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ANISOTROPIC BIANCHI TYPE-III COSMOLOGICAL MODEL IN SCALE COVARIANT THEORY OF GRAVITATION

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ABSTRACT

The spatially homogeneous and anisotropic Bianchi type-III metric is considered in the context of sting source in the scale covariant theory of gravitation framed by Canuto et al. [1977], with the help of special law of variation for Hubble's parameter proposed by Berman [1983]. In this theory, a cosmological model with a negative constant deceleration parameter is derived. Finally,also discussed some physical properties.

Keywords: *Bianchi type –III metric, Cosmic string, Scale covariant theory of gravitation.*

1. INTRODUCTION

Canuto *et al.*[6] formulated a scale covariant theory of gravitation which also admits a variable *G* and which is a viable alternative to general relativity within the frame work of the scale-covariant theory, the cosmological constant $\wedge$ appears as a variable parameter.Einstein's General Relativity Theory on the other hand does not admit the possibility of variable *G* or variable $\wedge$ .The field equations in scale-covariant theory with zero cosmological constant are

$$R_{ij} - \frac{1}{2} R g_{ij} + f_{ij}(\phi) = -8\pi G(\phi) T_{ij}, \tag{1}$$

and

$$\phi^2 f_{ij} = 2\phi\phi_{i;j} - 4\phi_i\phi_j - g_{ij}\left(\phi\phi^k_{;k} - \phi^k\phi_k\right). \tag{2}$$

In which ϕ is the scale function satisfying $0 < \phi < \infty$ and other symbols have their usual meanings as in Riemannian geometry. Where R_{ij} is the Ricci tensor, R is the Ricci scalar, G the gravitational constant and T_{ij} is the energy-momentum tensor. A semicolon denotes covariant derivative and denotes the ordinary derivative with respect to x^i. A particular feature of this theory is that no independent equation for ϕ exists.

The possibilities that have been considered for gauge functions ϕ are

$$\phi(t) = \left(\frac{t_0}{t}\right)^{\varepsilon} \quad , \varepsilon = \pm 1, \pm\frac{1}{2}, \tag{3}$$

where t_0 is the constant. The form $\phi \approx t^{\frac{1}{2}}$ is the one most supported to fit observations. In the work of Canuto *et al.* [6], Beesham [2, 3, 4], Reddy *et al.* [16], Mishra, [11] and Raju *et al.* [15] a detailed analysis of scale covariant theory is contained.

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For topological limitations, the grand unified theories of elementary particles give astrophysical roles.After a series of spontaneous asymmetry breaking, the symmetries of elementary particles resulted from a large symmetry group is known as basic premise of grand unification. In a cosmological context this implies that the early universe has gone through a number of phase transitions. Such type of transitions lead to the formation of topologically stable defect such as, cosmic strings, vacuum domain walls, monopoles, textures and other 'hybrid'creatures. To study the results of cosmic strings is one of the most outstanding problems in cosmology as the vacuum strings generate density perturbations which are strong enough for the formation of galaxies. Moreover, the large scale network of strings during the early universe does not contradicts the present-day observations. These strings together with gravitational field and stress energy generate gravitational effect. Letelier[9] introduce the general relativistic treatment of strings and he constructed for classical massive stringsthe energy momentum tensor. He showed in Bianchi type I and Kantowski-Sachs space-time cosmological solution of massive string.Khadekar *et al*. [8], Adhav [1], Reddy *et al*. [17], Mete *et al*. [10], Pund *et al*. [14], Kanatavalli *et al*. [7], Nimkar *et al*. [13] and Mishra *et al*. [12] have been broadly discussed the gravitational effects of cosmic strings.

In this paper, we study Bianchi type-III cosmological model with a negative constant deceleration parameter with the help of Hubble's special law of variation proposed by Berman [5].

2. METRIC AND FIELD EQUATION

We consider the Bianchi type-III space-time in the form

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2ax}dy^2 - C^2dz^2, \tag{4}$$

where A, B, C are functions of time t alone and a is constant.

