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ON SECOND BANHATTI-SOMBOR INDICES

V. R. KULLI*

Department of Mathematics, Gulbarga University, Gulbarga - 585106, India.

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ABSTRACT

Recently, a novel class of degree based topological indices was introduced, the so called Banhatti-Sombor indices. In this paper, we introduce the second Banhatti-Sombor index, second reduced Banhatti-Sombor index, second δ-Banhatti-Sombor index of a graph and compute exact formulas for some chemical networks such as armchair polyhex nanotubes, zigzag polyhex nanotubes and carbon nanocone networks.

Keywords: *second Banhatti-Sombor index, second reduced Banhatti-Sombor index, second δ- Banhatti-Sombor index, nanotube, network.*

Mathematics Subject Classification: *05C05, 05C07, 05C90.*

1. INTRODUCTION

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. Let $d_G(u)$ be the degree of a vertex u in a graph G. Let $\delta(G)$ be the minimum degree among the vertices of G. For undefined terms and notations, we refer [1].

Chemical Graph Theory is branch of Mathematical Chemistry which has an important effect on the development of Chemical Sciences. A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. Topological indices are useful for establishing correlation between the structure of a molecular compound and its physicochemical properties. Numerous topological indices [2] have been considered in Theoretical Chemistry and have found some applications, especially in QSPR/QSAR research, see [3, 4].

In [5], Gutman proposed the Sombor indices and they are defined as

$$SO(G)=\sum_{uv\in E(G)}\left[d_G(u)^2+d_G(v)^2\right]^{\frac{1}{2}},$$

$$RSO(G)=\sum_{uv\in E(G)}\left[\left(d_G(u)-1\right)^2+\left(d_G(v)-1\right)^2\right]^{\frac{1}{2}}$$

$$ASO(G)=\sum_{uv\in E(G)}\left[\left(d_G(u)-\frac{2m}{n}\right)^2+\left(d_G(v)-\frac{2m}{n}\right)^2\right]^{\frac{1}{2}}$$

where $|V(G)| = n$ and $|E(G)| = m$.

Recently, some Sombor indices were studied, for example, in [6, 7, 8, 9, 10, 11, 12, 13 14, 15, 16, 17, 18, 19, 2o, 21, 22, 23, 24].

Inspired by work on Sombor indices, Kulli [25] introduced the first Banhatti-Sombor index, first reduced Banhatti-Sombor index and first δ-Banhatti-Sombor index of a graph and they are defined as

$$BSO_1(G)=\sum_{uv\in E(G)}\left[\frac{1}{d_G(u)^2}+\frac{1}{d_G(v)^2}\right]^{\frac{1}{2}},$$

Corresponding Author: V. R. Kulli*
Department of Mathematics, Gulbarga University, Gulbarga - 585106, India.

$$RBSO_1(G)=\sum_{uv\in E(G)}\left[\frac{1}{\left(d_G(u)-1\right)^2}+\frac{1}{\left(d_G(v)-1\right)^2}\right]^{\frac{1}{2}},$$

$$\delta BSO(G)=\sum_{uv\in E(G)}\left[\frac{1}{\left(d_G(u)-\delta(G)+1\right)^2}+\frac{1}{\left(d_G(v)-\delta(G)+1\right)^2}\right]^{\frac{1}{2}}.$$

We now introduce the second Banhatti-Sombor index, second reduced Banhatti-Sombor index and second δ- Banhatti-Sombor index of a graph and they are defined as

$$BSO_2(G)=\sum_{uv\in E(G)}\left[\frac{1}{d_G(u)^2}+\frac{1}{d_G(v)^2}\right]^{-\frac{1}{2}},$$

$$RBSO_2(G)=\sum_{uv\in E(G)}\left[\frac{1}{\left(d_G(u)-1\right)^2}+\frac{1}{\left(d_G(v)-1\right)^2}\right]^{-\frac{1}{2}},$$

$$\delta BSO_2(G)=\sum_{uv\in E(G)}\left[\frac{1}{\left(d_G(u)-\delta(G)+1\right)^2}+\frac{1}{\left(d_G(v)-\delta(G)+1\right)^2}\right]^{-\frac{1}{2}}.$$

In this paper, we compute the second Banhatti-Sombor index, second reduced Banhatti-Sombor index, second δ-Banhatti-Sombor index for certain networks. For nanotubes, networks, see [26].

2. OBSERVATIONS

(1) If $\delta(G) = 1$, then $\delta BSO_2(G)$ is the second Banhatti-Sombor index $BSO_2(G)$.
(2) If $\delta(G) = 2$, then $\delta BSO_2(G)$ is the second reduced Banhatti-Sombor index $RBSO_2(G)$.

