International Journal of Mathematical Archive-12(3), 2021, 1-5
Available online through www.ijma.info ISSN 2229 – 5046

MULTIPLICATIVE SOMBOR INDICES OF CERTAIN NANOTUBES

V. R. KULLI*

Department of Mathematics, Gulbarga University, Gulbarga - 585106, India.

(Received On: 11-01-21; Revised & Accepted On: 20-02-21)

ABSTRACT

A topological index is a numeric quantity from structural graph of a molecule. In this paper, we introduce the multiplicative Sombor index, multiplicative modified Sombor index, multiplicative reduced Sombor index and multiplicative reduced modified Sombor index of a graph. Furthermore we compute these multiplicative Sombor indices for certain nanotubes.

Mathematics Subject Classification: *05C05, 05C07, 05C35.*

Keywords: *multiplicative Sombor index, multiplicative modified Sombor index, multiplicative reduced Sombor index, nanotube.*

1. INTRODUCTION

We consider only finite, simple connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. For undefined term and notation, we refer the book [1].

A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical Graph Theory is a branch of Mathematical Chemistry, which has an important affect on the development of the Chemical Sciences. Topological indices are useful for establishing correlation between the structure of a molecular compound and its physicochemical properties. Several topological indices [2] have been considered in Theoretical Chemistry and have found some applications, especially in QSPR/QSAR study, see [3, 4, 5].

The Sombor index [6] was introduced and defined it as

$$SO(G) = \sum_{uv \in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2}.$$

The modified Sombor index was introduced by Kulli at al. in [7], defined as

$$^{m}SO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_G(u)^2 + d_G(v)^2}}.$$

Recently, some Sombor indices were studied in [8, 9, 10, 11, 12].

Inspired by work on Sombor indices, we introduce the multiplicative Sombor index, multiplicative modified Sombor index, general multiplicative first (a, b)-KA index of a graph as follows:

The multiplicative Sombor index of a graph G is defined as

$$SOII(G) = \prod_{uv \in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2}.$$

The multiplicative modified Sombor index of a graph G is defined as

$$^{m}SOII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_G(u)^2 + d_G(v)^2}}.$$

Corresponding Author: V. R. Kulli*
Department of Mathematics, Gulbarga University, Gulbarga - 585106, India.

The multiplicative first and second (*a, b*)-*KA* indices of a graph *G* were defined by Kulli in [13], as

$$KA_{a,b}^{1}II(G)=\prod_{uv\in E(G)}\left[d_G(u)^a+d_G(v)^a\right]^b,$$

$$KA_{a,b}^{2}II(G)=\prod_{uv\in E(G)}\left[d_G(u)^a\,d_G(v)^a\right]^b.$$

Motivated by the definition of the reduced indices, we introduce the multiplicative reduced Sombor index, multiplicative reduced modified Sombor index, general multiplicative reduced first (*a, b*)-*KA* index of a graph as follows:

The multiplicative reduced Sombor index of a graph *G* is defined as

$$RSOII(G)=\prod_{uv\in E(G)}\sqrt{\left(d_G(u)-1\right)^2+\left(d_G(v)-1\right)^2}.$$

The multiplicative reduced modified Sombor index of a graph *G* is defined as

$$^{m}RSOII(G)=\prod_{uv\in E(G)}\frac{1}{\sqrt{\left(d_G(u)-1\right)^2+\left(d_G(v)-1\right)^2}}.$$

The multiplicative reduced first and second (*a, b*)-*KA* indices of a graph *G* are defined as

$$RKA_{a,b}^{1}II(G)=\prod_{uv\in E(G)}\left[\left(d_G(u)-1\right)^a+\left(d_G(v)-1\right)^a\right]^b,$$

$$RKA_{a,b}^{2}II(G)=\prod_{uv\in E(G)}\left[\left(d_G(u)-1\right)^a\left(d_G(v)-1\right)^a\right]^b.$$

Recently some reduced indices were studied in [14, 15, 16] and also some multiplicative indices were studied in [17, 18, 19, 20, 21, 22].

In this paper, we compute the multiplicative Sombor index, multiplicative modified Sombor index, multiplicative reduced Sombor index, multiplicative reduced modified Sombor index, multiplicative first (*a, b*)-*KA* index, multiplicative reduced first (*a, b*)-*KA* index of $HC_5C_7[p, q]$ and $SC_5C_7[p, q]$ nanotubes. For nanotubes, see [23].

2. Results For HC_5C_7 [*p, q*] NANOTUBES

In this section, we focus on $HC_5C_7\,[p, q]$ nanotubes in which *p* is the number of heptagons in the first row and *q* rows of pentagons repeated alternately. The 2-dimensional lattice of nanotube $HC_5C_7[8, 4]$ is shown in Figure 1.

