International Journal of Mathematical Archive-12(4), 2021, 8-20
Available online through www.ijma.info ISSN 2229 – 5046

CO-FUZZY IDEALS OF FINITE Γ-NEAR RING

DR. D. BHARATHI[1], P. VENKATRAO*[2] AND K. BALA KOTESWARA RAO[3]

[1,2]Department of Mathematics, S. V. University, Tirupati – (A.P.), India.

[3]Department of Mathematics, P. R. R & V S Govt. College, Vidavaluru, India.

(Received On: 03-03-21; Revised & Accepted On: 12-04-21)

ABSTRACT

In this paper we introduce and study the concept of ideals of finite Γ-near rings, co-fuzzy ideals of finite Γ-near rings and related theorems. We prove intersection of two co-fuzzy ideals is always a co-fuzzy ideal but union of two co-fuzzy ideals need not be a co-fuzzy ideal of finite Γ-near ring and it is explained by a suitable example. We also introduce homomorphic image and homomorphic pre image of co-fuzzy ideals of finite Γ-near rings and related theorems.

Key Words : *finite Γ-near ring, fuzzy set, level subset, union of fuzzy sets, intersection of fuzzy sets, co-fuzzy ideals and homomorphism of co-fuzzy ideals.*

1. INTRODUCTION

The fuzzy set theory was developed by Zadeh.L.A. [12] in 1965. The fuzzification of algebraic structure was introduced by Rosenfield.A[7] and he introduced the notation of fuzzy of subgroups in 1971. Swamy K.L.N and Swamy U.M [8] studied fuzzy prime ideals [4,6] Later Satyanarayana. Bh [9] defined Γ-near rings and also he studied ideal theory in Γ-near rings. The notation of fuzzy ideals and its properties were applied to various areas like semi groups [11, 10, 5] and semi rings [1,2] Jun.Y.B [3] considered the fuzzification of ideals of Γ-near rings and investigated the related properties.

In this paper, we have defined co-fuzzy ideal concept of finite $\Gamma-$near ring with less than or equal and maximum conditions and also investigated several properties with the new definitions. Throughout this chapter $\mathcal{N}$ stands for zero symmetric finite $\Gamma -$ near ring.

2. PRELIMINARIES

In this section we recall some of the fundamental definitions, which are necessary for this paper.

Definition 2.1: A triplet $(\mathcal{N},+,\cdot)$ is said to be a near ring if

1. $(\mathcal{N},+)$ is a group
2. $(\mathcal{N},\cdot)$ is a semi-group
3. $(a+b)\cdot c = a\cdot c + b\cdot c$ (Right distribution law) $\forall\, a,b,c \in \mathcal{N}$.

Definition 2.2: A near ring $(\mathcal{N},+,\cdot)$ is said to be finite near ring if $\mathcal{N}$ has finite number of elements.

Definition 2.3: Let $\mathcal{N}$ be a non-empty finite set. If

1. $(\mathcal{N},+)$ is a group. (Not necessarily abelian group)
2. Γ is the set of binary operations on $\mathcal{N}$ such that $(\mathcal{N},+,\alpha)$ is a near ring. Where $\alpha \in \Gamma$.
3. $a\alpha(b\beta c) = (a\alpha b)\beta c, \forall\, a,b,c \in \mathcal{N}$ and $\alpha,\beta \in \Gamma$. Then $(\mathcal{N},+,\Gamma)$ is called finite Γ-near ring.

Corresponding Author: P. Venkatrao*[2],
[2]Department of Mathematics, S. V. University, Tirupati - (A.P.), India.

Definition 2.4: If X be a non empty set and $f: X \to [0,1]$ is a mapping then the pair (X, f) is called fuzzy set and f is called fuzzy sub set of X.

Definition 2.5: Let f be a fuzzy sub set of the set X.Then the set $\{x \in X / f(x) < s\}$ Where $s \in [0,1]$ is called level sub set of f.It is denoted by f_s.This f_s is also s-cut of f.

$$\therefore f_s = \{x \in X / f(x) < s\}$$

Definition 2.6: Let f and g be two fuzzy sub sets of the set X. If $f(x) \leq g(x),\ \forall\, x \in X$ then f is said to be contained in g.It is denoted by $f \subseteq g$.

Definition 2.7: Let f and g be two fuzzy sub sets of the set X.Then their intersection and union are denoted by $f \cap g$ and $f \cup g$ respectively and defined as follows,

$$(f \cap g)(x) = Min\,\{f(x), g(x)\},\ \forall x \in X.$$

and

$$(f \cup g)(x) = Max\,\{f(x), g(x)\},\ \forall x \in X.$$

Definition 2.8: Let M_1 and M_2 be two non empty sets and $\mu: M_1 \to M_2$ is a mapping. If f is a fuzzy sub set of M_1 then g be a fuzzy sub set of M_2 defined by

$$g(y) = \begin{cases} \underset{z \in \mu^{-1}(y)}{Inf} f(z) & if\ \mu^{-1}(y) \neq \phi \\ 0 & if\ \mu^{-1}(y) \neq \phi \end{cases}$$

Where $\mu^{-1}(y) = \{x \in M_1 / \mu(x) = y\}$

If g is a fuzzy sub set of M_2 then f be a fuzzy sub set of M_1 defined by

$$f(x) = g(\mu(x)),\ \forall\, x \in M_1.$$

Definition 2.9: For any sub set A of the set X, the co-fuzzy characteristic set δ_A is defined as follows,

$$\delta_A(x) = \begin{cases} 0 & if\ x \in A \\ 1 & if\ x \notin A \end{cases}$$

3. CO - FUZZY IDEALS OF FINITE Γ-NEAR RINGS

In this section we define ideals and co-fuzzy ideals of a finite Γ-near rings. We prove the intersection of two co-fuzzy ideals of a finite Γ-near ring is always a co-fuzzy ideal. But, the union of two co-fuzzy ideals of a finite Γ-near ring need not be a co-fuzzy ideal and it is explained by a suitable example. We also prove some of related theorems on ideals and co-fuzzy ideals of a finite Γ-near rings.

Definition 3.1: A sub set S of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$ is said to be a left ideal of $\mathcal{N}$ if

1. $x + y \in S,\quad \forall\, x, y \in S$
2. $y + x - y \in S,\quad \forall\, x \in S$ and $y \in \mathcal{N}$
3. $a\alpha(x + b) - a\alpha b \in S,\ \forall\, x \in S, a, b \in \mathcal{N}$ and $\alpha \in \Gamma$.

