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INTUITIONISTIC FUZZY GENERALIZED SEMI STAR CLOSED SETS

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ABSTRACT

*In this paper, we define and study about a new class of Intuitionistic fuzzy closed sets called Intuitionistic fuzzy g*s closed sets in Intuitionistic fuzzy topological space, which are stronger forms of IFGS closed sets. The study centers around general properties of IFg*s closed sets. Furthermore we study the relationship of IFg*s closed sets with already defined IFCSs in IFTS. We also introduce the concept of IFg*s open sets.*

Key words: *IFg*s closed sets, IFg*s open sets.*

AMS Mathematics Subject Classification – *54A40.*

I. INTRODUCTION

The concept of fuzzy sets was introduced by L. A Zadeh [16]. Whereas it was C. L. Chang [4] who established a generalization of Fuzzy Sets in the topological space as Fuzzy Topological Space. Further in recent times enough research is being done in Intuitionistic Fuzzy Sets and Intuitionistic Fuzzy Topological Spaces which are also generalizations to Fuzzy Sets. These two concepts were introduced by Atanassov [2] and Coker [5] respectively.

In this paper we, introduce the concept of Intuitionistic fuzzy g*s closed (open) sets in Intuitionistic fuzzy topological space, which happen to lie between the class of all IF Semi closed (IF Semi open) sets and the class of all IFGS closed (IFGS open) sets. We study some general properties of IFg*sC (IFg*sO) sets. Also we discuss the relationship between these sets with other existing Intuitionistic fuzzy closed and open sets.

II. PRELIMINARIES

Definition2.1: [2] An intuitionistic fuzzy set(IFS) A is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x\in X\}$where the functions $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x\in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x\in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X. An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x\in X\}$.

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Definition2.2:[2] Let A and B be two IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x\in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x\in X\}$. Then,

(a) $A\subseteq B$ ifand only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu(x)$ for all $x\in X$,
(b) $A = B$ if and only if $A\subseteq B$ and $A\supseteq B$,
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x\in X\}$,
(d) $A\cup B = \{\langle x, \mu_A(x)\vee\mu_B(x), \nu_A(x)\wedge\nu_B(x)\rangle : x\in X\}$,
(e) $A\cap B = \{\langle x, \mu_A(x)\wedge\mu_B(x), \nu_A(x)\vee\nu_B(x)\rangle : x\in X\}$.

The intuitionistic fuzzy sets $\tilde{\mathbf{0}} = \langle x, 0, 1\rangle$and $\tilde{\mathbf{1}} = \langle x, 1, 0\rangle$are respectively the empty set and the whole set of X.

Definition 2.3: [5] Anintuitionistic fuzzy topology (IFT) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $\tilde{\mathbf{0}}, \tilde{\mathbf{1}}\in \tau$,
(ii) $G1 \cap G2 \in \tau$ for any $G1, G2 \in \tau$,
(iii) $\cup G_i\in \tau$for any family $\{G_i : i\in I\}\in\tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS) in X. The complement A^c of an IFOS in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.4: [15] Two IFSs A and B are said to be q-coincident (A_qB) if and only if there exits an element $x\in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.5: [15] Two IFSs A and B are said to be not q-coincident (A_qB) if and only if $A\subseteq B^c$.

Definition 2.6: [6] An intuitionistic fuzzy point (IFP), written as $p(\alpha, \beta)$, is defined to be an IFS of X given by

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha,\beta) & if\ p = x \\ (0,1) & otherwise. \end{cases}$$

An intuitionistic fuzzy point $p_{(\alpha,\beta)}$ is said to belong to a IFS
$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x\in X\}$,denoted by $p_{(\alpha,\beta)}(x) \in A$ if $\alpha \le \mu_A$ and $\beta \ge \nu_A$.

Definition 2.7: [5] Let (X, τ) be an IFTS and $A = \langle x, \mu_A(X), \nu_A(x)\rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$Iint(A) = \cup \{G / G$ is an IFOS in X and $G\subseteq A\}$,
$Icl(A) = \cap\{K / K$ is an IFCS in X and $A \subseteq K\}$.
It is to be noted that for any IFS A in (X, τ), we have
$Icl(A^c) = (Iint(A))^c$ and $Iint(A^c) = (Icl(A))^c$.