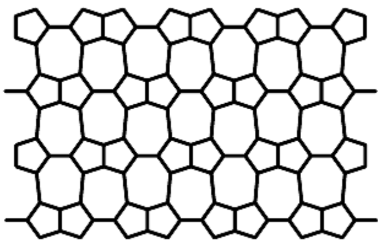

Figure-1: 2-*D* lattice of HC_5C_7 [8, 4] nanotube.

Let *G* be the graph of a nanotube $HC_5C_7[p, q]$. By calculation, *G* has $4pq$ vertices and $6pq - p$ edges. Also by calculation, there are two types of edges based on the degree of the vertices of each edge as given in Table 1.

$d_G(u), d_G(v)\backslash uv \in E(G)$	(2, 3)	(3, 3)
Number of edges	$4p$	$6pq - 5p$

Table-1: Edge partition of *G*

In the following Theorem, we compute the general multiplicative first (*a, b*)-*KA* index of a nanotue $HC_5C_7[p, q]$.

Theorem 1: The general first (a, b)-KA index of $HC_5C_7[p, q]$ nanotubes is given by

$$KA_{a,b}^{1}II\left(HC_5C_7\left[p,q\right]\right)=\left(2^a+3^a\right)^{4bp}\times\left(2\times3^a\right)^{b(6pq-5p)}.$$

Proof: Let $G = HC_5C_7[p, q]$. By definition and by using Table 1, we deduce

$$\begin{aligned}KA_{a,b}^{1}II\left(HC_5C_7\left[p,q\right]\right)&=\prod_{uv\in E(G)}\left[d_G(u)^a+d_G(v)^a\right]^b\\&=\left(2^a+3^a\right)^{b4p}\times\left(3^a+3^a\right)^{b(6pq-5p)}\\&=\left(2^a+3^a\right)^{4bp}\times\left(2\times3^a\right)^{b(6pq-5p)}.\end{aligned}$$

Using Theorem 2, we establish the following results.

Corollary 1.1: The multiplicative Sombor index of $HC_5C_7[p, q]$ is

$$SOII\left(HC_5C_7\left[p,q\right]\right)=13^{2p}\times\left(\sqrt{18}\right)^{(6pq-5p)}.$$

Corollary 1.2: The modified Sombor index of $HC_5C_7[p, q]$ is

$${}^{m}SOII\left(HC_5C_7\left[p,q\right]\right)=\left(\frac{1}{13}\right)^{2p}\times\left(\frac{1}{\sqrt{18}}\right)^{6pq-5p}.$$

In the following Theorem, we determine the general multiplicative reduced first (a, b)-KA index of $HC_5C_7[p, q]$.

Theorem 2: The general reduced first (a, b)-KA index of $HC_5C_7[p, q]$ is given by

$$RKA_{a,b}^{1}II\left(HC_5C_7\left[p,q\right]\right)=\left(1+2^a\right)^{4pb}\times\left(2\times2^a\right)^{b(6pq-5p)}.$$

Proof: Let $G = HC_5C_7[p, q]$. By definition and by using Table 1, we deduce

$$\begin{aligned}RKA_{a,b}^{1}II\left(HC_5C_7\left[p,q\right]\right)&=\prod_{uv\in E(G)}\left[\left(d_G(u)-1\right)^a+\left(d_G(v)-1\right)^a\right]^b\\&=\left((2-1)^a+(3-1)^a\right)^{b\times4p}\times\left((3-1)^a+(3-1)^a\right)^{b(6pq-5p)}\\&=\left(1+2^a\right)^{4pb}\times\left(2\times2^a\right)^{b(6pq-5p)}.\end{aligned}$$

The following results are obtained from Theorem 2.

Corollary 2.1: The multiplicative reduced Sombor index of $HC_5C_7[p, q]$ is

$$RSOII\left(HC_5C_7\left[p,q\right]\right)=5^{2p}\times\left(\sqrt{8}\right)^{(6pq-5p)}.$$

Corollary 2.2: The multiplicative reduced modified Sombor index of $HC_5C_7[p, q]$ is

$${}^{m}SOII\left(HC_5C_7\left[p,q\right]\right)=\left(\frac{1}{5}\right)^{2p}\times\left(\frac{1}{\sqrt{8}}\right)^{6pq-5p}.$$

2. RESULTS FOR $SC_5C_7[p,q]$ NANOTUBES

In this section, we focus on $SC_5C_7[p, q]$ nanotubes, in which p is the number of heptagons in the first row and q rows of vertices and edges are repeated alternately. The 2-dimensional lattice of nanotube $SC_5C_7[8,4]$ is shown in Figure 2.

