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COMPUTATION OF MULTIPLICATIVE BANHATTI-SOMBOR INDICES OF CERTAIN BENZENOID SYSTEMS

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ABSTRACT:

Recently, a novel class of degree based topological indices was introduced, the so called Banhatti-Sombor indices. In this paper, we introduce the multiplicative first Banhatti-Sombor index, multiplicative first reduced Banhatti-Sombor index, multiplicative first δ-Banhatti-Sombor index of a graph and compute exact formulas for some nanostructures.

Keywords: *multiplicative first Banhatti-Sombor index, multiplicative first reduced Banhatti-Sombor index, multiplicative first δ- Banhatti-Sombor index, nanostructure.*

Mathematics Subject Classification: *05C05, 05C07, 05C90.*

1. INTRODUCTION

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. Let $d_G(u)$ be the degree of a vertex u in a graph G. For undefined terms and notations, we refer [1].

Chemical Graph Theory is branch of Mathematical Chemistry which has an important effect on the development of Chemical Sciences. A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. Topological indices are useful for establishing correlation between the structure of a molecular compound and its physicochemical properties. Numerous topological indices [2] have been considered in Theoretical Chemistry and have found some applications, especially in QSPR/QSAR research, see [3, 4].

In [5], Kulli introduced the first Banhatti-Sombor index, first reduced Banhatti-Sombor index, first δ-Banhatti-Sombor index and they are defined as

$$BSO_1(G) = \sum_{uv \in E(G)} \left[\frac{1}{d_G(u)^2} + \frac{1}{d_G(v)^2} \right]^{\frac{1}{2}},$$

$$RBSO_1(G) = \sum_{uv \in E(G)} \left[\frac{1}{\left(d_G(u)-1\right)^2} + \frac{1}{\left(d_G(v)-1\right)^2} \right]^{\frac{1}{2}}, \text{ if } \delta(G) \geq 2$$

$$\delta BSO_1(G) = \sum_{uv \in E(G)} \left[\frac{1}{\left(d_G(u)-\delta(G)+1\right)^2} + \frac{1}{\left(d_G(v)-\delta(G)+1\right)^2} \right]^{\frac{1}{2}},$$

where $\delta(G)$ is the minimum degree among the vertices of G.

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Recently, some Sombor indices were studied, for example, in [6, 7, 8, 9, 10, 11, 12, 13, 14].

Inspired by work on Sombor indices, we put forward the multiplicative first Banhatti-Sombor index, multiplicative first reduced Banhatti-Sombor index and multiplicative first δ-Banhatti-Sombor index of a graph and they are defined as

$$BSO_1II(G)=\prod_{uv\in E(G)}\left[\frac{1}{d_G(u)^2}+\frac{1}{d_G(v)^2}\right]^{\frac{1}{2}},$$

$$RBSO_1II(G)=\prod_{uv\in E(G)}\left[\frac{1}{\left(d_G(u)-1\right)^2}+\frac{1}{\left(d_G(v)-1\right)^2}\right]^{\frac{1}{2}},\ \text{if}\ \ \delta(G)\geq 2$$

$$\delta BSO_1II(G)=\prod_{uv\in E(G)}\left[\frac{1}{\left(d_G(u)-\delta(G)+1\right)^2}+\frac{1}{\left(d_G(v)-\delta(G)+1\right)^2}\right]^{\frac{1}{2}}$$

where $\delta(G)$ is the minimum degree among the vertices of G.

In this paper, we compute the multiplicative first Banhatti-Sombor index, multiplicative first reduced Banhatti-Sombor index, multiplicative first δ-Banhatti-Sombor index of some families of benzenoid systems.

2. OBSERVATIONS

(1) If $\delta(G) = 1$, then $\delta BSO_1 II(G)$ is the multiplicative first Banhatti-Sombor index $BSO_1 II(G)$.
(2) If $\delta(G) = 2$, then $\delta BSO_1 II(G)$ is the multiplicative first reduced Banhatti-Sombor index $RBSO_1II(G)$.

3. TRIANGULAR BENZENOIDS

In this section, we consider a family of triangular benzenoids. This family of benzenoids is denoted by T_p, where p is the number of hexagons in the base graph. Clearly T_p has $\frac{1}{2}p(p-1)$ hexagons. The graph of T_4 is shown in Figure 1.

