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HOMOMORPHISM AND ANTI HOMOMORPHISM FUNCTIONS IN BIPOLAR VALUED VAGUE SUBSEMIRINGS OF A SEMIRING

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ABSTRACT

In this paper, bipolar valued vague subsemiring of a semiring is studied by homomorphism and anti homomorphism and some properties are discussed. These properties are useful to further research.

Key Words: *Fuzzy subset, vague subset, bipolar valued fuzzy subset, bipolar valued vague subset, bipolar valued vague subsemiring, bipolar valued vague normal subsemiring, intersection, image and preimage.*

INTRODUCTION

In 1965, Zadeh [13] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval valued fuzzy sets, vague sets, soft sets etc. Grattan-Guiness [7] discussed about fuzzy membership mapped onto interval and many valued quantities. Vague set is an extension of fuzzy set and it is appeared as a unique case of context dependent fuzzy sets. The vague set was introduced by W.L.Gau and D.J.Buehrer [6]. Lee [8] introduced the notion of bipolar valued fuzzy sets. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [−1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [8, 9]. Fuzzy subgroup was introduced by Azriel Rosenfeld [3]. RanjitBiswas [11] introduced the vague groups. Cicily Flora. S and Arockiarani.I [5] have introduced a new class of generalized bipolar vague sets. Anitha.M.S., et.al. [1] defined as bipolar valued fuzzy subgroups of a group and Balasubramanian.A *et.al* [4] have defined the bipolar interval valued fuzzy subgroups of a group. K.Murugalingam and K.Arjunan[10] have discussed about interval valued fuzzy subsemiring of a semiring and then bipolar valued multi fuzzy subsemirings of a semiring have been introduced by Yasodara.B and KE.Sathappan [12]. Anitha.K., *et.al.* [2] defined as bipolar valued vague subsemirings of a semiring. Here, the concept of bipolar valued vague subsemiring of a semiring is introduced and estaiblished some results. Homomorphism and anti homomorphism are applied in bipolar valued vague subsemiring of a semiring.

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1. PRELIMINARIES

Definition 1.1: [13] Let X be any nonempty set. A mapping M: X $\to$ [0, 1] is called a fuzzy subset of X.

Definition 1.2: [6] A vague set A in the universe of discourse U is a pair [t_A, $1-f_A$], where t_A : U$\to$ [0, 1] and f_A : U$\to$ [0, 1] are mappings, they are called truth membership function and false membership function respectively. Here t_A(x) is a lower bound of the grade of membership of x derived from the evidence for x and f_A(x) is a lower bound on the negation of x derived from the evidence against x and $t_A(x) + f_A(x) \leq 1$, for all x$\in$U.

Definition 1.3: [6] The interval [$t_A(x)$, $1-f_A(x)$] is called the vague value of x in A and it is denoted by $V_A(x)$, i.e., $V_A(x)$ = [$t_A(x)$, $1-f_A(x)$].

Example 1.4: A = { < a, [0.4, 0.6] >, < b, [0.6, 0.8] >, < c, [0.3, 0.9] >} is a vague subset of X = {a, b, c }.

Definition 1.5: [8] A bipolar valued fuzzy set (BVFS) A in X is defined as an object of the form A = { < x, $A^+(x)$, $A^-(x)$ >/ x$\in$X}, where A^+ : X$\to$ [0, 1] and A^- : X$\to$ [−1, 0]. The positive membership degree $A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A.

Example 1.6: A = {< a, 0.4, −0.2 >, < b, 0.6, −0.5 >, < c, 0.3, −0.7 >} is a bipolar valued fuzzy subset of X = {a, b, c }.