Definition 3.2: A sub set S of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$ is said to be a right ideal of $\mathcal{N}$ if

1. $x + y \in S,\quad \forall\, x, y \in S$
2. $y + x - y \in S,\ \forall\, x \in S$ and $y \in \mathcal{N}$
3. $x\alpha a \in S,\quad \forall\, x \in S, a \in \mathcal{N}$ and $\alpha \in \Gamma$.

Definition 3.3: A sub set S of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$ is said to be an ideal of $\mathcal{N}$ if

1. $x + y \in S,\quad \forall\, x, y \in S$
2. $y + x - y \in S,\quad \forall\, x \in S$ and $y \in \mathcal{N}$
3. $a\alpha(x + b) - a\alpha b \in S,\ \forall\, x \in S, a, b \in \mathcal{N}$ and $\alpha \in \Gamma$.
4. $x\alpha a \in S,\quad \forall\, x \in S, a \in \mathcal{N}$ and $\alpha \in \Gamma$.

Definition 3.4: A fuzzy sub set f of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$ is said to be a co-fuzzy left ideal of $\mathcal{N}$ if

1. $f(x + y) \leq \text{Max}\,\{f(x), f(y)\}$,
2. $f(y + x - y) \leq f(x)$,
3. . $f(a\alpha(x + b) - a\alpha b) \leq f(x), \forall\, x, y, a, b \in \mathcal{N}$ and $\alpha \in \Gamma$.

Definition 3.5: A fuzzy sub set f of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$ is said to be a co-fuzzy right ideal of $\mathcal{N}$ if

1. $f(x+y) \le \text{Max}\{f(x), f(y)\}$,
2. $f(y+x-y) \le f(x)$,
3. $f(x\alpha a) \le f(x),\ \forall\, x, y, a \in \mathcal{N}$ and $\alpha \in \Gamma$.

Definition 3.6: A fuzzy sub set f of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$ is said to be a co-fuzzy ideal of $\mathcal{N}$ if

1. $f(x+y) \le \text{Max}\{f(x), f(y)\}$,
2. $f(y+x-y) \le f(x)$,
3. $f(a\alpha(x+b) - a\alpha b) \le f(x)$,
4. $f(x\alpha a) \le f(x),\ \forall\, x, y, a, b \in \mathcal{N}$ and $\alpha \in \Gamma$.

Definition 3.7: A L-fuzzy sub set f_L of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$ is said to be L-fuzzy ideal of $\mathcal{N}$ if

1. $f_L(x+y) \le \text{Max}\{f_L(x), f_L(y)\}$
2. $f_L(y+x-y) \le f_L(x)$
3. $f_L(a\alpha(x+b) - a\alpha b) \le f_L(x)$
4. $f_L(x\alpha a) \le f_L(x)\ \ \forall\, x, y, a, b \in \mathcal{N}$ and $\alpha \in \Gamma$.

Here L is a complete lattice satisfying infinite distribute laws.

Definition 3.8: A finite Γ-near ring $(\mathcal{N}, +, \Gamma)$ is said to be zero symmetric if $x\alpha 0 = 0\alpha x = 0\ \ \forall x \in \mathcal{N}$

Note 3.9:

1. $f(x+y) \le \text{Max}\{f(x), f(y)\}$
2. $f(x-y) \le \text{Max}\{f(x), f(-y)\}$
3. $f(0) \le \text{Max}\{f(x), f(-x)\}$
4. $f(0) = f(x\alpha 0) \le f(x),\ \ \forall x, y \in \mathcal{N}$

Example 3.10: Let $S = \{1,2,3,4\}$ and $\mathcal{N}$ = P(S) is the power set of S.

Let $\Gamma = \{\{1\}, \{2,3\}\}$

Define f: $\mathcal{N} \to [0,1]$ such that $f(A) = \begin{cases} \frac{1}{2} & if \quad A \ne \phi \\ \frac{1}{3} & if \quad A = \phi \end{cases}$

Here f is both co-fuzzy left and co-fuzzy right ideal of finite Γ-near ring $(\mathcal{N}, \Delta, \Gamma)$.

Hence f is co-fuzzy ideal of finite Γ-near ring $(\mathcal{N}, \Delta, \Gamma)$.

Example 3.11: Let $\mathcal{N}$ be the set of all 2 × 2 matrices defined over Z_5.Where $Z_5 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}\}$ is the set of all residue classes modulo 5 and $\Gamma = \{\bar{1}, \bar{2}, \bar{3}\}$.Then $(\mathcal{N}, +, \Gamma)$ is finite Γ-near ring.

Define f: $\mathcal{N} \to [0,1]$ such that $f(A) = \begin{cases} 0.3 & if \quad A = \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} \\ 0.5 & if \quad other\ wise \end{cases}$

Then f is a co-fuzzy right ideal of $\mathcal{N}$. But not co-fuzzy left ideal of $\mathcal{N}$.

Define f: $\mathcal{N} \to [0,1]$ such that $f(A) = \begin{cases} 0.2 & if\ A = \begin{bmatrix} 0 & a \\ 0 & b \end{bmatrix} \\ 0.6 & otherwise \end{cases}$

Then f is a co-fuzzy left ideal of $\mathcal{N}$. But not co-fuzzy right ideal of $\mathcal{N}$.

Now, we prove some important theorems on co-fuzzy ideals.

Theorem 3.12: A fuzzy sub set f of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$ is a co-fuzzy ideal of $\mathcal{N}$ if and only if for each $\delta \in Im(f)$, the level set f_δ of f is an ideal of $\mathcal{N}$.

Proof: Let f be a co-fuzzy ideal of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$.

Now, we have to prove that the level set f_δ of f is an ideal of $\mathcal{N}$ $\forall \delta \in im(f)$

By the definition of the level set $f_\delta = \{X \in \mathcal{N}, \ f(x) < \delta\}$