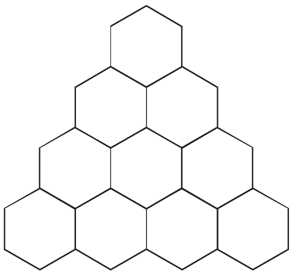

Figure-1: The graph of T_4

Let G be the graph of a triangular benzenoid T_p. The graph G has $p^2 + 4p + 1$ vertices and $\frac{3p(p+3)}{2}$ edges. From Figure 1, we see that the vertices of G are either of degree 2 or 3. Therefore $\delta(G) = 2$. By calculation, we obtain that G has three types of edges based on degrees of end vertices of each edge as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = 6$.
$E_2 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_2| = 6p - 6$.
$E_3 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_3| = \frac{3p(p-1)}{2}$.

In the following theorem, we compute the multiplicative first Banhatti-Sombor index of T_p.

Theorem 1: The multiplicative first Banhatti-Sombor index of a triangular benzenoid T_p is given by

$$BSO_1II(T_p)=\left(\frac{1}{2}\right)^{\frac{6}{2}}\times\left(\frac{13}{36}\right)^{\frac{1}{2}(6p-6)}\times\left(\frac{2}{9}\right)^{\frac{3}{4}p(p-1)}.$$

Proof: By using definition and cardinalities of the edge partition of T_p, we have

$$BSO_1II\left(T_p\right)=\left(\frac{1}{2^2}+\frac{1}{2^2}\right)^{\frac{6}{2}}\times\left(\frac{1}{2^2}+\frac{1}{3^2}\right)^{\frac{1}{2}(6p-6)}\times\left(\frac{1}{3^2}+\frac{1}{3^2}\right)^{\frac{3}{4}p(p-1)}$$

$$=\left(\frac{1}{2}\right)^{\frac{6}{2}}\times\left(\frac{13}{36}\right)^{\frac{1}{2}(6p-6)}\times\left(\frac{2}{9}\right)^{\frac{3}{4}p(p-1)}.$$

In the following theorem, we determine the multiplicative first reduced Banhatti-Sombor index of T_p.

Theorem 2: The multiplicative first reduced Banhatti-Sombor index of a triangular benzenoid T_p is given by

$$RBSO_1II\left(T_p\right)=(2)^{\frac{6}{2}}\times\left(\frac{5}{4}\right)^{\frac{1}{2}(6p-6)}\times\left(\frac{1}{2}\right)^{\frac{3}{4}p(p-1)}.$$

Proof: From definition and by using cardinalities of the edge partition of T_p, we obtain

$$RBSO_1II\left(T_p\right)=\left(\frac{1}{1^2}+\frac{1}{1^2}\right)^{\frac{6}{2}}\times\left(\frac{1}{1^2}+\frac{1}{2^2}\right)^{\frac{1}{2}(6p-6)}\times\left(\frac{1}{2^2}+\frac{1}{2^2}\right)^{\frac{3}{4}p(p-1)}$$

$$=(2)^{\frac{6}{2}}\times\left(\frac{5}{4}\right)^{\frac{1}{2}(6p-6)}\times\left(\frac{1}{2}\right)^{\frac{3}{4}p(p-1)}.$$

In the following theorem, we determine the multiplicative first δ-Banhatti-Sombor index of T_p.

Theorem 3: The multiplicative first δ-Banhatti-Sombor index of a triangular benzenoid T_p is given by

$$\delta BSO_1II\left(T_p\right)=(2)^{\frac{6}{2}}\times\left(\frac{5}{4}\right)^{\frac{1}{2}(6p-6)}\times\left(\frac{1}{2}\right)^{\frac{3}{4}p(p-1)}.$$

Proof: By observation (2) and Theorem 2, the result follows.

4. BENZENOID RHOMBUS

In this section, we consider a family of benzenoid rhombus. This family of benzenoids is denoted by R_p. The benzenoid rhombus R_p is obtained from two copies of a triangular benzenoid T_p by identifying hexagons in one of their base rows. The graph of R_4 is depicted in Figure 2.