Definition 1.7: [5] A bipolar valued vague subset A in X is defined as an object of the form A = {$\langle$x,[$t_A^+(x), 1-f_A^+(x)$], [$-1-f_A^-(x), t_A^-(x)$]$\rangle$ /x$\in$X }, where t_A^+: X$\to$ [0, 1], f_A^+ :X$\to$ [0, 1], t_A^- : X$\to$ [−1, 0] and f_A^- : X$\to$ [−1, 0] are mapping such that $t_A(x) + f_A(x) \leq 1$ and $-1 \leq t_A^- + f_A^-$. The positive interval membership degree [$t_A^+(x), 1-f_A^+(x)$] denotes the satisfaction region of an element x to the property corresponding to a bipolar valued vague subset A and the negative interval membership degree [$-1-f_A^-(x), t_A^-(x)$] denotes the satisfaction region of an element x to some implicit counter-property corresponding to a bipolar valued vague subset A. Bipolar valued vague subset A is denoted as A = {$\langle$ x, $V_A^+(x)$, $V_A^-(x)$ $\rangle$ / x$\in$X }, where $V_A^+(x)$, = [$t_A^+(x), 1-f_A^+(x)$] and $V_A^-(x)$ = [$-1-f_A^-(x), t_A^-(x)$].

Note that. [0] = [0, 0], [1] = [1, 1] and [−1] = [−1, −1].

Example 1.8: [A] = { < a, [0.4, 0,6], [−0.5, −0.2] >, < b, [0.2, 0.4], [−0.6, −0.3] > < c, [0.1, 0.6], [−0.6, −0.2] > } is a bipolar valued vague subset of X = {a, b, c }.

Definition 1.9: [5] Let A = $\langle V_A^+, V_A^- \rangle$ and B = $\langle V_B^+, V_B^- \rangle$ be two bipolar valued vague subsets of a set X. We define the following relations and operations:

(i) [A] $\subset$ [B] if and only if V_A^+(u) $\leq$ V_B^+(u) and V_A^-(u) $\geq$ V_B^-(u), $\forall$ u$\in$X.

(ii) [A] = [B] if and only if V_A^+(u) = V_B^+(u) and V_A^-(u) = V_B^-(u), $\forall$ u$\in$X.

(iii) [A]$\cap$[B] = {$\langle$ u, rmin (V_A^+(u), V_B^+(u)), rmax (V_A^-(u), V_B^-(u)) $\rangle$ / u$\in$X }.

(iv) [A]$\cup$[B] = {$\langle$ u, rmax (V_A^+(u), V_B^+(u)), rmin (V_A^-(u), V_B^-(u)) $\rangle$ / u$\in$X}. Here rmin (V_A^+(u), V_B^+(u)) = [min { $t_A^+(x), t_B^+(x)$ }, min {$1-f_A^+(x)$, $1-f_B^+(x)$}], rmax (V_A^+(u), V_B^+(u)) = [max { $t_A^+(x), t_B^+(x)$ }, max {$1-f_A^+(x)$, $1-f_B^+(x)$}], rmin (V_A^-(u), V_B^-(u)) = [min {$-1-f_A^-(x)$, $-1-f_B^-(x)$}, min { $t_A^-(x), t_B^-(x)$ }], rmax (V_A^-(u), V_B^-(u)) = [max {$-1-f_A^-(x)$, $-1-f_B^-(x)$ }, max { $t_A^-(x), t_B^-(x)$ }].

Definition 1.10: [2] Let R be a semiring. A bipolar valued vague subset A of R is said to be a bipolar valued vague subsemiring of R (BVVSSR) if the following conditions are satisfied,

(i) V_A^+(x+y) $\geq$ rmin{V_A^+(x), V_A^+(y)}

(ii) V_A^+(xy) $\geq$ rmin{V_A^+(x), V_A^+(y) }

(iii) V_A^-(x+y) $\leq$ rmax{V_A^-(x), V_A^-(y)}

(iv) V_A^-(xy) $\leq$ rmax{V_A^-(x), V_A^-(y)} for all x and y in R.

Example 1.11: Let R = Z_3 = { 0, 1, 2 } be a semiring with respect to the ordinary addition and multiplication. Then A = { < 0, [0.5, 0.7], [– 0.8, – 0.5] >, < 1, [0.4, 0.6], [–0.7, –0.4] >, <2, [0.4, 0.6], [–0.7, –0.4] > } is a bipolar valued vague subsemiring of R.

Definition 1.12: Let R be a semiring. A bipolar valued vague subsemiring A = $\langle V_A^+ , V_A^- \rangle$ of R is said to be a bipolar valued vague normal subsemiring of R if V_A^+(x+y) = V_A^+(y+x), V_A^+(xy) = V_A^+(yx), V_A^-(x+y) = V_A^-(y+x) and V_A^-(xy) = V_A^-(yx) for all x and y in R.