1. Let $x, y \in f_\delta$
   Then $f(x) < \delta$ and $f(y) < \delta$
   Now $f(x+y) \leq \text{Max}\{f(x), f(y)\} < \delta$
   $\therefore f(x+y) < \delta$
   $\Rightarrow x + y \in f_\delta$
2. Let $x \in f_\delta$ and $y \in \mathcal{N}$
   Then $f(x) < \delta$
   Now $f(y + x - y) \leq f(x) < \delta$
   $\therefore f(y + x - y) < \delta$
   $\Rightarrow y + x - y \in f_\delta$
3. Let $x \in f_\delta$, $a, b \in \mathcal{N}$ and $\alpha \in \Gamma$
   Then $f(x) < \delta$
   Now $f(a\alpha(x+b) - a\alpha b) \leq f(x) < \delta$
   $\therefore f(a\alpha(x+b) - a\alpha b) < \delta$
   $\Rightarrow a\alpha(x+b) - a\alpha b \in f_\delta$
   Then the level set f_δ of f is a left ideal of $\mathcal{N}$, $\forall \ \delta \in Im(f)$
4. Let $x \in f_\delta$, $a \in \mathcal{N}$ and $\alpha \in \Gamma$
   Then $f(x) < \delta$
   We have $f(x\alpha a) \leq f(x) < \delta$
   $\therefore f(x\alpha a) < \delta$
   $\Rightarrow x\alpha a \in f_\delta$
   Then the level set f_δ of f is a right ideal of $\mathcal{N}$.
   Hence the level set f_δ of f is a ideal of $\mathcal{N}$, $\forall \delta \in \text{Im}(f)$
   Conversely assume that f_δ is an ideal of $\mathcal{N}$
   Now we prove that f is a co-fuzzy ideal of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$
1. Let $x, y \in \mathcal{N}$
   If possible, let there exists $x_0, y_0 \in \mathcal{N}$ such that
   $f(x_0 + y_0) > Max\{f(x_0), f(y_0)\}$
   Let $\delta = \frac{1}{2}\{f(x_0 + y_0) + Max\{f(x_0), f(y_0)\}\}$
   Then $\delta < \frac{1}{2}\{f(x_0 + y_0) + f(x_0 + y_0)\}$
   $\Rightarrow \delta < f(x_0 + y_0)$
   $\Rightarrow x_0 + y_0 \notin f_\delta$
   And $\delta > \frac{1}{2}\{2\, Max\{f(x_0), f(y_0)\}\}$
   $\Rightarrow \delta > Max\{f(x_0), f(y_0)\}$
   $\Rightarrow \delta > f(x_0)$ and $\delta > f(y_0)$
   $\Rightarrow x_0, y_0 \in f_\delta$
   $\therefore x_0 + y_0 \notin f_\delta$ and $x_0, y_0 \in f_\delta$
   This is a contradiction to f_δ is an ideal of $\mathcal{N}$
   Then our assumption is wrong.
   Hence $f(x+y) \leq \text{Max}\{f(x), f(y)\}$, $\forall x, y \in \mathcal{N}$

2. Let $x, y \in \mathcal{N}$
   If possible, let there exists $x_0, y_0 \in \mathcal{N}$ such that
   $f(y_0 + x_0 - y_0) > f(x_0)$
   Let $\delta = \frac{1}{2}\{f(y_0 + x_0 - y_0) + f(x_0)\}$
   Then $\delta < \frac{1}{2}\{2f(y_0 + x_0 - y_0)\}$ and $\delta > \frac{1}{2}\{2f(x_0)\}$
   $\Rightarrow \delta < f(y_0 + x_0 - y_0)$ and $\delta > f(x_0)$
   $\Rightarrow y_0 + x_0 - y_0 \notin f_\delta$ and $x_0 \in f_\delta$
   This is a contradiction to f_δ is an ideal of $\mathcal{N}$
   Then our assumption is wrong and hence $f(y + x - y) \le f(x)$,
   $\forall\, x, y \in \mathcal{N}$
3. Let $x, a, b \in \mathcal{N}$ and $\alpha \in \Gamma$
   If possible, let there exists $x_0, a_0, b_0 \in \mathcal{N}$ and $\alpha \in \Gamma$ such that
   $f(a_0\alpha(x_0 + b_0) - a_0\alpha b_0) > f(x_0)$
   Let $\delta = \frac{1}{2}\{f(a_0\alpha(x_0 + b_0) - a_0\alpha b_0) + f(x_0)\}$
   Then $\delta < \frac{1}{2}\{2f(a_0\alpha(x_0 + b_0) - a_0\alpha b_0)\}$ *and* $\delta > \frac{1}{2}\{2f(x_0)\}$
   $\Rightarrow \delta < f(a_0\alpha(x_0 + b_0) - a_0\alpha b_0)$ and $\delta > f(x_0)$
   $\Rightarrow a_0\alpha(x_0 + b_0) - a_0\alpha b_0 \notin f_\delta$ and $x_0 \in f_\delta$
   This is a contradiction to f_δ is an ideal of $\mathcal{N}$
   Then our assumption is wrong and hence
   $f(a\alpha(x + b) - a\alpha b) \le f(x)$, $\forall\, x, a, b \in \mathcal{N}$ and $\alpha \in \Gamma$.
   Then f is a co-fuzzy left ideal of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$.
4. Let $x, a \in \mathcal{N}$ and $\alpha \in \Gamma$
   If possible, let there exists $x_0, a_0 \in \mathcal{N}$ and $\alpha \in \Gamma$ such that
   $f(x_0\alpha a_0) > f(x_0)$
   Let $\delta = \frac{1}{2}\{f(x_0\alpha a_0) + f(x_0)\}$
   Then $\delta < f(x_0\alpha a_0)$ and $\delta > f(x_0)$
   $\Rightarrow x_0\alpha a_0 \notin f_\delta$ and $x_0 \in f_\delta$
   This is a contradiction to f_δ is an ideal of $\mathcal{N}$.
   Then our assumption is wrong and hence $f(x\alpha a) \le f(x)$, $\forall\, x, a \in \mathcal{N}$ and $\alpha \in \Gamma$
   Then f is a co- fuzzy right ideal of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$
   Hence f is a co-fuzzy ideal of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$.

Theorem 3.13: Let $S(\neq \phi)$ be a non empty sub set of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$. Then the fuzzy set $f_S: \mathcal{N} \to [0,1]$ defined by $f_S(x) = \begin{cases} 0 & if\ x \in S \\ 1 & if\ x \notin S \end{cases}$ is a co-fuzzy ideal of $\mathcal{N}$ if and only if S is an ideal of $\mathcal{N}$.

Proof: Let $S(\neq \phi)$ be a non empty sub set of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$

Define $f_S: \mathcal{N} \to [0,1]$ such that $f_S(x) = \begin{cases} 0 & if\ x \in S \\ 1 & if\ x \notin S \end{cases}$

Let f_S be a co-fuzzy ideal of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$

Now we prove that S is an ideal of $\mathcal{N}$

1. Let $x, y \in S$
   Then $f_S(x) = 0$ and $f_S(y) = 0$
   Now $f_S(x + y) \le \text{Max}\{f_S(x), f_S(y)\} = 0$
   $\therefore f_S(x + y) = 0$
   $\Rightarrow x + y \in S$