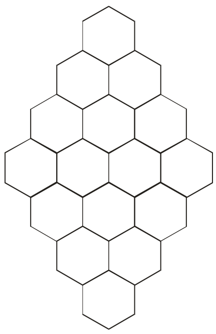

Figure-2: The graph of R_4

Let G be the graph of a benzenoid rhombus R_p. The graph G has $2p^2 + 4p$ vertices and $3p^2+4p-1$ edges. From Figure 2, it is easy to see that the vertices of R_p are either of degree 2 or 3. Thus $\delta(R_p) = 2$. By calculation, we obtain that G has three types of edges based on degrees of end vertices of each edge as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = 6$.
$E_2 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_2| = 8p - 8$.
$E_3 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_3| = 3p^2 - 4p + 1$.

In the following theorem, we compute the multiplicative first Banhatti-Sombor index of R_p.

Theorem 4: The multiplicative first Banhatti-Sombor index of a benzenoid rhombus R_p is given by

$$BSO_1II(R_p)=\left(\frac{1}{2}\right)^{\frac{6}{2}}\times\left(\frac{13}{36}\right)^{\frac{1}{2}(8p-8)}\times\left(\frac{2}{9}\right)^{\frac{1}{2}(3p-4p+1)}.$$

Proof: From definition and by cardinalities of the edge partition of R_p, we deduce

$$BSO_1II(R_p)=\left(\frac{1}{2^2}+\frac{1}{2^2}\right)^{\frac{6}{2}}\times\left(\frac{1}{2^2}+\frac{1}{3^2}\right)^{\frac{1}{2}(8p-8)}\times\left(\frac{1}{3^2}+\frac{1}{3^2}\right)^{\frac{1}{2}(3p^2-4p+1)}$$

[

$$=\left(\frac{1}{2}\right)^{\frac{6}{2}}\times\left(\frac{13}{36}\right)^{\frac{1}{2}(8p-8)}\times\left(\frac{2}{9}\right)^{\frac{1}{2}(3p-4p+1)}.$$

In the following theorem, we determine the multiplicative first reduced Banhatti-Sombor index of R_p.

Theorem 5: The multiplicative first reduced Banhatti-Sombor index of a benzenoid rhombus R_p is given by

$$RBSO_1II(R_p)=(2)^{\frac{6}{2}}\times\left(\frac{5}{4}\right)^{\frac{1}{2}(8p-8)}\times\left(\frac{1}{2}\right)^{\frac{1}{2}(3p^2-4p+1)}.$$

Proof: By using definition and by cardinalities of the edge partition of R_p, we derive

$$RBSO_1II(R_p)=\left(\frac{1}{1^2}+\frac{1}{1^2}\right)^{\frac{6}{2}}\times\left(\frac{1}{1^2}+\frac{1}{2^2}\right)^{\frac{1}{2}(8p-8)}\times\left(\frac{1}{2^2}+\frac{1}{2^2}\right)^{\frac{1}{2}(3p^2-4p+1)}$$

$$=(2)^{\frac{6}{2}}\times\left(\frac{5}{4}\right)^{\frac{1}{2}(8p-8)}\times\left(\frac{1}{2}\right)^{\frac{1}{2}(3p^2-4p+1)}.$$

In the following next theorem, we compute the multiplicative first δ-Banhatti-Sombor index of R_p.

Theorem 6: The multiplicative first δ-Banhatti-Sombor index of R_p is given by

$$\delta BSO_1II(R_p)=(2)^{\frac{6}{2}}\times\left(\frac{5}{4}\right)^{\frac{1}{2}(8p-8)}\times\left(\frac{1}{2}\right)^{\frac{1}{2}(3p^2-4p+1)}.$$

Proof: By observation (2) and Theorem 5, the result follows.

5. BENZENOID HOURGLASS

In this section, we consider a family of benzenoid hourglass, which is denoted by X_p. This family is obtained from two copies of a triangular benzenoid T_p by overlapping hexagons. The graph of benzenoid hourglass is presented in Figure 3.

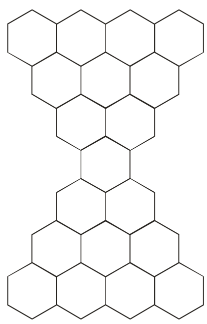

Figure-3: The graph of benzenoid hourglass

Let G be the graph of a benzenoid hourglass X_p. This graph G has $2(p^2 + 4p - 2)$ vertices and $3p^2+9p-4$ edges. From Figure 3, we see that the vertices of X_p are either of degree 2 or 3. Thus $\delta(X_p) = 2$.