Definition 1.13: [3] Let R and R^l be any two semirings. Then the function f: R → R^l is said to be an antihomomorphism if f(x+y) = f(y)+f(x) and f(xy) = f(y)f(x) for all x and y in R.

Definition 1.14: Let X and X^l be any two sets. Let f : X→ X^l be any function and let A = $\langle V_A^+ , V_A^- \rangle$ be a bipolar valued vague subset in X, V = $\langle V_V^+ , V_V^- \rangle$ be a bipolar valued vague subset in f(X) = X^l, defined by V_V^+(y) = $r\sup_{x\in f^{-1}(y)} V_A^+(x)$ and V_V^-(y) = $r\inf_{x\in f^{-1}(y)} V_A^-(x)$, for all x in X and y in X^l. A is called a preimage of V under f and is defined as V_A^+(x) = V_V^+(f(x)), V_A^-(x) = V_V^-(f(x)) for all x in X and is denoted by f^{-1}(V).

2. SOME THEOREMS

Theorem 2.1: Let R and R^l be any two semirings. The homomorphic image of a bipolar valued vague subsemiring of R is a bipolar valued vague subsemiring of R^l.

Proof: Let f : R→ R^l be a homomorphism. Let V = f(A) = $\langle V_V^+ , V_V^- \rangle$, where A = $\langle V_A^+ , V_A^- \rangle$ is a bipolar valued vague subsemiring of R. We have to prove that V is a bipolar valued vague subsemiring of R^l. Now for f(x), f(y) in R^l, V_V^+(f(x)+f(y)) = V_V^+(f(x+y)) ≥ V_A^+(x+y) ≥ rmin{V_A^+(x), V_A^+(y)} = rmin{V_V^+(f(x)), V_V^+(f(y)) } which implies that V_V^+(f(x)+f(y)) ≥ rmin{V_V^+(f(x)), V_V^+(f(y))}. And V_V^+(f(x)f(y)) = V_V^+(f(xy)) ≥ V_A^+(xy) ≥ rmin{V_A^+(x), V_A^+(y)}= rmin { V_V^+(f(x)), V_V^+(f(y)) } which implies that V_V^+(f(x)f(y)) ≥ rmin{ V_V^+(f(x)), V_V^+(f(y)) }. Also V_V^-(f(x)+f(y)) = V_V^-(f(x+y)) ≤ V_A^-(x+y) ≤ rmax {V_A^-(x), V_A^-(y) } = rmax{ V_V^-(f(x)), V_V^-(f(y))} which implies that V_V^-(f(x)+f(y)) ≤ rmax{V_V^-(f(x)), V_V^-(f(y))}. And V_V^-(f(x)f(y)) = V_V^-(f(xy)) ≤ V_A^-(xy) ≤ rmax{V_A^-(x), V_A^-(y)} = rmax{V_V^-(f(x)), V_V^-(f(y))} which implies that V_V^-(f(x)f(y)) ≤ rmax{V_V^-(f(x)), V_V^-(f(y)) }. Hence V is a bipolar valued vague subsemiring of R^l.

2.2 Theorem: Let R and R^l be any two semirings. The homomorphic preimage of a bipolar valued vague subsemiring of R^l is a bipolar valued vague subsemiring of R.

Proof: Let f: R → R^l be a homomorphism. Let V = f(A) = $\langle V_V^+ , V_V^- \rangle$ where V is a bipolar valued vague subsemiring of R^l. We have to prove that A = $\langle V_A^+ , V_A^- \rangle$ is a bipolar valued vague subsemiring of R. Let x and y in R. Now V_A^+(x+y) = V_V^+(f(x+y)) = V_V^+(f(x)+f(y)) ≥ rmin {V_V^+(f(x)), V_V^+(f(y))}= rmin{V_A^+(x), V_A^+(y)} which implies that V_A^+(x+y) ≥ rmin{V_A^+(x), V_A^+(y)}. And V_A^+(xy) = V_V^+(f(xy)) = V_V^+(f(x)f(y)) ≥ rmin {V_V^+(f(x)), V_V^+(f(y))}= rmin{V_A^+(x), V_A^+(y)} which implies that V_A^+(xy) ≥ rmin {V_A^+(x), V_A^+(y)}. Also V_A^-(x+y) = V_V^-(f(x+y)) = V_V^-(f(x)+f(y)) ≤ rmax{V_V^-(f(x)), V_V^-(f(y))} = rmax{V_A^-(x), V_A^-(y)} which implies that V_A^-(x+y) ≤ rmax{V_A^-(x), V_A^-(y)}. And V_A^-(xy) = V_V^-(f(xy)) = V_V^-(f(x)f(y)) ≤ rmax{V_V^-(f(x)), V_V^-(f(y))} = rmax{V_A^-(x), V_A^-(y)} which implies that V_A^-(xy) ≤ rmax{V_A^-(x), V_A^-(y)}. Hence A is a bipolar valued vague subsemiring of R.

