{M(q, p, t)\}\} = \psi\{M(q, p, t)\}$

$\Rightarrow \psi\{M(q, p, t)\} = 1$ or $\emptyset\{M(q, p, t)\} = 1$

$\Rightarrow M(q, p, t) = 1$

Hence $p = q$

**Corollary 3.2:** Let $(X, M, *)$ be a fuzzy metric space and P, Q, S, T are four self - mappings defined on X with $P(X) \subseteq T(X)$ and $Q(X) \subseteq S(X)$ such that.

$$M(Px, Qy, t) \geq \mathcal{F}\{\psi\, M_a(x, y, t), M_a(x, y, t)\} \text{ for all } x, y \in X \text{ where } t > 0,$$

Where

$$M_a(x, y, t) \geq \min\{M(Sx, Ty, t), M(Px, Sx, t), M(Qy, Ty, t), M(Px, Ty, t), M(Sx, Qy, t)\}$$

Suppose that the pair (P, S) or (Q, T) satisfy E.A. property and one of the subspace P(X), Q(X), S(X), T(X) is closed in X. Then the pairs (P, S) and (Q, T) have a point of coincidence in X. and if pairs (P, S) and (Q, T) are weakly compatible. Then P, Q, S, T has a unique common fixed point.

 

**Corollary 3.3**: Let (X, M, $*$) be a fuzzy metric space ,and P, Q :X $\rightarrow$X are self-mappings such that

$$\psi\{M(Px, Qy, t)\} \geq \mathcal{F}\{\psi\{M_a(x, y, t)\}, \emptyset\{M_a(x, y, t)\}\} \text{ for all } x, y \in X \text{ where } t > 0$$

Where

$$M_a(x, y, t) \geq \emptyset\ \{\min\{M(Qx, Qy, t), M(Px, Qx, t), M(Py, Qy, t), M(Px, Qy, t), M(Qx, Py, t)\}\}$$

Suppose that the pair (P, Q) satisfy E.A. property and subspace P(X), is closed in X. Then the pair (P, Q) has a point of coincidence in X. Moreover if the pair (P, Q) is weakly compatible. Then P and Q have a unique common fixed point.

Now we consider following example in the support ofour result.

**Example 3.4:** Let $\mathcal{F}(x, y)$ be any function of class $\mathcal{A}$, X = [0, 2] and d: X × X→ [0, ∞) be usual metric defined by d(x, y) =$|x-y|$, For $*$ is minimum t-norm and for all x, y ∈ X,

$$M(x, y, t) = \frac{t}{t+d(x,y)}$$

Then (X, M,$*$) is a GV- fuzzy metric space.

Suppose that P, Q, S and T are four self-mappings of X, defined as.

$$P(x) = \begin{cases} \frac{3}{4} & \text{for } 0 \leq x \leq 1 \\ \frac{x+2}{4} & \text{for } 1 < x \leq 2 \end{cases} \quad \text{and} \quad Q(x) = \begin{cases} x & \text{for } 0 \leq x \leq 1 \\ \frac{1}{2} & \text{for } x = 1 \\ \frac{2x}{5} & \text{for } 1 < x \leq 2 \end{cases}$$

$$S(x) = \begin{cases} x & \text{for } 0 \leq x \leq 1 \\ \frac{3x}{4} & \text{for } 1 < x \leq 2 \end{cases} \quad \text{and} \quad T(x) = \begin{cases} \frac{3x}{2} & \text{for } 0 \leq x \leq 1 \\ \frac{x+1}{5} & \text{for } 1 < x \leq 2 \end{cases}$$

Clearly P(X), Q(X), S(X) and T(X) are closed and P(X) ⊆ T(X) & Q(X) ⊆ S(X). The sequence $\{x_n\}$ and $\{y_n\}$ are defined by $x_n = 1 + \frac{2}{n+3}$ , and $y_n = 1 + \frac{1}{2n}$, in X such that

$$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Sx_n = \frac{3}{4} \text{ and } \lim_{n\to\infty} Qy_n = \lim_{n\to\infty} Ty_n = \frac{2}{5}$$

So the pairs (P, S) and (Q, T) satisfies E.A. property. And also weakly compatible. The monotonic increasing function $\psi(t) = \sqrt{t}$. Now we check the contractive condition (3.1.1) for all x, y ∈ X,

**Case-I:** If x = 0, y = 0 and x = 1, y = 1 then contractive condition (3.1.1) is satisfied.

We will discuss in the following two cases.