3. RESULTS FOR ARMCHAIR POLYHEX NANOTUBES

Carbon polyhex nanotubes exist in nature with remarkable stability and posses very interesting thermal, electrical and mechanical properties. Cylindrical surface of these nanotubes is made up of entirely hexagons. We consider the family of armchair polyhex nanotubes which is denoted by $TUAC_6\,[p,q]$. A 2-dimensional network $TUAC_6\,[p,q]$ is shown in Figure 1.

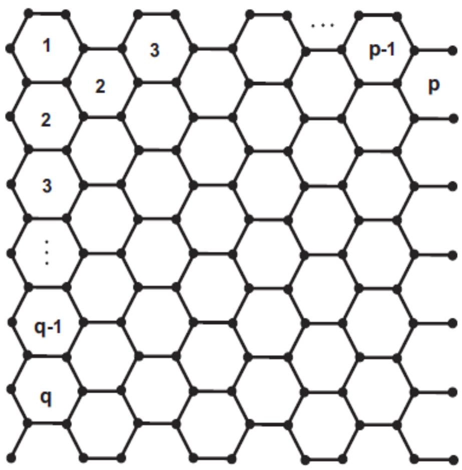

Figure-1

Let G be the graph $TUAC_6\,[p,q]$. By calculation, G has $2p(q+1)$ vertices and $3pq + 2p$ edges. In G, there are three types of edges based on degrees of end vertices of each edge. The edge partition of G is given in Table 1.

$d_G(u),d_G(v)\backslash uv \in E(G)$	(2, 2)	(2, 3)	(3, 3)
Number of edges	p	$2p$	$3pq - p$

Table-1: Edge partition of $TUAC_6\,[p,q]$

In the following theorem, we compute the second Banhatti-Sombor index of $TUAC_6[p,q]$.

Theorem 1: The second Banhatti-Sombor index of $TUAC_6[p,q]$ is given by

$$BSO_2(TUAC_6[\mathrm{p,q}]) = \frac{9}{\sqrt{2}}pq + \left(\sqrt{2} + \frac{12}{\sqrt{13}} - \frac{3}{\sqrt{2}}\right)p.$$

Proof: By using definition and cardinalities of the edge partition of $TUAC_6[p,q]$, we have

$$BSO_2(TUAC_6[\mathrm{p,q}]) = \left(\frac{1}{2^2}+\frac{1}{2^2}\right)^{-\frac{1}{2}} p + \left(\frac{1}{2^2}+\frac{1}{3^2}\right)^{-\frac{1}{2}} 2p + \left(\frac{1}{3^2}+\frac{1}{3^2}\right)^{-\frac{1}{2}} (3pq - p)$$

$$= \frac{9}{\sqrt{2}}pq + \left(\sqrt{2} + \frac{12}{\sqrt{13}} - \frac{3}{\sqrt{2}}\right)p.$$

In the following theorem, we determine the second reduced Banhatti-Sombor index of $TUAC_6[p,q]$.

Theorem 2: The second reduced Banhatti-Sombor index of $TUAC_6[p,q]$ is given by

$$RBSO_2(TUAC_6[\mathrm{p,q}]) = 3\sqrt{2}pq + \left(\frac{1}{\sqrt{2}} + \frac{4}{\sqrt{5}} - \sqrt{2}\right)p$$

Proof: From definition and by using cardinalities of the edge partition of $TUAC_6[p,q]$, we obtain

$$RBSO_2(TUAC_6[\mathrm{p,q}]) = \left(\frac{1}{1^2}+\frac{1}{1^2}\right)^{-\frac{1}{2}} p + \left(\frac{1}{1^2}+\frac{1}{2^2}\right)^{-\frac{1}{2}} 2p + \left(\frac{1}{2^2}+\frac{1}{2^2}\right)^{-\frac{1}{2}} (3pq - p)$$

$$= 3\sqrt{2}pq + \left(\frac{1}{\sqrt{2}} + \frac{4}{\sqrt{5}} - \sqrt{2}\right)p.$$

In the following theorem, we determine the second δ-Sombor index of $TUAC_6[p,q]$.

Theorem 3: The second δ-Sombor index of $TUAC_6[p,q]$ is given by

$$\delta BSO_2(TUAC_6[\mathrm{p,q}]) = 3\sqrt{2}pq + \left(\frac{1}{\sqrt{2}} + \frac{4}{\sqrt{5}} - \sqrt{2}\right)p.$$

Proof: By observation (2) and Theorem 2, the result follows.