2.3 Theorem: Let R and R^{l} be any two semirings. The antihomomorphic image of a bipolar valued vague subsemiring of R is a bipolar valued vague subsemiring of R^{l}.

Proof: Let f : R $\rightarrow$ R^{l} be an antihomomorphism. Let V = f(A) = $\langle V_V^+, V_V^- \rangle$ where A = $\langle V_A^+, V_A^- \rangle$ is a bipolar valued vague subsemiring of R. We have to prove that V is a bipolar valued vague subsemiring of R^{l}. Now for f(x), f(y) in R^{l}, V_V^+ (f(x)+f(y)) = V_V^+ (f(y+x)) $\geq$ V_A^+ (y+x) $\geq$ rmin {V_A^+ (x), V_A^+ (y)} = rmin {V_V^+ (f(x)), V_V^+ (f(y))} which implies that V_V^+ (f(x)+f(y)) $\geq$ rmin{ V_V^+ (f(x)), V_V^+ (f(y)) }. And V_V^+ (f(x)f(y)) = V_V^+ (f(yx)) $\geq$ V_A^+ (yx) $\geq$ rmin {V_A^+ (x), V_A^+ (y)} = rmin {V_V^+ (f(x)), V_V^+ (f(y)) } which implies that V_V^+ (f(x)f(y)) $\geq$ rmin {V_V^+ (f(x)), V_V^+ (f(y))}. Also V_V^- (f(x)+f(y)) = V_V^- (f(y+x)) $\leq$ V_A^- (y+x) $\leq$ rmax{ V_A^- (x), V_A^- (y)} = rmax{V_V^- (f(x)), V_V^- (f(y))} which implies that V_V^- (f(x)+f(y)) $\leq$ rmax{V_V^- (f(x)), V_V^- (f(y))}. And V_V^- (f(x)f(y)) = V_V^- (f(yx)) $\leq$ V_A^- (yx) $\leq$ rmax{V_A^- (x), V_A^- (y)} = rmax{V_V^- (f(x)), V_V^- (f(y))} which implies that V_V^- (f(x)f(y)) $\leq$ rmax{ V_V^- (f(x)), V_V^- (f(y))}. Hence V is a bipolar valued vague subsemiring of R^{l}.

2.4 Theorem: Let R and R^{l} be any two semirings. The antihomomorphic preimage of a bipolar valued vague subsemiring of R^{l} is a bipolar valued vague subsemiring of R.

Proof: Let f : R $\rightarrow$ R^{l} be an antihomomorphism. Let V = f(A) = $\langle V_V^+, V_V^- \rangle$ where V is a bipolar valued vague subsemiring of R^{l}. We have to prove that A = $\langle V_A^+, V_A^- \rangle$ is a bipolar valued vague subsemiring of R. Let x and y in R. Now V_A^+ (x+y) = V_V^+ (f(x+y)) = V_V^+ (f(y)+f(x)) $\geq$ rmin{ V_V^+ (f(x)), V_V^+ (f(y))} = rmin{V_A^+ (x), V_A^+ (y)} which implies that V_A^+ (x+y) $\geq$ rmin{V_A^+ (x), V_A^+ (y)}. And V_A^+ (xy) = V_V^+ (f(xy)) = V_V^+ (f(y)f(x)) $\geq$ rmin {V_V^+ (f(x)), V_V^+ (f(y))} = rmin{ V_A^+ (x), V_A^+ (y)} which implies that V_A^+ (xy) $\geq$ rmin {V_A^+ (x), V_A^+ (y)}. Also V_A^- (x+y) = V_V^- (f(x+y)) = V_V^- (f(y)+f(x)) $\leq$ rmax{V_V^- (f(x)), V_V^- (f(y))} = rmax{V_A^- (x), V_A^- (y)} which implies that V_A^- (x+y) $\leq$ rmax{V_A^- (x), V_A^- (y)}. And V_A^- (xy) = V_V^- (f(xy)) = V_V^- (f(y)f(x)) $\leq$ rmax{V_V^- (f(x)), V_V^- (f(y))} = rmax {V_A^- (x), V_A^- (y)} which implies that V_A^- (xy) $\leq$ rmax{V_A^- (x), V_A^- (y)}. Hence A is a bipolar valued vague subsemiring of R.