Proposition 2.8: [5] For any IFSs A and B in (X, τ), we have

(1) $Iint(A) \subseteq A$
(2) $A \subseteq Icl(A)$
(3) A is an IFCS in X $\Leftrightarrow Icl(A)= A$
(4) A is an IFOS in X $\Leftrightarrow Iint(A) = A$
(5) $A \subseteq B \Rightarrow Iint(A) \subseteq Iint(B)$ and $Icl(A) \subseteq Icl(B)$
(6) $Iint(Iint(A)) = Iint(A)$
(7) $Icl(Icl(A)) = Icl(A)$
(8) $Icl(A \cup B) = Icl(A) \cup Icl(B)$
(9) $Iint(A\cap B) = Iint(A) \cap Iint(B)$

Proposition 2.9: [3] For any IFS A in (X, τ), we have

(1) $Iint(\tilde{\mathbf{0}}) = \tilde{\mathbf{0}}$and $Icl(\tilde{\mathbf{0}}) = \tilde{\mathbf{0}}$,
(2) $Iint(\tilde{\mathbf{1}},) = \tilde{\mathbf{1}},$ and$Icl(\tilde{\mathbf{1}},) = \tilde{\mathbf{1}},$
(3) $(Iint(A)))^C = Icl(A^C)$,
(4) $(Icl(A))^C = Iint(A^C)$

Definition 2.10: [8]An IFS $A = \{\langle x, \mu_A(X), \nu_A(x)\rangle : x \in X\}$in an IFTS (X, τ) issaid to be an

(i) intuitionistic fuzzy semi closed set (IFSCS) if $Iint(Icl(A)) \subseteq A$,
(ii) intuitionistic fuzzy pre closed set(IFPCS) if $Icl(Iint(A)) \subseteq A$,
(iii) intuitionistic fuzzy αclosed set(IFαCS) if $Icl(Iint(Icl(A))) \subseteq A$,
(iv) intuitionistic fuzzy regular closed set (IFRCS) if $Icl(Iint(A)) = A$,

Definition 2.11: [16] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy α generalized closed set (IFGCS)if$Icl(A)\subseteq U$ whenever $A\subseteq U$ and U is an IFOS in X.

Definition 2.12: [11] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized closed set (IFαGCS) if Iαcl(A)⊆U whenever A⊆U and U is an IFOS in X.

Definition 2.13: [14] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi closed set (IFGSCS) if *IScl*(A) ⊆ U, whenever A ⊆ U and U is an IFOS in X.

Definition 2.14: [1] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi regular closed set (IFGSRCS) if *IScl*(A) ⊆ U whenever A ⊆ U and U is an IFROS in X.

Definition 2.15: [7] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized star closed set (IFG*CS) if *Icl*(A) ⊆ U whenever A ⊆ U and U is an IFGOS in X.

Definition 2.16: [12] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized star closed set (IFG*CS) if *IScl*(A) ⊆ U whenever A ⊆ U and U is an IFGSO in X.

Definition 2.17: [8] An IFS A in an IFTS (X, τ) is said to be an
(i) intuitionistic fuzzy semi open set (IFSOS) if A⊆ *Icl*(*Iint*(A)),
(ii) intuitionistic fuzzy pre-open set(IFPOS) if A ⊆ *Iint*(*Icl*(A)),
(iii) intuitionistic fuzzy α open set(IFαOS) if A⊆ *Iint*(*Icl*(*Iint*(A))),
(iv) intuitionistic fuzzy regular open set (IFROS) if A = *Iint(Icl(A)).*

Definition 2.18: [12] Let A = {⟨x, μ_A, ν_A⟩ be IFS in an IFTS (X, τ). Then the semi interior and semi closure of A are defined a
ISint(A) =∪{G / G is an IFSOS in X and G ⊆A},
IScl(A) = ∩{K / K is an IFSCS in X and A⊆K}.
It is to be noted that for any IFS A in (X, τ), we have
IScl(A^c) = (*ISint*(A))^c and
ISint(A)^c = (*IScl*(A))^c.

Result 2.19: [12] Let A be IFS in (X, τ), then
IScl(A)=A∪*Iint*(*Icl*(A)) and
ISint(A)=A∩ *Icl*(*Iint*(A))

III. INTUITIONISTIC FUZZY G*S-CLOSED SET

In this section we introduce Intuitionistic fuzzy g*semi-closed sets and study some of their Properties.

Definition 3.1: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is called an intuitionistic fuzzy g*s -closed if *IScl* (A) ⊆ Q whenever A ⊆ Q and Q is intuitionistic fuzzy g-open in X.

Example3.2: Let X = {a, b} and intuitionistic fuzzy set M is defined as follows
M = { < x, (0.5,0.6) ,(0.5,0.4) >}
Let τ = {$\tilde{\mathbf{0}}$, M, $\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X.
Then P = {< x, (0.3, 0.4), (0.7, 0.6) >} is intuitionistic fuzzy g*s closed set.