2. $x \in S$ and $y \in \mathcal{N}$
   Then $f_S(x) = 0$
   Now $f_S(y + x - y) \leq f_S(x) = 0$
   $\therefore f_S(y + x - y) = 0$
   $\Rightarrow y + x - y \in S$
3. Let $x \in S,\ a, b \in \mathcal{N}$ and $\alpha \in \Gamma$
   Then $f_S(x) = 0$
   Now $f_S(a\alpha(x + b) - a\alpha b) \leq f_S(x) = 0$
   $\therefore f_S(a\alpha(x + b) - a\alpha b) = 0$
   $\Rightarrow a\alpha(x + b) - a\alpha b \in S$
   Then S is a left ideal of $\mathcal{N}$
4. Let $x \in S,\ a \in \mathcal{N}$ and $\alpha \in \Gamma$
   Then $f_S(x) = 0$
   We have $f_S(x\alpha a) \leq f_S(x) = 0$
   $\therefore f_S(x\alpha a) \leq 0$
   $\Rightarrow f_S(x\alpha a) = 0$
   $\Rightarrow x\alpha a \in S$
   Then S is a right ideal of $\mathcal{N}$
   Hence S is an ideal of $\mathcal{N}$

Conversely assume that S is an ideal of $\mathcal{N}$

We prove that f_S is a co-fuzzy ideal of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$

1. Let $x, y \in \mathcal{N}$
   (i) *Let* $x, y \in S$
   Then $f_s(x) = 0$ *and* $f_s(y) = 0$ *and* $x + y \in S$
   $\Rightarrow f_s(x + y) = 0 \leq \max\{f_s(x), f_s(y)\}$
   $\therefore f_s(x + y) \leq \max\{f_s(x), f_s(y)\}$
   (ii) *Let* $x, y \notin S$ *and* $x + y \in S$
   Then $f_s(x) = f_s(y) = 1$ *and* $f_s(x + y) = 0$
   Now $f_s(x + y) = 0 \leq \max\{f_s(x), f_s(y)\}$
   $\therefore f_s(x + y) \leq \max\{f_s(x), f_s(y)\}$
   (iii) *Let* $x, y \notin S$ *and* $x + y \notin S$
   Then $f_s(x) = f_s(y) = f_s(x + y) = 1$
   Now $f_s(x + y) = 1 \leq \max\{f_s(x), f_s(y)\}$
   $\therefore f_s(x + y) \leq \max\{f_s(x), f_s(y)\}$
2. Let $x, y \in \mathcal{N}$
   (i) Let $x \in S$ and $y \in \mathcal{N}$
   Then $f_s(x) = 0$ and $y + x - y \in S$
   $\Rightarrow f_S(y + x - y) = 0 \leq f_S(x)$
   $\therefore f_S(y + x - y) \leq f_S(x)$
   (ii) Let $x \notin S$ and $y \in \mathcal{N}$
   Then $f_S(x) = 1$
   Now $f_S(y + x - y) \leq 1 \leq f_S(x)$
   $\therefore f_S(y + x - y) \leq f_S(x)$
3. Let $x, a, b \in \mathcal{N}$ and $\alpha \in \Gamma$
   (i) Let $x \in S$, $a, b \in \mathcal{N}$ and $\alpha \in \Gamma$
   Then $f_S(x) = 0$ and $a\alpha(x + b) - a\alpha b \in S$
   $\Rightarrow f_S(a\alpha(x + b) - a\alpha b) = 0$
   Now $f_S(a\alpha(x + b) - a\alpha b) = 0 \leq f_S(x)$
   $\therefore f_S(a\alpha(x + b) - a\alpha b) \leq f_S(x)$

(ii) Let $x \notin S$, $a, b \in \mathcal{N}$ and $\alpha \in \Gamma$
Then $f_S(x) = 1$
Now $f_S(a\alpha(x+b) - a\alpha b) \leq 1 = f_S(x)$
$\therefore f_S(a\alpha(x+b) - a\alpha b) \leq f_S(x)$
Thus f_S is a co-fuzzy left ideal of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$

4. Let $x, a \in \mathcal{N}$ and $\alpha \in \Gamma$

(i) Let $x \in S$, $a \in \mathcal{N}$ and $\alpha \in \Gamma$
Then $f_S(x) = 0$ and $x\alpha a \in S$
$\Rightarrow f_S(x\alpha a) = 0$
Now $f_S(x\alpha a) = 0 \leq f_S(x)$
$\therefore f_S(x\alpha a) \leq f_S(x)$

(ii) Let $x \notin S$, $a \in \mathcal{N}$ and $\alpha \in \Gamma$
Then $f_S(x) = 1$
Now $f_S(x\alpha a) \leq 1 \leq f_S(x)$
$\therefore f_S(x\alpha a) \leq f_S(x)$

Thus f_S is a co-fuzzy right ideal of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$

Hence f_S is a co-fuzzy ideal of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$

Theorem 3.14: If f is a co-fuzzy ideal of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$ then the set $S = \{x \in \mathcal{N} / f(x) = f(0)\}$ is an ideal of $\mathcal{N}$

Proof: Let f is a co-fuzzy ideal of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$
Given that $S = \{x \in \mathcal{N} / f(x) = f(0)\}$
We know that $f(0) \leq f(x)$, $\forall x \in \mathcal{N}$
Now we prove that S is an ideal of $\mathcal{N}$

1. Let $x, y \in S$
Then $f(x) = f(0)$ and $f(y) = f(0)$
Since f is a fuzzy ideal, $f(x+y) \leq \text{Max}\{f(x), f(y)\} = f(0)$
$f(x+y) \leq f(0)$
$\Rightarrow f(x+y) = f(0)$
$\Rightarrow x + y \in S$
2. Let $x \in S$ and $y \in \mathcal{N}$
Then $f(x) = f(0)$
Since f is a fuzzy ideal, $f(y + x - y) \leq f(x) = f(0)$
$\therefore f(y + x - y) \leq f(0)$
$\Rightarrow f(y + x - y) = f(0)$
$\Rightarrow y + x - y \in S$
3. Let $x \in S, a, b \in \mathcal{N}$ and $\alpha \in \Gamma$
Then $f(x) = f(0)$
Since f is a fuzzy left ideal, $f(a\alpha(x+b) - a\alpha b) \leq f(x) = f(0)$
$\therefore f(a\alpha(x+b) - a\alpha b) \leq f(0)$
$\Rightarrow f(a\alpha(x+b) - a\alpha b) = f(0)$
$\Rightarrow a\alpha(x+b) - a\alpha b \in S$
Thus S is a left ideal of $\mathcal{N}$.
4. Let $x \in S$, $a \in \mathcal{N}$ and $\alpha \in \Gamma$
Then $f(x) = f(0)$
Since f is a fuzzy right ideal, $f(x\alpha a) \leq f(x) = f(0)$
$\therefore f(x\alpha a) \leq f(0)$
$\Rightarrow f(x\alpha a) = f(0)$
$\Rightarrow x\alpha a \in S$
Thus S is a right ideal of $\mathcal{N}$.
Hence S is an ideal of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$.