2.5 Theorem: Let R and R^{l} be any two semirings. The homomorphic image of a bipolar valued vague normal subsemiring of R is a bipolar valued vague normal subsemiring of R^{l}.

Proof: Let f : R $\rightarrow$ R^{l} be a homomorphism. Let V = f(A) = $\langle V_V^+, V_V^- \rangle$ where A = $\langle V_A^+, V_A^- \rangle$ is a bipolar valued vague normal subsemiring of R. We have to prove that V is a bipolar valued vague normal subsemiring of R^{l}. By Theorem 2.1, V is a bipolar valued vague subsemiring of R^{l}. Now for f(x), f(y) in R^{l}, V_V^+ (f(x)+f(y)) = V_V^+ (f(x+y)) $\geq$ V_A^+ (x+y) = V_A^+ (y+x) $\leq$ V_V^+ (f(y+x)) = V_V^+ (f(y)+f(x)) which implies that V_V^+ (f(x)+f(y)) = V_V^+ (f(y)+f(x)). And V_V^+ (f(x)f(y)) = V_V^+ (f(xy)) $\geq$ V_A^+ (xy) = V_A^+ (yx) $\leq$ V_V^+ (f(yx)) = V_V^+ (f(y)f(x)) which implies that V_V^+ (f(x)f(y)) = V_V^+ (f(y)f(x)). Also V_V^- (f(x)+f(y)) = V_V^- (f(x+y)) $\geq$ V_A^- (x+y) = V_A^- (y+x) $\leq$ V_V^- (f(y+x)) = V_V^- (f(y)+f(x)) which implies that V_V^- (f(x)+f(y)) = V_V^- (f(y)+f(x)). And V_V^- (f(x)f(y)) = V_V^- (f(xy)) $\geq$ V_A^- (xy) = V_A^- (yx) $\leq$ V_V^- (f(yx)) = V_V^- (f(y)f(x)) which implies that V_V^- (f(x)f(y)) = V_V^- (f(y)f(x)). Hence V is a bipolar valued vague normal subsemiring of R^{l}.

2.6 Theorem: Let R and R^{l} be any two semirings. The homomorphic preimage of a bipolar valued vague normal subsemiring of R^{l} is a bipolar valued vague normal subsemiring of R.

Proof: Let f : R $\rightarrow$ R^{l} be a homomorphism. Let V = f(A) = $\langle V_V^+, V_V^- \rangle$ where V is a bipolar valued vague normal subsemiring of R^{l}. We have to prove that A = $\langle V_A^+, V_A^- \rangle$ is a bipolar valued vague normal subsemiring of R. By Theorem 2.2, A = $\langle V_A^+, V_A^- \rangle$ is a bipolar valued vague subsemiring of R. Let x and y in R. Now V_A^+ (x+y) = V_V^+ (f(x+y)) = V_V^+ (f(x)+f(y)) = V_V^+ (f(y)+f(x)) = V_V^+ (f(y+x)) = V_A^+ (y+x) which implies that V_A^+ (x+y) = V_A^+ (y+x).