The energy momentum tensor for cosmic strings is given by Letelier, [9]

$$T_i^j = \rho\, u_i u^j - \lambda\, x_i x^j, \tag{5}$$

where ρ is the rest energy density of cloud of strings with particles attached to them, $\rho = \rho_p + \lambda$, ρ_p being the rest energy density of particles attached to the strings and λ the tension density of the system of strings. As pointed out by Letelier [9], λ may be positive or negative, u^i describes the system four-velocity and x^i represents a direction of anisotropy, i.e. the direction of the strings.

We have

$$u^i u_i = -x^i x_i = 1 \text{ , and } u^i x_i = 0 \text{ .}$$

We consider

$$\rho = \rho_p + \lambda,$$

where ρ_p is the rest energy density of the particles attached to the string.

Here ρ and λ are the functions of t only.

Using the co-moving coordinate system, the non-vanishing components T_j^i can be obtained as

$$T_1^1 = 0, T_2^2 = 0, T_3^3 = \lambda, T_4^4 = \rho \tag{6}$$

Using the equations (1), (2) and (6) the field equations of metric (4) are

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} + \left(\frac{B_4}{B} + \frac{C_4}{C} - \frac{A_4}{A}\right)\frac{\phi_4}{\phi} + \frac{\phi_{44}}{\phi} - \left(\frac{\phi_4}{\phi}\right)^2 = 0, \tag{7}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} + \left(\frac{A_4}{A} - \frac{B_4}{B} + \frac{C_4}{C}\right)\frac{\phi_4}{\phi} + \frac{\phi_{44}}{\phi} - \left(\frac{\phi_4}{\phi}\right)^2 = 0, \tag{8}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} - \frac{a^2}{A^2} + \left(\frac{A_4}{A} + \frac{B_4}{B} - \frac{C_4}{C}\right)\frac{\phi_4}{\phi} + \frac{\phi_{44}}{\phi} - \left(\frac{\phi_4}{\phi}\right)^2 = -8\pi G\lambda, \tag{9}$$

$$\frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} + \frac{B_4C_4}{BC} - \frac{a^2}{A^2} + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)\frac{\phi_4}{\phi} = 8\pi G\rho, \tag{10}$$

$$\frac{A_4}{A} - \frac{B_4}{B} = 0 \tag{11}$$

$$\dot{\rho} + \rho\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) - \lambda\frac{\dot{C}}{C} = 0 \tag{12}$$

Where the subscript '4' after *A*, *B* and *C* denotes ordinary differentiation with respect to *t*.

From equation (11), we have

$$A = B\ . \tag{13}$$

With the help of equation (13), the set of equation (7) - (10) reduces to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \left(\frac{C_4}{C}\right)\frac{\phi_4}{\phi} + \frac{\phi_{44}}{\phi} - \left(\frac{\phi_4}{\phi}\right)^2 = 0\,, \tag{14}$$

$$2\frac{B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 - \frac{a^2}{B^2} + \left(2\frac{B_4}{B} - \frac{C_4}{C}\right)\frac{\phi_4}{\phi} + \frac{\phi_{44}}{\phi} - \left(\frac{\phi_4}{\phi}\right)^2 = -8\pi G\lambda\,, \tag{15}$$

$$\left(\frac{B_4}{B}\right)^2 + 2\frac{B_4 C_4}{BC} - \frac{a^2}{B^2} + \left(2\frac{B_4}{B} + \frac{C_4}{C}\right)\frac{\phi_4}{\phi} = 8\pi G\rho\ . \tag{16}$$

The Spatial Volume *V* and the average scale factor a for the metric (4) is defined as

$$a(t) = (B^2 C)^{1/3}\,, \tag{17}$$

$$V = a^3 = (B^2 C)\,. \tag{18}$$

The Hubble parameter is given by

$$H = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{1}{3}\left(2\frac{B_4}{B} + \frac{C_4}{C}\right), \tag{19}$$

where $H_1 = H_2 = \frac{B_4}{B}$ *and* $H_3 = \frac{C_4}{C}$ are the directional Hubble parameters in the direction of x, y *and* $z-axes$ respectively.