Theorem 3.15: Let $S(\neq \phi)$ is an ideal of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$. Then for all $t \in (0, 1]$ there exists a co-fuzzy ideal f of $\mathcal{N}$ Such that $\mathcal{N}_f = S$, Where $\mathcal{N}_f = \{x \in \mathcal{N} / f(x) = f(0)\}$.

Proof: Given that $S(\neq \phi)$ is an ideal of $\mathcal{N}$.Define $f\colon \mathcal{N} \to [0,1]$ such that $f(x) = \begin{cases} 0 & if\ x \in S \\ t & if\ x \notin S \end{cases}$

We prove that f is a co-fuzzy ideal of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$

1. Let $x, y \in \mathcal{N}$
   (i) Let $x, y \in S$
   Then $f(x) = f(y) = 0$
   Since S is an ideal of $\mathcal{N}$, $x + y \in S$
   $\Rightarrow f(x + y) = 0$
   $\Rightarrow f(x + y) = 0 \leq \text{Max}\{f(x), f(y)\}$
   $\therefore f(x + y) \leq \text{Max}\{f(x), f(y)\}$
   (ii) Let $x, y \notin S$ and $x + y \in S$
   Then $f(x) = f(y) = t$ and $f(x + y) = 0$
   Now $f(x + y) = 0 \leq \text{Max}\{f(x), f(y)\}$
   $\therefore f(x + y) \leq \text{Max}\{f(x), f(y)\}$
   (iii) Let $x, y \notin S$ and $x + y \notin S$
   Then $f(x) = f(y) = t$ and $f(x + y) = t$
   Now $f(x + y) = t \leq \text{Max}\{f(x), f(y)\}$
   $\therefore f(x + y) \leq \text{Max}\{f(x), f(y)\}$
2. Let $x, y \in \mathcal{N}$
   (i) Let $x \in S$ and $y \in \mathcal{N}$
   Then $f(x) = 0$
   Since S is an ideal of $\mathcal{N}$, $y + x - y \in S$
   $\Rightarrow f(y + x - y) = 0 \leq f(x)$
   $\Rightarrow f(y + x - y) \leq f(x)$
   (ii) Let $x \notin S$ and $y \in \mathcal{N}$
   Then $f(x) = t$ and $y + x - y \notin S$
   $\Rightarrow f(y + x - y) = t \leq f(x)$
   $\Rightarrow f(y + x - y) \leq f(x)$
3. Let $x, a, b \in \mathcal{N}$ and $\alpha \in \Gamma$
   (i) Let $x \in S$, $a, b \in \mathcal{N}$ and $\alpha \in \Gamma$
   Then $f(x) = 0$ and $a\alpha(x + b) - a\alpha b \in S$
   $\Rightarrow f(a\alpha(x + b) - a\alpha b) = 0 \leq f(x)$
   $\Rightarrow f(a\alpha(x + b) - a\alpha b) \leq f(x)$
   (ii) Let $x \notin S$ and $a\alpha(x + b) - a\alpha b \in S$
   Then $f(x) = t$ and $f(a\alpha(x + b) - a\alpha b) = 0$
   $\Rightarrow f(a\alpha(x + b) - a\alpha b) = 0 \leq t = f(x)$
   $\Rightarrow f(a\alpha(x + b) - a\alpha b) \leq f(x)$
   (iii) Let $x \notin S$ and $a\alpha(x + b) - a\alpha b \notin S$
   Then $f(x) = t$ and $f(a\alpha(x + b) - a\alpha b) = t$
   $\Rightarrow f(a\alpha(x + b) - a\alpha b) = t = f(x)$
   $\Rightarrow f(a\alpha(x + b) - a\alpha b) \leq f(x)$
   Thus f is a co-fuzzy left ideal of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$.
4. Let $x, a \in \mathcal{N}$ and $\alpha \in \Gamma$
   (i) Let $x \in S$, $a \in \mathcal{N}$ and $\alpha \in \Gamma$
   Then $f(x) = 0$ and $x\alpha a \in S$
   $\Rightarrow f(x\alpha a) = 0$
   $\Rightarrow f(x\alpha a) = 0 \leq f(x)$
   $\Rightarrow f(x\alpha a) \leq f(x)$
   (ii) Let $x \notin S$ and $x\alpha a \in S$
   Then $f(x) = t$ and $f(x\alpha a) = 0$
   $\Rightarrow f(x\alpha a) = 0 \leq t = f(x)$
   $\Rightarrow f(x\alpha a) \leq f(x)$
   (iii) Let $x \notin S$ and $x\alpha a \notin S$
   Then $f(x) = t$ and $f(x\alpha a) = t$
   $\Rightarrow f(x\alpha a) = t = f(x)$
   $\Rightarrow f(x\alpha a) \leq f(x)$
   Thus f is a co-fuzzy right ideal of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$.
   Hence f is a co-fuzzy ideal of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$.

Finally, $\mathcal{N}_f = \{x \in \mathcal{N} / f(x) = f(0)\}$
$= \{x \in \mathcal{N} / f(x) = 0\}$
$= S$
$\therefore \mathcal{N}_f = S$

Theorem 3.16: If f and g are two co-fuzzy ideals of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$, then their intersection $(f \cap g)$ is also co- fuzzy ideal of $\mathcal{N}$.

Proof: Given that f and g are two co-fuzzy ideals of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$

Now we prove that $(f \cap g)$ is co-fuzzy ideal of $\mathcal{N}$.

Let $x, y, a, b \in \mathcal{N}$ and $\alpha \in \Gamma$

1. $(f \cap g)(x + y) = Min\{f(x + y), g(x + y)\}$
$\leq Min\{\text{Max}\{f(x), f(y)\}, \text{Max}\{g(x), g(y)\}\}$
$\leq Max\{\text{Min}\{f(x), g(x)\}, \text{Min}\{f(y), g(y)\}\}$
$= Max\{(f \cap g)(x), (f \cap g)(y)\}$
$\therefore (f \cap g)(x + y) \leq Max\{(f \cap g)(x), (f \cap g)(y)\}$
2. $(f \cap g)(y + x - y) = Min\{f(y + x - y), g(y + x - y)\}$
$\leq Min\{f(x), g(x)\}$
$= (f \cap g)(x)$
$\therefore (f \cap g)(y + x - y) \leq (f \cap g)(x)$
3. $(f \cap g)(a\alpha(x + b) - a\alpha b) = Min\{f(a\alpha(x + b) - a\alpha b), g(a\alpha(x + b) - a\alpha b)\}$
$\leq Min\{f(x), g(x)\}$
$= (f \cap g)(x)$
$\therefore (f \cap g)(a\alpha(x + b) - a\alpha b) \leq (f \cap g)(x)$
Thus $(f \cap g)$ is a co-fuzzy left ideal of $\mathcal{N}$
4. $(f \cap g)(x\alpha a) = Min\{f(x\alpha a), g(x\alpha a)\}$
$\leq Min\{f(x), g(x)\}$
$= (f \cap g)(x)$
$\therefore (f \cap g)(x\alpha a) \leq (f \cap g)(x)$
Thus $(f \cap g)$ is a co-fuzzy right ideal of $\mathcal{N}$.
Hence $(f \cap g)$ is a co-fuzzy ideal of $\mathcal{N}$.