**Case-II:** If x, y ∈ (0, 1)

$$\psi\{M(Px, Qy, t)\} = \psi\{M(\tfrac{3}{4}, x, t)\} = \psi\{\frac{t}{t+\left|\frac{3-4x}{4}\right|}\} > \frac{t}{t+\left|\frac{3-4x}{4}\right|} \tag{i}$$

and

$$M_a(x, y, t) \geq \emptyset\left[\min\left[\{M(Sx, Ty, t), M(Px, Sx, t), M(Qy, Ty, t), M(Px, Ty, t), M(Sx, Qy, t)\}\right]\right]$$

$$\geq \emptyset\left[\min\left[\frac{t}{t+\frac{x}{2}}, \frac{t}{t+\left|\frac{3-4x}{4}\right|}, \frac{t}{t+\frac{x}{2}}, \frac{t}{t+\left|\frac{3-6x}{4}\right|}, 1\right]\right]$$

$$\geq \emptyset\left[\frac{t}{t+\left|\frac{3-4x}{4}\right|} > \frac{t}{t+\left|\frac{3-4x}{4}\right|}\right] \tag{ii}$$

Thus for all values of t > 0, contractive condition (3.1.1) is satisfied.

**Case-III:** If x, y ∈(1, 2]

$$\psi\{M(Px, Qy, t)\} = \psi\{M(\tfrac{X+2}{4}, \tfrac{2x}{5}, t)\} = \psi\{\frac{t}{t+\frac{10-3x}{20}}\} \geq \frac{t}{t+\frac{10-3x}{20}} \tag{iii}$$

and

$$M_a(x, y, t) \geq \emptyset\left[\min\left[\{M(Sx, Ty, t), M(Px, Sx, t), M(Qy, Ty, t), M(Px, Ty, t), M(Sx, Qy, t)\}\right]\right]$$

$$\geq \emptyset\left[\min\left[\frac{t}{t+\frac{11x-4}{20}}, \frac{t}{t+\frac{x-1}{2}}, \frac{t}{t+\frac{x-1}{5}}, \frac{t}{t+\frac{x+6}{20}}, \frac{t}{t+\frac{7x}{20}}\right]\right]$$

$$\geq \emptyset\left[\frac{t}{t+\frac{11x-4}{20}} > \frac{t}{t+\frac{11x-4}{20}}\right] \tag{iv}$$

Thus for all values of t > 0, $\frac{t}{t+\frac{10-3x}{20}} \geq \frac{t}{t+\frac{11x-4}{20}}$ hence contractive condition (3.1.1) is satisfied.

Therefore all conditions of theorem 3.1 are satisfied. Hence 0 is unique common fixed point of self - mappings P, Q, S, and T.

**REFERENCES**

1. Aamri, M. and Moutawakil, D. “Some new common fixed point theorems under strict Contractive conditions”, J. Math. Anal. Appl., 270(2002), 181-188.
2. A. George and P.Veeramani, “On some results in fuzzy metric spaces and fuzzy sets and system“vol. 64 (1994) 395-399.
3. G.Jungckand B.E. Rhoades, “fixed points for set valued functions without continuity”, Indian Journal of pure and applied mathematics, Vol.29, pp.227-238, 1998.
4. I.A. Zadeh, fuzzy sets, Inform and control,, 8(1965) 338-353.
5. I. Kromosil, and J. Michelek, “fuzzy metric space and statistical metric space”, kybernetica Vol. 11, pp.326-334, 1975.
6. Kamal Wadhwa & Ashlekhadubey “some common fixed point theorems in fuzzy metric space by using CLRg property” Global journal of pure and applied mathematics (2017), pp 6327-6342.
7. W. sintunavarat, and P. Kumam “Common fixed point theorems for a pair of weakly compatible mappings in fuzzy metric space”. Hindwani publishing corporation journal of applied mathematics, Vol. 2011, 1-14.

***Source of support: Nil, Conflict of interest: None Declared.***