And V_A^+(xy) = V_V^+(f(xy)) = V_V^+(f(x)f(y)) = V_V^+(f(y)f(x)) = V_V^+(f(yx)) = V_A^+(yx) which implies that V_A^+(xy) = V_A^+(yx). Also V_A^-(x+y) = V_V^-(f(x+y)) = V_V^-(f(x)+f(y)) = V_V^-(f(y)+f(x)) = V_V^-(f(y+x)) = V_A^-(y+x) which implies that V_A^-(x+y) =V_A^-(y+x). And V_A^-(xy) = V_V^-(f(xy)) = V_V^-(f(x)f(y)) = V_V^-(f(y)f(x)) = V_V^-(f(yx)) = V_A^-(yx) which implies that V_A^-(xy) = V_A^-(yx). Hence A is a bipolar valued vague normal subsemiring of R.

2.7 Theorem: Let R and R^l be any two semirings. The antihomomorphic image of a bipolar valued vague normal subsemiring of R is a bipolar valued vague normal subsemiring of R^l.

Proof: Let f : R $\to$ R^l be an antihomomorphism. Let V = f(A) = $\langle V_V^+, V_V^- \rangle$ where A = $\langle V_A^+, V_A^- \rangle$ is a bipolar valued vague normal subsemiring of R. We have to prove that V is a bipolar valued vague normal subsemiring of R^l. By Theorem 2.3, V is a bipolar valued vague subsemiring of R^l. Now for f(x), f(y) in R^l, V_V^+(f(x)+f(y)) = V_V^+(f(y+x)) $\geq$ V_A^+(y+x) = V_A^+(x+y) $\leq$ V_V^+(f(x+y)) = V_V^+(f(y)+f(x)) which implies that V_V^+(f(x)+f(y)) = V_V^+(f(y)+f(x)). And V_V^+(f(x)f(y)) = V_V^+(f(yx)) $\geq$ V_A^+(yx) = V_A^+(xy)$\leq$ V_V^+(f(xy)) = V_V^+(f(y)f(x)) which implies that V_V^+(f(x)f(y)) = V_V^+(f(y)f(x)). Also V_V^-(f(x)+f(y)) = V_V^-(f(y+x)) $\leq$ V_A^-(y+x) = V_A^-(x+y) $\geq$ V_V^-(f(x+y)) = V_V^-(f(y)+f(x)) which implies that V_V^-(f(x)+f(y)) = V_V^-(f(y)+f(x)). And V_V^-(f(x)f(y)) = V_V^-(f(yx))$\leq$ V_A^-(yx) = V_A^-(xy)$\geq$ V_V^-(f(xy)) = V_V^-(f(y)f(x)) which implies that V_V^-(f(x)f(y)) = V_V^-(f(y)f(x)). Hence V is a bipolar valued vague normal subsemiring of R^l.

2.8 Theorem: Let R and R^l be any two semirings. The antihomomorphic preimage of a bipolar valued vague normal subsemiring of R^l is a bipolar valued vague normal subsemiring of R.

Proof: Let f: R $\to$ R^l be an antihomomorphism. Let V = f(A) = $\langle V_V^+, V_V^- \rangle$ where V is a bipolar valued vague normal subsemiring of R^l. We have to prove that A = $\langle V_A^+, V_A^- \rangle$ is a bipolar valued vague normal subsemiring of R. By Theorem 2.4, A = $\langle V_A^+, V_A^- \rangle$ is a bipolar valued vague subsemiring of R. Let x and y in R.

Now V_A^+(x+y) = V_V^+(f(x+y)) = V_V^+(f(y)+f(x)) = V_V^+(f(x)+f(y)) = V_V^+(f(y+x)) = V_A^+(y+x) which implies that V_A^+(x+y) = V_A^+(y+x). And V_A^+(xy) = V_V^+(f(xy)) = V_V^+(f(y)f(x)) = V_V^+(f(x)f(y)) = V_V^+(f(yx)) = V_A^+(yx) which implies that V_A^+(xy) = V_A^+(yx). Also V_A^-(x+y) = V_V^-(f(x+y)) = V_V^-(f(y)+f(x)) = V_V^-(f(x)+f(y)) = V_V^-(f(y+x)) = V_A^-(y+x) which implies that V_A^-(x+y) = V_A^-(y+x). And V_A^-(xy) = V_V^-(f(xy)) = V_V^-(f(y)f(x)) = V_V^-(f(x)f(y)) = V_V^-(f(yx)) = V_A^-(yx) which implies that V_A^-(xy) = V_A^-(yx). Hence A is a bipolar valued vague normal subsemiring of R.

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