Remark 3.17: The union of two co-fuzzy ideals of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$ need not be a co-fuzzy ideal of $\mathcal{N}$. Now, we prove a sufficient condition for the union of two co-fuzzy ideals of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$ to be a co-fuzzy ideal of $\mathcal{N}$.

Theorem 3.18: If f and g are two co-fuzzy ideals of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$, then their union$(f \cup g)$ is fuzzy ideal of $\mathcal{N}$ if $f \subseteq g$ or $g \subseteq f$.

Proof: Given that f and g are two co-fuzzy ideals of a finite Γ-near ring $(\mathcal{N}, +, \Gamma)$ such that $f \subseteq g$ or $g \subseteq f$,
Without loss of generality, We assume that $f \subseteq g$
Now we prove that $(f \cup g)$ is co-fuzzy ideal of $\mathcal{N}$.
Let $x, y, a, b \in \mathcal{N}$ and $\alpha \in \Gamma$

1. $(f \cup g)(x + y) = Max\{f(x + y), g(x + y)\}$
$\leq Max\{\text{Max}\{f(x), f(y)\}, \text{Max}\{g(x), g(y)\}\}$
$\leq Max\{\text{Max}\{f(x), g(x)\}, \text{Max}\{f(y), g(y)\}\}$
$= Max\{(f \cup g)(x), (f \cup g)(y)\}$
$\therefore (f \cup g)(x + y) \leq Max\{(f \cup g)(x), (f \cup g)(y)\}$
2. $(f \cup g)(y + x - y) = Max\{f(y + x - y), g(y + x - y)\}$
$\leq Max\{f(x), g(x)\}$
$= (f \cup g)(x)$
$\therefore (f \cup g)(y + x - y) \leq (f \cup g)(x)$
3. $(f \cup g)(a\alpha(x + b) - a\alpha b) = Max\{f(a\alpha(x + b) - a\alpha b), g(a\alpha(x + b) - a\alpha b)\}$
$\leq Max\{f(x), g(x)\}$
$= (f \cup g)(x)$
$\therefore (f \cup g)(a\alpha(x + b) - a\alpha b) \leq (f \cup g)(x)$
Thus $(f \cup g)$ is a co-fuzzy left ideal of $\mathcal{N}$

4. $(f \cup g)(x\alpha a) = Max\,\{f(x\alpha a), g(x\alpha a)\}$
$\leq Max\,\{f(x), g(x)\}$
$= (f \cup g)(x)$
$\therefore (f \cup g)(x\alpha a) \leq (f \cup g)(x)$
Thus $(f \cup g)$ is a co-fuzzy right ideal of $\mathcal{N}$
Hence $(f \cup g)$ is a co-fuzzy ideal of $\mathcal{N}$

Remark 3.19: The converse of the above theorem need not be true. i.e, even if $(f \cup g)$ is a co-fuzzy ideal of a finite Γ-near ring, either one may not contained in other.

Example: Let $S = \{1,2,3,4\}$ and $\mathcal{N} = P(S)$, Where $P(S)$ is the power set of S.
Then $(\mathcal{N}, \Delta, \cap)$ is a finite near ring.
Let us take $\Gamma = \{\{1\}, \{1,2,3\}\}$, then $(\mathcal{N}, \Delta, \Gamma)$ is a finite Γ-near ring.
Define $f\colon \mathcal{N} \to [0,1]$ such that $f(A) = \begin{cases} 0.6 & if\ A \neq \phi \\ 0.3 & if\ A = \phi \end{cases}$
and define $g\colon \mathcal{N} \to [0,1]$ such that $g(A) = \begin{cases} 0.7 & if\ A \neq \phi \\ 0.2 & if\ A = \phi \end{cases}$
Then $(f \cup g)\colon \mathcal{N} \to [0,1]$ such that $(f \cup g)(A) = \begin{cases} 0.7 & if\ A \neq \phi \\ 0.3 & if\ A = \phi \end{cases}$ is a co-fuzzy ideal of a finite Γ-near ring $(\mathcal{N}, \Delta, \Gamma)$. But $f \nsubseteq g$ and $g \nsubseteq f$.

4. CO-FUZZY HOMOMORPHISM OF FINITE Γ-NEAR RINGS

In this section, we define co-fuzzy homomorphism between two finite Γ-near rings and we prove that the homomorphic image of co-fuzzy ideal is a co-fuzzy ideal and inverse image of a co-fuzzy ideal is also a co-fuzzy ideal.

Definition 4.1: Let $(\mathcal{N}_1, +, \Gamma)$ and $(\mathcal{N}_2, +, \Gamma)$ be two finite Γ-near rings. Then the function $f\colon \mathcal{N}_1 \to \mathcal{N}_2$ is said to be a homomorphism from $\mathcal{N}_1$ to $\mathcal{N}_2$ if

1. $f(a+b) = f(a) + f(b)$
2. $f(a\alpha b) = f(a)\alpha f(b),\ \forall\, a, b \in \mathcal{N}_1$ and $\alpha \in \Gamma$

Definition 4.2: Let $(\mathcal{N}_1, +, \Gamma_1)$ be a finite Γ_1-near ring and $(\mathcal{N}_2, +, \Gamma_2)$ be a finite Γ_2-near ring, and $f\colon \mathcal{N}_1 \to \mathcal{N}_2$ and $g\colon \Gamma_1 \to \Gamma_2$ be two functions. Then the pair (f, g) is said to be a homomorphism from $\mathcal{N}_1$ to $\mathcal{N}_2$ if

1. $f(a+b) = f(a) + f(b)$
2. $f(a\alpha b) = f(a)g(\alpha)f(b), \forall\, a, b \in \mathcal{N}_1$ and $\alpha \in \Gamma_1$