4. ZIGZAG POLYHEX NANOTUBES

The zigzag polyhex nanotube is denoted by $TUZC_6[p, q]$, where p is the number of hexagons in a row whereas q is the number of hexagons in a column. A 2-dimensional network of $TUZC_6[p, q]$ is depicted in Figure 2.

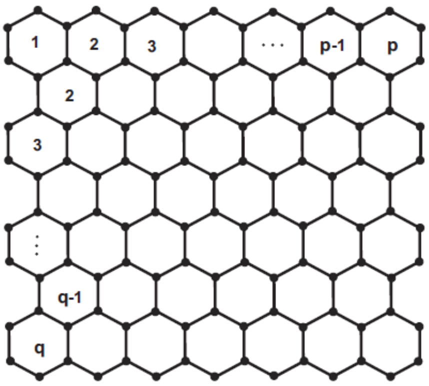

Figure-2

Let G be a graph of a (p, q) dimensional zigzag polyhex nanotube. The graph G has $2p(q+1)$ vertices and $3pq + 2p$ edges. In G, there are two types of edges based on degrees of end vertices of each edge. By calculation, the edge partition of G is given in Table 2.

$d_G(u), d_G(v) \setminus uv \in E(G)$	(2, 3)	(3, 3)
Number of edges	$4p$	$3pq - 2p$

Table-2: Edge partition of $TUZC_6[p, q]$

In the following theorem, we compute the second Banhatti-Sombor index of $TUZC_6$ [p, q].

Theorem 4: The second Banhatti-Sombor index of $TUZC_6$ [p, q] is given by

$$BSO_2\left(TUZC_6[\mathrm{p},\mathrm{q}]\right)=\frac{9}{\sqrt{2}}pq+\left(\frac{24}{\sqrt{13}}-\frac{6}{\sqrt{2}}\right)p$$

Proof: By using definition and by cardinalities of the edge partition of $TUZC_6$ [p, q], we have

$$\begin{aligned}BSO_2\left(TUZC_6[\mathrm{p},\mathrm{q}]\right)&=\left(\frac{1}{2^2}+\frac{1}{3^2}\right)^{-\frac{1}{2}}4p+\left(\frac{1}{3^2}+\frac{1}{3^2}\right)^{-\frac{1}{2}}(3pq-2p)\\&=\frac{9}{\sqrt{2}}pq+\left(\frac{24}{\sqrt{13}}-\frac{6}{\sqrt{2}}\right)p.\end{aligned}$$

In the following theorem, we determine the second reduced Banhatti-Sombor index of $TUZC_6$ [p, q].

Theorem 5: The second reduced Banhatti-Sombor index of $TUZC_6$ [p, q] is given by

$$RBSO_2\left(TUZC_6[\mathrm{p},\mathrm{q}]\right)=3\sqrt{2}pq+\left(\frac{8}{\sqrt{5}}-2\sqrt{2}\right)p$$

Proof: From definition and by using cardinalities of the edge partition of $TUZC_6$ [p, q], we obtain

$$\begin{aligned}RBSO_2\left(TUZC_6[\mathrm{p},\mathrm{q}]\right)&=\left(\frac{1}{1^2}+\frac{1}{2^2}\right)^{-\frac{1}{2}}4p+\left(\frac{1}{2^2}+\frac{1}{2^2}\right)^{-\frac{1}{2}}(3pq-2p)\\&=3\sqrt{2}pq+\left(\frac{8}{\sqrt{5}}-2\sqrt{2}\right)p.\end{aligned}$$

In the following next theorem, we compute the second δ-Banhatti-Sombor index of $TUZC_6$ [p, q].

Theorem 6: The second δ-Banhatti-Sombor index of $TUZC_6$ [p, q] is given by

$$\delta BSO_2\left(TUAC_6[\mathrm{p},\mathrm{q}]\right)=3\sqrt{2}pq+\left(\frac{8}{\sqrt{5}}-2\sqrt{2}\right)p.$$

Proof: By observation (2) and Theorem 5, we get the desired result.

5. CARBON NANOCONE NETWORKS

An n-dimensional one-pentagonal nanocone is denoted by CNC_5 [n], where n is the number of hexagons layers encompassing the conical surface of the nanocone and 5 denote that there is a pentagon on the tip called its core. A 6-dimensional one-pentagonal nanocone network is depicted in Figure 3.