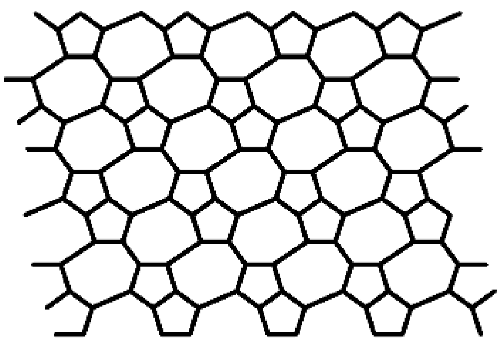

Figure-2: 2-D lattice of nanotube $SC_5C_7[8,4]$

Let G be the graph of $SC_5C_7[p, q]$. By calculation, we obtain that G has $4pq$ vertices and $6pq - p$ edges. Also by calculation, there are three types of edges based on the degree of end vertices of each edge as given in Table 2.

$d_G(u), d_G(v) \backslash uv \in E(G)$	(2, 2)	(2, 3)	(3, 3)
Number of edges	q	$6q$	$6pq - p - 7q$

Table-2: Edge partition of G

In the following theorem, we compute the general multiplicative first (a,b)-KA index of $SC_5C_7[p,q]$.

Theorem 3: The general multiplicative first (a, b)-KA index of $SC_5C_7[p, q]$ nanotubes is given by

$$KA^1_{a,b}II\left(SC_5C_7\left[p,q\right]\right) = \left(2\times 2^a\right)^{bq} \times \left(2^a + 3^a\right)^{6bq} \times \left(2\times 3^a\right)^{b(6pq-p-7q)}.$$

Proof: Let $G = SC_5C_7[p, q]$. By definition and Table 2, we deduce

$$KA^1_{a,b}II\left(SC_5C_7\left[p,q\right]\right) = \prod_{uv\in E(G)} \left[d_G(u)^a + d_G(v)^a\right]^b.$$

$$= \left(2^a + 2^a\right)^{bq} \times \left(2^a + 3^a\right)^{b6q} \times \left(3^a + 3^a\right)^{b(6pq-p-7q)}.$$

$$= \left(2\ \times 2^a\right)^{bq} \times \left(2^a + 3^a\right)^{6bq} \times \left(2\times 3^a\right)^{b(6pq-p-7q)}.$$

From Theorem 3, the following results are established.

Corollary 3.1: The multiplicative Sombor index of $SC_5C_7[p, q]$ is

$$SOII\left(SC_5C_7\left[p,q\right]\right) = \left(\sqrt{8}\right)^q \times \left(\sqrt{13}\right)^{6q} \times \left(\sqrt{18}\right)^{6pq-p-7q}.$$

Corollary 3.2: The multiplicative modified Sombor index of $SC_5C_7[p, q]$ is

$${}^mSOII\left(SC_5C_7\left[p,q\right]\right) = \left(\frac{1}{\sqrt{8}}\right)^q \times \left(\frac{1}{\sqrt{13}}\right)^{6q} \times \left(\frac{1}{\sqrt{18}}\right)^{6pq-p-7q}.$$

In the following theorem, we determine the general multiplicative reduced first (a, b)-KA index of $SC_5C_7[p, q]$.

Theorem 4: The general multiplicative reduced first (a, b)-KA index of $SC_5C_7[p, q]$ is given by

$$RKA^1_{a,b}II\left(SC_5C_7\left[p,q\right]\right) = 2^{bq} \times \left(1 + 2^a\right)^{b6q} \times \left(2\times 2^a\right)^{b(6pq-p-7q)}.$$

Proof: Let $G = SC_5C_7[p, q]$. By definition and using Table 2, we obtain

$$RKA^1_{a,b}II\left(SC_5C_7\left[p,q\right]\right) = \prod_{uv\in E(G)} \left[\left(d_G(u)-1\right)^a + \left(d_G(v)-1\right)^a\right]^b.$$

$$= \left[(2-1)^a + (2-1)^a\right]^{bq} \times \left[(2-1)^a + (3-1)^a\right]^{b6q} \times \left[(3-1)^a + (3-1)^a\right]^{b(6pq-p-7q)}.$$

$$= 2^{bq} \times \left(1 + 2^a\right)^{b6q} \times \left(2\times 2^a\right)^{b(6pq-p-7q)}.$$

From Theorem 4, we establish the following results.