The average anisotropic parameter is defined as

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \text{ where } \Delta H_i = H_i - H\ . \tag{20}$$

The expansion scalar θ and shear scalar σ are given by

$$\theta = 2\frac{B_4}{B} + \frac{C_4}{C}\,, \tag{21}$$

and

$$\sigma^2 = \frac{3}{2}A_m H^2\,. \tag{22}$$

3. SOLUTION OF THE FIELD EQUATION

The field equations (14) to (16) are three equations in five unknown B, C, ϕ, ρ & λ. Hence to get a determinate solution, we assume that shear scalar σ is proportional to expansion scalar θ gives,

$$\sigma \prec \theta\,,$$

$$\sigma = K_1\theta\,. \tag{23}$$

Using equation (20) and (21) we get,

$$B = C^{\frac{\sqrt{3}K_1+1}{1-2\sqrt{3}K_1}}\,. \tag{24}$$

We consider the constant deceleration parameter for the model is defined $q = -\dfrac{aa_{44}}{{a_4}^2}$ = constant, (25)

where a is the average scale factor.

From equation (24) average scale factor a is obtained

$$a = (K_2 t + K_3)^{\frac{1}{1+q}}, \text{ for } q \neq -1, \tag{26}$$

where $K_2 (\neq 0)$ and K_3 are constants of integrations.

Further on solving equations (14) - (16) and using (17) and (26) we get metric coefficients as

$$A = (K_2 t + K_3)^{\left(\frac{\sqrt{3}k_1 + 1}{q+1}\right)} \tag{27}$$

$$B = (K_2 t + K_3)^{\left(\frac{\sqrt{3}k_1 + 1}{q+1}\right)} \tag{28}$$

$$C = (K_2 t + K_3)^{\left(\frac{1 - 2\sqrt{3}\,K_1}{1+q}\right)} \tag{29}$$

Using equation (27) - (29) Bianchi type-III cosmological model (4) takes the form

$$ds^2 = dt^2 - (K_2 t + K_3)^{2\left(\sqrt{3}\,K_1 + 1\right)/1+q} dx^2 - (K_2 t + K_3)^{2\left(\sqrt{3}\,K_1 + 1\right)/1+q} e^{-2ax} dy^2 - (K_2 t + K_3)^{\left(\frac{\left(1-2\sqrt{3}\,K_1\right)}{1+q}\right)} dz^2$$

Through a proper choice of coordinates and constants of integration, the above equation reduces to

$$ds^2 = dt^2 - T^{\frac{2(\sqrt{3k_1}+1)}{1+q}} dx^2 - T^{\frac{2(\sqrt{3k_1}+1)}{1+q}} e^{-2ax} dy^2 - T^{2\left(\frac{1-2\sqrt{3k_1}}{1+q}\right)} dz^2 \tag{30}$$

where $T = K_2 t + K_3$

4. PHYSICAL AND KINEMETICAL PROPERTIES FOR THE MODEL

The energy density ρ and tension density λ for the model (30) are given

$$\rho = \lambda = \frac{K_4}{T^{2\left(\frac{1}{1+q}\right)\left(\sqrt{3}K_1 + 1\right)}} \tag{31}$$

Spatial Volume $V = (T)^{\frac{3}{(1+q)}}$ (32)

Expansion Scalar $\theta = \dfrac{3K_2}{(1+q)} \dfrac{1}{(T)}$ (33)

Hubble Parameter $H = \dfrac{K_2}{(1+q)} \dfrac{1}{(T)}$ (34)

Shear Scalar. $\sigma = \dfrac{3K_2\ K_1}{(1+q)} \dfrac{1}{(T)}$ (35)

It may be observed that at initial moment, when *T=0* the spatial volume will be zero while the energy density diverges. when $T \to 0$,then expansion scalar θ,shear scalar σ^2 and the Hubble's parameter H tends to ∞.

Also, Since $\lim\limits_{T \to \infty} \left(\dfrac{\sigma}{\theta}\right) \neq 0$ and hence the model does not approach isotropy for large values of *T*. The scalar field ϕ increases indefinitely as time $T \to \infty$ and is free from initial singularity.

Graphs are plotted for particular values of the physical parameters and Time.