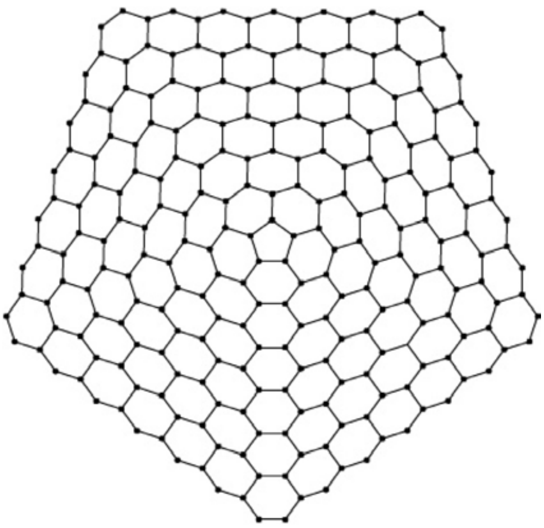

Figure-3

Let G be an n-dimensional one-pentagonal nanocone network $CNC_5[n]$, $n \geq 2$. Then G has $5(n+1)^2$ vertices and $\frac{15}{2}n^2 + \frac{25}{2}n + 5$ edges. In G, there are three types of edges based on degrees of end vertices of each edge. By algebraic method, this edge partition is given in Table 3.

$d_G(u), d_G(v) \setminus uv \in E(G)$	(2, 2)	(2, 3)	(3, 3)
Number of edges	5	$10n$	$\frac{15}{2}n^2 + \frac{5}{2}n$

Table-3: Edge partition of $CNC_5[n]$

In the following theorem, we compute the second Banhatti-Sombor index of $CNC_5[n]$.

Theorem 7: The second Banhatti-Sombor index of $CNC_5[n]$ is given by

$$BSO_2(CNC_5[\mathrm{n}]) = \frac{45}{2\sqrt{2}}n^2 + \left(\frac{60}{\sqrt{13}} + \frac{15}{2\sqrt{2}}\right)n + 5\sqrt{2}.$$

Proof: By using definition and cardinalities of the edge partition of $CNC_5[n]$, we have

$$BSO_2(CNC_5[\mathrm{n}]) = \left(\frac{1}{2^2} + \frac{1}{2^2}\right)^{-\frac{1}{2}} 5 + \left(\frac{1}{2^2} + \frac{1}{3^2}\right)^{-\frac{1}{2}} 10n + \left(\frac{1}{3^2} + \frac{1}{3^2}\right)^{-\frac{1}{2}} \left(\frac{15}{2}n^2 + \frac{5}{2}n\right)$$

$$= \frac{45}{2\sqrt{2}}n^2 + \left(\frac{60}{\sqrt{13}} + \frac{15}{2\sqrt{2}}\right)n + 5\sqrt{2}.$$

In the following theorem, we compute the second reduced Banhatti-Sombor index of $CNC_5[n]$.

Theorem 8: The second reduced Banhatti-Sombor index of $CNC_5[n]$ is given by

$$RBSO_2(CNC_5[\mathrm{n}]) = \frac{15}{\sqrt{2}}n^2 + \left(\frac{20}{\sqrt{5}} + \frac{5}{\sqrt{2}}\right)n + \frac{5}{\sqrt{2}}.$$

Proof: By using definition and cardinalities of the edge partition of $CNC_5[n]$, we have

$$RBSO_2(CNC_5[\mathrm{n}]) = \left(\frac{1}{1^2} + \frac{1}{1^2}\right)^{-\frac{1}{2}} 5 + \left(\frac{1}{1^2} + \frac{1}{2^2}\right)^{-\frac{1}{2}} 10n + \left(\frac{1}{2^2} + \frac{1}{2^2}\right)^{-\frac{1}{2}} \left(\frac{15}{2}n^2 + \frac{5}{2}n\right)$$

$$= \frac{15}{\sqrt{2}}n^2 + \left(\frac{20}{\sqrt{5}} + \frac{5}{\sqrt{2}}\right)n + \frac{5}{\sqrt{2}}.$$

In the next theorem, we compute the second δ-Banhatti-Sombor index of $CNC_5[n]$.

Theorem 9: The second δ-Banhatti-Sombor index of $CNC_5[n]$ is given by

$$\delta BSO_2(CNC_5[\mathrm{n}]) = \frac{15}{\sqrt{2}}n^2 + \left(\frac{20}{\sqrt{5}} + \frac{5}{\sqrt{2}}\right)n + \frac{5}{\sqrt{2}}.$$

Proof: By observation (2) and Theorem 8, the result follows.

6. CONCLUSION

In this study, we have introduced the second Banhatti-Sombor index, second reduced Banhatti-Sombor index, second δ-Banhatti-Sombor index of a graph and have computed exact formulas for armchair polyhex nanotubes, zigzag polyhex nanotubes and carbon nanocone networks.

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