Corollary 4.1: The multiplicative reduced Sombor index of $SC_5C_7[p, q]$ is

$$RSOII\left(SC_5C_7\left[p,q\right]\right) = \left(\sqrt{2}\right)^q \times (5)^{3q} \times \left(\sqrt{8}\right)^{6pq-p-7q}.$$

Corollary 4.2: The multiplicative reduced modified Sombor index of $SC_5C_7[p, q]$ is

$${}^mRSOII\left(SC_5C_7\left[p,q\right]\right) = \left(\frac{1}{\sqrt{2}}\right)^q \times \left(\frac{1}{5}\right)^{3q} \times \left(\frac{1}{\sqrt{8}}\right)^{6pq-p-7q}.$$

4. CONCLUSION

In this study, we have introduced the multiplicative Sombor index, multiplicative modified Sombor index, multiplicative reduced Sombor index, multiplicative reduced modified Sombor index, multiplicative first (a, b)-KA index, multiplicative reduced first (a, b)-KA index of a molecular graph. Also these multiplicative indices for $HC_5C_7[p, q]$ and $SC_5C_7[p, q]$ nanotubes are computed.

REFERENCES

1. V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
2. V.R.Kulli, Graph indices, in *Hand Book of Research on Advanced Applications of Application Graph Theory in Modern Society*, M. Pal. S. Samanta and A. Pal, (eds.) IGI Global, USA (2020) 66-91.
3. I. Gutman and O.E. Polansky, *Mathematical Concepts in Organic Chemistry,* Springer, Berlin (1986).
4. V.R.Kulli, *Multiplicative Connectivity Indices of Nanostructures*, LAP LEMBERT Academic Publishing (2018).
5. R.Todeschini and V. Consonni, *Hand Book of Molecular Descriptors for Chemoinformatics*, Wiley-VCH, Weinheim, (2000).
6. I. Gutman Geometric approach to degree based topological indices Sombor indices, *MATCH Common. Math. Comput. Chem* 86(2021) 11-16.
7. V.R.Kulli and I.Gutman, Computation of Sombor indices of certain networks, *SSRG International Journal of Applied Chemistry,* 8(1) (2021) 1-5.
8. K.C.Das, A.S.Cevik, I.N.Cangul and Y.Shang, On Sombor index, *Symmetry,* 13 (2021) 140.
9. I.Gutman, Some basic properties of Sombor indices, *Open Journal of Discrete Applied Mathematics,* 4(1) (2021) 1-3.
10. V.R.Kulli Sombor indices of certain graph operators, *International Journal of Engineering Sciences and Research Technology,* 10(1) (2021) 127-134.
11. I.Milovanovic, E.Milovanovic and M.Matejic, On some mathematical properties of Sombor indices, *Bull. Int. Math. Virtual Inst.* 11(2) (2021) 341-353.
12. T.Reti, T.Doslic and A.Ali, On the Sombor index of graphs, *Contributions of Mathematics,* 3 (2021) 11-18.
13. V.R.Kulli, Multiplicative (a, b)-KA indices of certain dendrimer nanostars, *International Journal of Recent Scientific Research* 10(11-E) (2019) 36010-36013.
14. S.Ediz, On the reduced first Zagreb index of graphs, Pacific J. Appl. Math. 8(2) (2016) 99-102.
15. B. Furtula, I. Gutman and S. Ediz, On difference of Zagreb indices, *Discrete Appl. Math*. 178, (2014) 83-88.
16. V.R.Kulli General reduced second Zagreb index of certain networks, International Journal of Current Research in Life Sciences, 7(11) (2018) 2827-2833.
17. W.Gao, Y. Wang and W. Wang, and Li Shi, The first multiplication atom bond connectivity index of molecular structures in drugs, *Saudi Pharmaceutical Journal*, (2017), http://dx.doi.org/10.1016/j.jsps.2017.04.021.
18. V.R.Kulli, Multiplicative connectivity KV indices of dendrimers, *Journal of Mathematics and Informatics*, 15(2019) 1-7.
19. V.R.Kulli, Multiplicative Gourava indices of armchair and zigzag polyhex nanotubes, *Journal of Mathematics and Informatics*, 17(2019) 107-112.
20. V.R.Kulli, Computation of Multiplicative (a, b)-status index of certain graphs, *Journal of Mathematics and Informatics*, 18(2020) 50-55.
21. V.R.Kulli, Computing some Multiplicative temperature indices of certain networks, *Journal of Mathematics and Informatics*, 18(2020) 139-143.
22. V.R.Kulli, Computation of Multiplicative minus F-indices of titania nanotubes, *Journal of Mathematics and Informatics*, 18(2020) 135-140.
23. V.R.Kulli, On the product connectivity Revan index of certain nanotubes, *Journal of Computer and Mathematical Sciences,* 8(10) (2017) 562-567.

Source of support: Nil, Conflict of interest: None Declared.