Theorem 3.3: Every intuitionistic fuzzy semi closed set is intuitionistic fuzzy g*s closed but converse is not true.

Proof: Let A is intuitionistic fuzzy semi closed set. Let A ⊆ U and U is intuitionistic fuzzy g-open sets in X. Since A is intuitionistic fuzzy semi closed set we have A = *IScl*(A). Hence *IScl*(A) ⊆ U, whenever A ⊆ U and U is intuitionistic fuzzy g open in X. Therefore A is intuitionistic fuzzy g*s-closed set.

Example 3.4: Let X = {a, b} and intuitionistic fuzzy set P is defined as followsP = {< x, (0.3, 0.4), (0.7, 0.6) >}
Let τ = {$\tilde{\mathbf{0}}$, P,$\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X.
Then the intuitionistic fuzzy set Q = {<x, (0.3, 0.2) ,(0.7, 0.8) >} is intuitionistic fuzzy g*s closed set. But *Iint* (*Icl*(Q))is not a subset of Q . Therefore Q is not a intuitionistic fuzzy semiclosed set.

Theorem 3.5: Every intuitionistic fuzzy closed set is intuitionistic fuzzy g*s-closed but converse is not true.

Proof: Let A is intuitionistic fuzzy closed set. Let A ⊆ U and U is intuitionistic fuzzy g-open sets in X. Since A is intuitionistic fuzzy -closed set we have A= *Icl*(A). But *IScl*(A) ⊆*Icl*(A) , therefore *IScl*(A) ⊆ U. whenever A ⊆ U and U is intuitionisticfuzzy g-open in X. Hence A is intuitionistic fuzzy g*s-closed set.

Example 3.6: Let X = {a, b} and intuitionistic fuzzy set A is defined as A = { < x,(0.2,0.3), (0.8,0.7) >}
Let $\tau = \{\tilde{\mathbf{0}}, A, \tilde{\mathbf{1}}\}$ be an intuitionistic fuzzy topology on X.
Then the intuitionistic fuzzy set Q = {<x, (0.6, 0.7), (0.4, 0.3) >} is intuitionistic fuzzy g*s –closed set. But *Icl*(Q) ≠ Q, therefore it is not intuitionistic fuzzy closed.

Theorem 3.7: Every intuitionistic fuzzy α-closed set is intuitionistic fuzzy g*s closed but converse is not true.

Proof: Let A is intuitionistic fuzzy α-closed set. Let A ⊆ U and U is intuitionistic fuzzy g-open sets in X. We know that every α closed set is semi closed set. By Theorem 3.3, Every intuitionistic fuzzy semi closed set is IFg*s closed set. Therefore A is intuitionistic fuzzy g*s-closed set.

Example 3.8: Let X = {a, b} and $\tau = \{\tilde{\mathbf{0}}, A, \tilde{\mathbf{1}}\}$ be an IFTS on X, where A= < x, (0.1, 0.3), (0.6, 0.5) >.
Then the IFS S= < x, (0.5, 0.4),(0.2,0.3) > is an IF g*s-closed set.
But *Icl*(*Iint*(*Icl*(S)))⊈ S. Therefore S is not IF α-closed set.

Theorem 3.9: Every intuitionistic fuzzy regular -closed set is intuitionistic fuzzy g*s-closed but converse is not true.

Proof: Let A be intuitionistic fuzzy regular closed set. We know that every intuitionistic fuzzy regular closed set is intuitionistic fuzzy closed set. Then by Theorem 3.6, we get A is intuitionistic fuzzy g*s- closed set.

Example 3.10: Let X = {a, b} and intuitionistic fuzzy set O is defined as O = {< x, (0.2, 0.4), (0.3, 0.6) >}.
Let $\tau = \{\tilde{\mathbf{0}}, O, \tilde{\mathbf{1}}\}$be an intuitionistic fuzzy topology on X.
Then the intuitionistic fuzzy set A = {< x, (0.1, 0.2), (0.4, 0.5) >} is intuitionistic fuzzy g*s closed set.
But *Icl*(*Iint* (A)) ≠ A, therefore it is not intuitionistic fuzzy regular-closed set .

Theorem 3.11: Every intuitionistic fuzzy g*-closed set is intuitionistic fuzzy g*s-closed but converse is not true.

Proof: Let A is intuitionistic fuzzy g*-closed set. Let A ⊆ U and U is intuitionistic fuzzy g*-open sets in X.
By definition of intuitionistic fuzzy g*-closed set, cl(A) ⊆ U. Note that *IScl*(A) ⊆ cl(A) is always true.
Now we have *IScl*(A) ⊆ U, whenever A⊆ U, U is IFg* closed set. Hence A is intuitionistic fuzzy g*s-closed set.