Definition 4.3: Let $(\mathcal{N}_1, +, \Gamma)$ and $(\mathcal{N}_2, +, \Gamma)$ be two finite Γ-near rings and $f\colon \mathcal{N}_1 \to \mathcal{N}_2$ is a homomorphism from $\mathcal{N}_1$ to $\mathcal{N}_2$. If μ is a fuzzy sub set of $\mathcal{N}_1$ then its image $f(\mu)$ is a fuzzy sub set of $\mathcal{N}_2$ is defined by

$$\big(f(u)\big)(y) = \begin{cases} \inf_{z \in f^{-1}(y)} \mu(z) & if \quad f^{-1}(y) \neq \phi \\ 0 & if \quad f^{-1}(y) = \phi \end{cases} \quad \forall y \in N_2$$

Definition 4.4: Let $(\mathcal{N}_1, +, \Gamma)$ and $(\mathcal{N}_2, +, \Gamma)$ be two finite Γ-near rings. The function $f\colon \mathcal{N}_1 \to \mathcal{N}_2$ is a homomorphism from $\mathcal{N}_1$ to $\mathcal{N}_2$. If σ is a co-fuzzy sub set of $\mathcal{N}_2$ then its inverse image $f^{-1}(\sigma)$ is a fuzzy sub set of $\mathcal{N}_1$ defined by $\big(f^{-1}(\sigma)\big)(x) = \sigma\big(f(x)\big),\ \forall\, x \in \mathcal{N}_1$.

Theorem 4.5: Let $(\mathcal{N}_1, +, \Gamma)$ and $(\mathcal{N}_2, +, \Gamma)$ be two finite Γ-near rings. The function $f\colon \mathcal{N}_1 \to \mathcal{N}_2$ is a homomorphism from $\mathcal{N}_1$ to $\mathcal{N}_2$. If σ is a co-fuzzy ideal of $\mathcal{N}_2$ then its inverse image $f^{-1}(\sigma)$ is a co-fuzzy ideal of $\mathcal{N}_1$.

Proof: Let $f\colon \mathcal{N}_1 \to \mathcal{N}_2$ is a homomorphism from the finite Γ-near rings $(\mathcal{N}_1, +, \Gamma)$ to $(\mathcal{N}_2, +, \Gamma)$ and $\sigma\colon \mathcal{N}_2 \to [0,1]$ is a co-fuzzy sub set of $\mathcal{N}_2$.

Let σ be a co-fuzzy ideal of $\mathcal{N}_2$

Now we prove that $f^{-1}(\sigma)$ is a co-fuzzy ideal of $\mathcal{N}_1$

Let $x, y, a, b \in \mathcal{N}_1$ and $\alpha \in \Gamma$

1. $$\begin{aligned}(f^{-1}(\sigma))(x+y) &= \sigma(f(x+y))\\ &= \sigma(f(x)+f(y))\\ &\le Max\{\sigma(f(x)), \sigma(f(y))\}\\ &= Max\{(f^{-1}(\sigma))(x), (f^{-1}(\sigma))(y)\}\end{aligned}$$

$\therefore (f^{-1}(\sigma))(x+y) \le Max\{(f^{-1}(\sigma))(x), (f^{-1}(\sigma))(y)\}$

2. $$\begin{aligned}(f^{-1}(\sigma))(y+x-y) &= \sigma(f(y+x-y))\\ &= \sigma(f(y)+f(x)-f(y))\\ &\le \sigma(f(x))\\ &= (f^{-1}(\sigma))(x)\end{aligned}$$

$\therefore (f^{-1}(\sigma))(y+x-y) \le (f^{-1}(\sigma))(x)$

3. $$\begin{aligned}(f^{-1}(\sigma))(a\alpha(x+b)-a\alpha b) &= \sigma(f(a\alpha(x+b)-a\alpha b))\\ &= \sigma\left(f(a)\alpha(f(x)+f(b)) - f(a)\alpha f(b)\right)\\ &\le \sigma(f(x))\\ &= (f^{-1}(\sigma))(x)\end{aligned}$$

$\therefore (f^{-1}(\sigma))(a\alpha(x+b)-a\alpha b) \le (f^{-1}(\sigma))(x)$

Thus $(f^{-1}(\sigma))$ is a co-fuzzy left ideal of $\mathcal{N}_1$

4. $$\begin{aligned}(f^{-1}(\sigma))(x\alpha a) &= \sigma(f(x\alpha a))\\ &= \sigma(f(x)\alpha f(a))\\ &\le \sigma(f(x))\\ &= (f^{-1}(\sigma))(x)\end{aligned}$$

$\therefore (f^{-1}(\sigma))(x\alpha a) \le (f^{-1}(\sigma))(x)$

Thus $(f^{-1}(\sigma))$ is a co-fuzzy right ideal of $\mathcal{N}_1$

Hence $(f^{-1}(\sigma))$ is a co-fuzzy ideal of $\mathcal{N}_1$.

Theorem 4.6: Let $(\mathcal{N}_1, +, \Gamma)$ and $(\mathcal{N}_2, +, \Gamma)$ be two finite Γ-near rings. The function $f\colon \mathcal{N}_1 \to \mathcal{N}_2$ is a onto homomorphism from $\mathcal{N}_1$ to $\mathcal{N}_2$. If μ is a co-fuzzy ideal of $\mathcal{N}_1$ then its image $f(\mu)$ is a co-fuzzy ideal of $\mathcal{N}_2$.

Proof: Let $f\colon \mathcal{N}_1 \to \mathcal{N}_2$ is an onto homomorphism from the finite Γ-near rings $(\mathcal{N}_1, +, \Gamma)$ to $(\mathcal{N}_2, +, \Gamma)$ and $\mu\colon \mathcal{N}_1 \to [0,1]$ is a fuzzy sub set of $\mathcal{N}_1$

Let μ be a co-fuzzy ideal of $\mathcal{N}_1$

Now we prove that $f(\mu)$ is a co-fuzzy ideal of $\mathcal{N}_2$

We have

$$(f(u))(y) = \begin{cases} \inf_{z \in f^{-1}(y)} \mu(z) & if \quad f^{-1}(y) \neq \phi \\ 0 & if \quad f^{-1}(y) = \phi \end{cases} \quad \forall y \in N_2$$

Let $x, y, a, b \in \mathcal{N}_2$ and $\alpha \in \Gamma$

Since f is onto from $\mathcal{N}_1$ to $\mathcal{N}_2$ then there exists $x_0, y_0, a_0, b_0 \in \mathcal{N}_1$ such that $f(x_0) = x$, $f(y_0) = y$, $f(a_0) = a$ and $f(b_0) = b$

1. $\big(f(\mu)\big)(x+y) = \text{Inf}_{z\in f^{-1}(x+y)}\,\mu(z)$ (1)