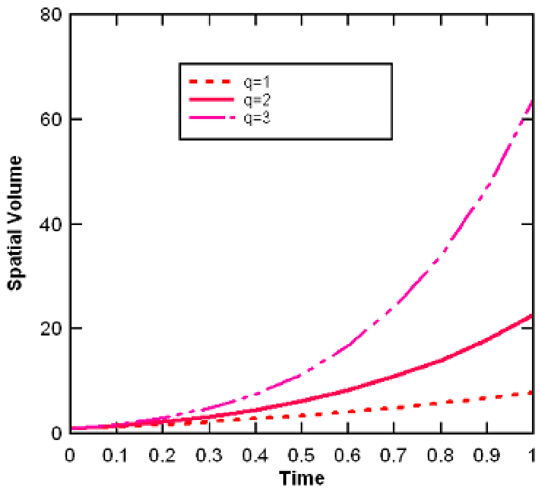

Fig.-1: Plot of Spatial Volume Vs. Time

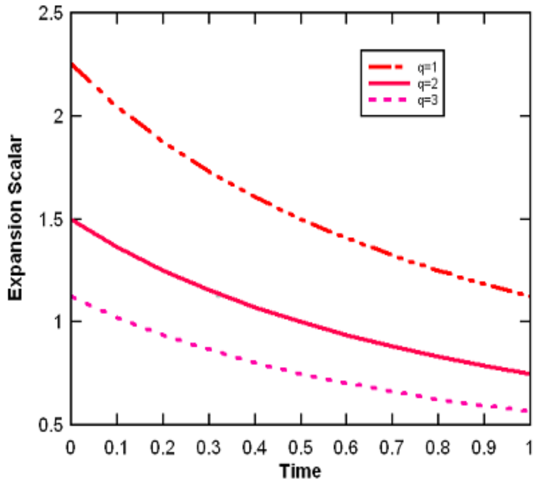

Fig.-2: Plot of Expansion Scalar Vs. Time

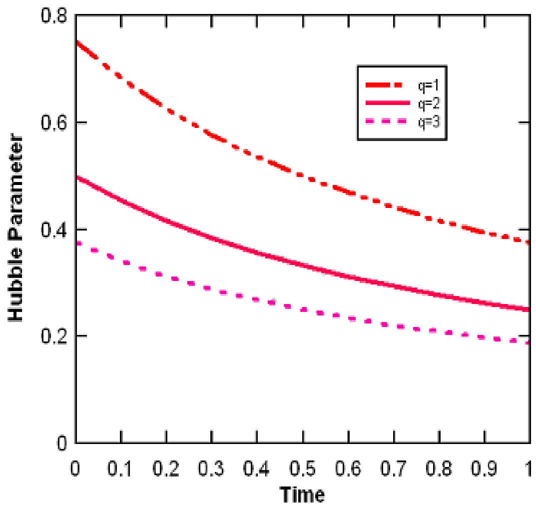

Fig.-3: Plot of Hubble Parameter Vs. Time

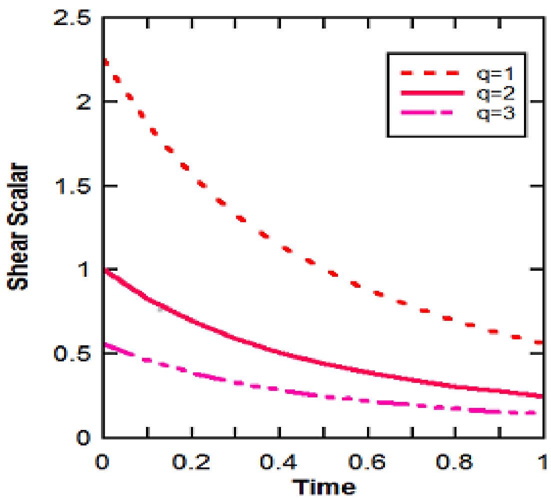

Fig.-4: Plot of Shear Scalar Vs. Time

This shows that there is an inverse relation between spatial volume and each Expansion Scalar, Hubble parameter and Shear Scalar. If we take $T = K_2 t + K_3$.

5. CONCLUSION

Basically our motivation to investigate Bianchi type-III cosmological model in the presence of cosmic string in the scale covariant theory of gravitation formulated by Canuto *et al.* [6] with the help of special law of variation for the Hubble's parameter proposed by Berman [5].The model is free from initial singularities and they are expanding, shearing and non- rotating in the standard way. Also, it is interesting to note that as T gradually increases, the scalar expansion θ, Hubble parameter $H$and shear scalar σ^2 decrease and finally they vanish when $T \to \infty$.

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