By algebraic method, we find that G has three types of edges based on degrees of end vertices of each as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = 8$.
$E_2 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_2| = 12p - 16$.
$E_3 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_3| = 3p^2 - 3p + 4$.

In the following theorem, we compute the multiplicative first Banhatti-Sombor index of X_p.

Theorem 7: The multiplicative first Banhatti-Sombor index of a benzenoid hourglass X_p is given by

$$BSO_1II(X_p) = \left(\frac{1}{2}\right)^{\frac{8}{2}} \times \left(\frac{13}{36}\right)^{\frac{1}{2}(12p-16)} \times \left(\frac{2}{9}\right)^{\frac{1}{2}(3p^2-3p+4)}.$$

Proof: From definition and by cardinalities of the edge partition of X_p, we obtain

$$BSO_1II(X_p) = \left(\frac{1}{2^2}+\frac{1}{2^2}\right)^{\frac{8}{2}} \times \left(\frac{1}{2^2}+\frac{1}{3^2}\right)^{\frac{1}{2}(12p-16)} \times \left(\frac{1}{3^2}+\frac{1}{3^2}\right)^{\frac{1}{2}(3p^2-3p+4)}$$
$$= \left(\frac{1}{2}\right)^{\frac{8}{2}} \times \left(\frac{13}{36}\right)^{\frac{1}{2}(12p-16)} \times \left(\frac{2}{9}\right)^{\frac{1}{2}(3p^2-3p+4)}.$$

In the following theorem, we compute the multiplicative first reduced Banhatti-Sombor index of X_p.

Theorem 8: The multiplicative first reduced Banhatti-Sombor index of a benzenoid hourglass is given by

$$.RBSO_1II(X_p) = (2)^{\frac{8}{2}} \times \left(\frac{5}{4}\right)^{\frac{1}{2}(12p-16)} \times \left(\frac{1}{2}\right)^{\frac{1}{2}(3p^2-3p+1)}.$$

Proof: From definition and by cardinalities of the edge partition of X_p, we deduce

$$RBSO_1II(X_p) = \left(\frac{1}{1^2}+\frac{1}{1^2}\right)^{\frac{8}{2}} \times \left(\frac{1}{1^2}+\frac{1}{2^2}\right)^{\frac{1}{2}(12p-16)} \times \left(\frac{1}{2^2}+\frac{1}{2^2}\right)^{\frac{1}{2}(3p^2-3p+1)}$$
$$= (2)^{\frac{8}{2}} \times \left(\frac{5}{4}\right)^{\frac{1}{2}(12p-16)} \times \left(\frac{1}{2}\right)^{\frac{1}{2}(3p^2-3p+1)}.$$

In the next theorem, we determine the multiplicative first δ-Banhatti-Sombor index of X_p.

Theorem 9: The multiplicative first δ-Banhatti-Sombor index of a benzenoid hourglass X_p is given by

$$\delta BSO_1II(X_p) = (2)^{\frac{8}{2}} \times \left(\frac{5}{4}\right)^{\frac{1}{2}(12p-16)} \times \left(\frac{1}{2}\right)^{\frac{1}{2}(3p^2-3p+1)}.$$

Proof: By observation (2) and from Theorem 8, we obtain the desired result.

6. JAGGED RECTANGLE BENZENOID SYSTEMS

In this section, we focus in the molecular graph structure of a jagged rectangle benzenoid system. This system is denoted by $B_{m,n}$ for all $m, n \in N$. Three chemical graphs of a jagged rectangle benzenoid system are presented in Figure 4.