Example 3.12: Let X = {a, b} and intuitionistic fuzzy set A is defined as G = {< x, (0.3, 0.4), (0.7, 0.5) >}

Let $\tau = \{\tilde{\mathbf{0}}, G, \tilde{\mathbf{1}}\}$be an intuitionistic fuzzy topology on X. Then the IFS S = <x, (0.3, 0.5), (0.6, 0.4) > is intuitionistic fuzzy g*s –closed set. Then take a intuitionistic fuzzy g-open set M= {<x, (0.6, 0.4),(0.4, 0.6). Now *Icl*(S) ⊈ M. Therefore S is not a intuitionistic fuzzy g*-closed.

Theorem 3.13: Every intuitionistic fuzzy g*s-closed set is intuitionistic fuzzy GS-closed but converse is not true.

Proof: Let A is intuitionistic fuzzy g*s-closed set. Let A⊆ U and U is intuitionist fuzzy open sets in X. Since every intuitionistic fuzzy open set is intuitionistic fuzzy g- open sets, U is intuitionistic fuzzy g- open sets such that A⊆ U.

Now by definition of intuitionistic fuzzy g*s-closed sets IScl(A) ⊆ U. We have *IScl*(A) ⊆U whenever A ⊆ U and U is intuitionistic fuzzy open in X. Therefore A is intuitionistic fuzzy GS-closed set.

Example 3.14: Let X = {a, b} and intuitionistic fuzzy sets O is defined as follows O = {< x, (0.5, 0.6), (0.5, 0.4) >}
Let $\tau = \{\tilde{\mathbf{0}}, O, \tilde{\mathbf{1}}\}$ be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set
P = { < x, (0.6, 0.6), (0.4, 0.4) >} is intuitionistic fuzzy GS-closed but it is not intuitionistic fuzzy g*s-closed .

Theorem3.15: Every intuitionistic fuzzy g*s-closed set is IFGSR –closed but converse is not true.

Proof: Let A is intuitionistic fuzzy g*s-closed set. Let A ⊆ U and U is intuitionistic fuzzy regular open sets in X. Since every intuitionistic fuzzy regular open set is intuitionistic fuzzy g- open sets, U is intuitionistic fuzzy g- open sets such that A ⊆U. By definition of intuitionistic fuzzy g*s-closed sets *IScl*(A) ⊆ U. We have *IScl*(A) ⊆ U, whenever A ⊆ U and U is intuitionistic fuzzy regular open in X. Therefore A is IFGSR- closed set.

Example 3.16: Let X = {a, b} and intuitionistic fuzzy sets O is defined as O = {< x, (0.3, 0.3), (0.4, 0.4 >}
Let $\tau = \{0, O, \tilde{\mathbf{1}}\}$ be an intuitionistic fuzzy topology on X. And let Q = {<x, (0.1, 0.1),(0.5,0.6)>} be any IFS in X.

Let U be any IFRO. Then *IScl*(Q) ⊆ U Hence Q is IFGSRC. Let P = {<x, (0.2, 0.2), (0.8, 0.8 >}. Then P is an IFGO set. Also Q⊆P, *IScl*(Q) = O ⊈ P. Hence Q is not IFg*sCS.

Theorem 3.17: Every intuitionistic fuzzy g*s-closed set is intuitionistic fuzzy SG –closed but converse is not true.

Proof: Let A is intuitionistic fuzzy g*s-closed set. Let A ⊆U and U is intuitionist fuzzy semi open sets in X.

Since every intuitionistic fuzzy semi open set is intuitionistic fuzzy g- open sets, U is intuitionistic fuzzy g- open sets such that A ⊆U. By definition of intuitionistic fuzzy g*s-closed sets *IScl*(A) ⊆ U. We have *IScl*(A) ⊆ U, whenever A ⊆U and U is intuitionistic fuzzy semi open in X. Therefore A is intuitionistic fuzzy SG-closed set.

Example 3.18: Let X = {a, b} and intuitionistic fuzzy sets O is defined as O = {< x, (0.3, 0.4), (0.4, 0.5> }
Let τ= {$\tilde{\mathbf{0}}$, O, $\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X. And let Q = {<x, (0.1,0.2),(0.5,0.6)>} be any IFS in X.
Then *IScl*(Q)= O. Clearly *IScl* (Q) ⊆ O, whenever Q ⊆O for all IFSOS O in X. Q is IFSG- closedset in (X,τ) but not intuitionistic fuzzy g*s-closed.