Let $\mu(x_0) = \text{Inf}_{z\in f^{-1}(x)}\,\mu(z)$ and $\mu(y_0) = \text{Inf}_{z\in f^{-1}(y)}\,\mu(z)$
$\Rightarrow f(x_0) = x$ and $f(y_0) = y$
Now $f(x_0+y_0) = f(x_0)+f(y_0) = x+y$
$\Rightarrow x_0+y_0 \in f^{-1}(x+y)$

From equation (1), $\big(f(\mu)\big)(x+y) = \text{Inf}_{z\in f^{-1}(x+y)}\,\mu(z)$

$$\begin{aligned}&\le \mu(x_0+y_0)\\ &\le Max\,\{\mu(x_0),\mu(y_0)\}\\ &= Max\,\{\text{Inf}_{z\in f^{-1}(x)}\,\mu(z),\text{Inf}_{z\in f^{-1}(y)}\,\mu(z)\}\\ &= Max\,\{\big(f(\mu)\big)(x),\big(f(\mu)\big)(y)\}\end{aligned}$$

$\therefore (f(\mu))(x+y) \le Max\,\{(f(\mu))(x),(f(\mu))(y)\}$

2. $\big(f(\mu)\big)(y+x-y) = \text{Inf}_{z\in f^{-1}(y+x-y)}\,\mu(z)$ (2)

Now $f(y_0+x_0-y_0) = f(y_0)+f(x_0)-f(y_0) = y+x-y$
$\therefore y_0+x_0-y_0 \in f^{-1}(y+x-y)$

From equation (2), $\big(f(\mu)\big)(y+x-y) = \text{Inf}_{z\in f^{-1}(y+x-y)}\,\mu(z)$

$$\begin{aligned}&\le \mu\,(y_0+x_0-y_0)\\ &\le \mu(x_0)\\ &= \text{Inf}_{z\in f^{-1}(x)}\,\mu(z)\\ &= \big(f(\mu)\big)(x)\end{aligned}$$

$\therefore (f(\mu))(y+x-y) \le (f(\mu))(x)$

3. $\big(f(\mu)\big)(a\alpha(x+b)-a\alpha b) = \text{Inf}_{z\in f^{-1}(a\alpha(x+b)-a\alpha b)}\,\mu(z)$ (3)

Now $f(a_0\alpha(x_0+b_0)-a_0\alpha b_0) = f(a_0)\alpha\big(f(x_0)+f(b_0)\big)-f(a_0)\alpha f(b_0)$
$= a\alpha(x+b)-a\alpha b$
$\therefore a_0\alpha(x_0+b_0)-a_0\alpha b_0 \in f^{-1}(a\alpha(x+b)-a\alpha b)$

From equation (3), $\big(f(\mu)\big)(a\alpha(x+b)-a\alpha b) = \text{Inf}_{z\in f^{-1}(a\alpha(x+b)-a\alpha b)}\,\mu(z)$

$$\begin{aligned}&\le \mu(a_0\alpha(x_0+b_0)-a_0\alpha b_0)\\ &\le \mu(x_0)\\ &= \text{Inf}_{z\in f^{-1}(x)}\,\mu(z)\\ &= \big(f(\mu)\big)(x)\end{aligned}$$

$\therefore (f(\mu))(a\alpha(x+b)-a\alpha b) \le (f(\mu))(x)$

Thus $\big(f(\mu)\big): N_2 \to [0,1]$ is a co-fuzzy left ideal of $\mathcal{N}_2$

4. $\big(f(\mu)\big)(x\alpha a) = \text{Inf}_{z\in f^{-1}(x\alpha a)}\,\mu(z)$ (4)

Now $f(x_0\alpha a_0) = f(x_0)\alpha f(a_0) = x\alpha a$
$\therefore x_0\alpha a_0 \in f^{-1}(x\alpha a)$

From equation (4), $\big(f(\mu)\big)(x\alpha a) = \text{Inf}_{z\in f^{-1}(x\alpha a)}\,\mu(z)$

$$\begin{aligned}&\le \mu(x_0\alpha a_0)\\ &\le \mu(x_0)\\ &= \text{Inf}_{z\in f^{-1}(x)}\,\mu(z)\\ &= \big(f(\mu)\big)(x)\end{aligned}$$

$\therefore (f(\mu))(x\alpha a) \le (f(\mu))(x)$

Then $\big(f(\mu)\big): \mathcal{N}_2 \to [0,1]$ is a co-fuzzy right ideal of $\mathcal{N}_2$
Hence $\big(f(\mu)\big): \mathcal{N}_2 \to [0,1]$ is a co-fuzzy ideal of $\mathcal{N}_2$.

5. CONCLUSION

In this article, we inspected the idea of co-fuzzy ideals of a finite Γ – near ring. We proved some necessary and sufficient conditions for a fuzzy subset of finite Γ – near ring to be co-fuzzy ideal of the ring. The intersection and union of co-fuzzy ideals and homomorphism theorems have been proved. This concept may be extended to Bipolar co-fuzzy ideals in finite Γ – near rings.

6. REFERENCES

1. Dutta, T.K., Sardar. S.K. and Sarbani Goswami,(2011) Fuzzy Ideals Extension in $\Gamma -$semirings, arXiv : 1104.4791v1 – [math. GM].
2. Jun. Y.B.Neggers.J. and Kim.H.S.,(1996) Normal L – Fuzzy ideals in semirings. Fuzzy Sets and Systems. Vol.82, 383 – 386.
3. Jun.Y.B.. On fuzzy prime ideals of G-rings. Soochow J. Math, 21(1). (1995). 41-48.
4. Jun.Y.B., Lee. C.Y., Fuzzy prime ideals in G-rings. Pusan Kyongnam Math J., 9(1), (1993), 105-111.
5. Magil.K.D.Jr. and S.subbiah(1975) Maximal ideals of semigroups of endomorphisms, Bull. Austral,Math. Soc. Vol.12, 211-225.
6. Malik. D.S and Mordeson N.John. (1990) Fuzzy prime ideals of a ring. Fuzzy Sets and Systems. Vol.37, 93-98.
7. Rosen field.A., Fuzzy groups, J. Math, Anal.Appl.35.512–517(1971).
8. Swamy, K.L.N and Swamy, U.M..(1988) Fuzzy Prime Ideals of Rings, Journal of Mathematical Analysis and Applications. Vol.134, No.1. 94–103.
9. Satyanarayana Bh., A note on G–near–rings, Indian J.Math., 41(3).(1999). 427-133.
10. Steinfeld, O., (1978) Quasi-ideals in rings and semigroups. Akademiaikiado, Budapest.
11. Yaqoob.N., Bipolar – Valued Fuzzy Ideals in LA – semigroups. Journal of Advanced Studies in Topology. 3(2012). 60–71.
12. Zadeh L.A., (1965) Fuzzy sets. Information and Control Vol.8. 338–353.