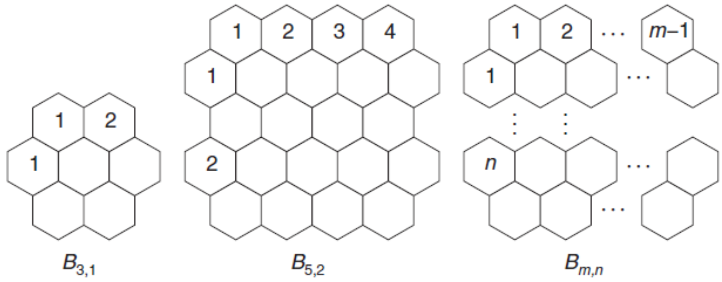

Figure-4

Let G be the graph of a jagged rectangle benzenoid system $B_{m,n}$. By calculation, we obtain that G has $4mn + 4m + 2n – 2$ vertices and $6mn + 5m + n – 4$ edges. From Figure 4, it is easy to see that the vertices of G are either of degree 2 or 3. Thus $\delta(G)=2$. By calculation, we obtain that the edge set of $B_{m,n}$ can be divided into three partitions as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = 2n + 4$.
$E_2 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_2| = 4m + 4n – 4$.
$E_3 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_3| = 6mn + m – 5n – 4$.

In the following theorem, we determine the multiplicative first Banhatti-Sombor index of $B_{m,n}$.

Theorem 10: The multiplicative first Banhatti-Sombor inverse index of $B_{m,n}$ is given by

$$BSO_1II\left(B_{m,n}\right)=\left(\frac{1}{2}\right)^{\frac{1}{2}(2n+4)}\times\left(\frac{13}{36}\right)^{\frac{1}{2}(4m+4n-4)}\times\left(\frac{2}{9}\right)^{\frac{1}{2}(6mn+m-n-4)}.$$

Proof: From definition and by cardinalities of the edge partition of $B_{m,n}$, we obtain

$$BSO_1II\left(B_{m,n}\right)=\left(\frac{1}{2^2}+\frac{1}{2^2}\right)^{\frac{1}{2}(2n+4)}\times\left(\frac{1}{2^2}+\frac{1}{3^2}\right)^{\frac{1}{2}(4m+4n-4)}\times\left(\frac{1}{3^2}+\frac{1}{3^2}\right)^{\frac{1}{2}(6mn+m-n-4)}$$
$$=\left(\frac{1}{2}\right)^{\frac{1}{2}(2n+4)}\times\left(\frac{13}{36}\right)^{\frac{1}{2}(4m+4n-4)}\times\left(\frac{2}{9}\right)^{\frac{1}{2}(6mn+m-n-4)}.$$

In the following theorem, we compute the multiplicative first reduced Banhatti-Sombor index of $B_{m,n}$.

Theorem 11: The multiplicative first reduced Banhatti-Sombor index of $B_{m,n}$ is given by

$$RBSO_1II\left(B_{m,n}\right)=(2)^{\frac{1}{2}(2n+4)}\times\left(\frac{5}{4}\right)^{\frac{1}{2}(4m+4n-4)}\times\left(\frac{1}{2}\right)^{\frac{1}{2}(6mn+m-5n-4)}.$$

Proof: From definition and by cardinalities of the edge partition of $B_{m,n}$, we obtain

$$RBSO_1II\left(B_{m,n}\right)=\left(\frac{1}{1^2}+\frac{1}{1^2}\right)^{\frac{1}{2}(2n+4)}\times\left(\frac{1}{1^2}+\frac{1}{2^2}\right)^{\frac{1}{2}(4m+4n-4)}\times\left(\frac{1}{2^2}+\frac{1}{2^2}\right)^{\frac{1}{2}(6mn+m-5n-4)}$$
$$=(2)^{\frac{1}{2}(2n+4)}\times\left(\frac{5}{4}\right)^{\frac{1}{2}(4m+4n-4)}\times\left(\frac{1}{2}\right)^{\frac{1}{2}(6mn+m-5n-4)}$$

In the following next theorem, we determine the multiplicative first δ-Banhatti-Sombor index of $B_{m,n}$

Theorem 12: The multiplicative first δ-Banhatti-Sombor index of $B_{m,n}$ is given by

$$\delta BSO_1II\left(B_{m,n}\right)=(2)^{\frac{1}{2}(2n+4)}\times\left(\frac{5}{4}\right)^{\frac{1}{2}(4m+4n-4)}\times\left(\frac{1}{2}\right)^{\frac{1}{2}(6mn+m-5n-4)}.$$

Proof: From observation (2) and Theorem 11, we get the desired result.

7. CONCLUSION

In this study, we have introduced the multiplicative first Banhatti-Sombor index, multiplicative first reduced Banhatti-Sombor index, multiplicative first δ-Banhatti-Sombor index of a graph and have computed exact formulas for certain benzenoid systems.

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