Theorem 3.19: Every intuitionistic fuzzy g*s-closed set is intuitionistic fuzzy GSP-closed but converse is not true.

Proof: Let A is intuitionistic fuzzy g*s-closed set. Let A⊆ U and U is intuitionistic fuzzy open sets in X. Since every intuitionistic fuzzy open set is intuitionistic fuzzy g-open, U is intuitionistic fuzzy g-open set Now by definition of intuitionistic fuzzy g*s-closed sets *IScl*(A) ⊆U. Note that *ISPcl*(A)⊆*IScl*(A) is always true. We have *ISPcl*(A)⊆U whenever A ⊆ U and U is intuitionistic fuzzy open in X. Therefore A is intuitionistic fuzzy GSP-closed set.

Example 3.20: Let X = {a, b} and τ={$\tilde{\mathbf{0}}$, U, V,$\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X,
where U={<x,(o.5,0.6),(0.5,0.4)>}, V= {<x,(0.6,0.7),(0.4,0.3).Then the intuitionistic fuzzy set
A ={<x,(0.6,0.6),(0.4,0.4)>} is IFGSP-closed but it is not intuitionistic fuzzy g*s- closed.

Remark 3.21: From the above discussion and known results we have the following diagram of implications;

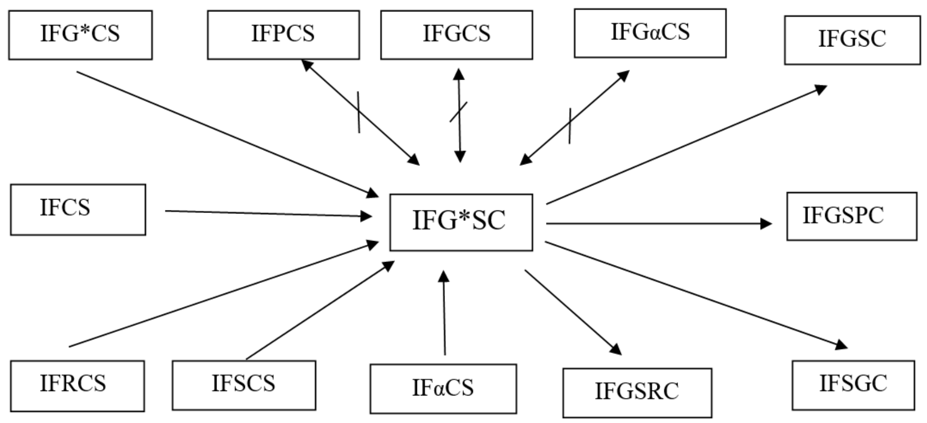

Figure 1: Relationship between intuitionistic fuzzy g*s closed set and other existing intuitionistic fuzzy closed sets.

Where A ⟶ B represents A implies B and A↮ B represents A and B are independent.
Remark 3.22: IFPCS and IFg*sCS are independent

Example 3.23: Let X={a,b}. Let τ = {$\tilde{\mathbf{0}}$, V, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X,
where V = {<x,(0.8,0.3) ,(0.2,0.6)>}. Consider a IFS W={<x,(0.2,0.4),(0.7,0.6)>}.Then W is IFPCS but not IFg*sCS.

Example 3.24: Let X={a,b}. Let τ = {$\tilde{\mathbf{0}}$, V, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X,
where V = {<x,(0.2,0.3) ,(0.8,0.7)>}. Consider a IFS W={<x,(0.6,0.7),(0.4,0.3)>}.Then W is IFg*sCS but not IFPCS.

Remark 3.25: IFGCS and IFg*sCS are independent.

Example 3.26: Let X={a,b}. Let τ = {$\tilde{\mathbf{0}}$, V, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X,
where V = {<x,(0.5,0.6) ,(05,0.4)>}. Consider a IFS W={<x,(0.6,0.6),(0.4,0.4)>}.Then W is IFGCS but not IFg*sCS.

Example3.27: Let X={a,b}. Let τ = {$\tilde{\mathbf{0}}$, V, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X,
where V = {<x,(0.2,0.4) ,(0.3,0.6)>}. Consider a IFS W={<x,(0.1,0.2),(0.4,0.5)>}.Then W is IFg*sCS but not IFGCS.

Remark 3.28: IFGαCS and IFg*sCS are independent.

Example 3.29: Let X={a,b}. Let τ = {$\tilde{\mathbf{0}}$, V, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X,
where V = {<x,(0.2,0.2) ,(0.8,0.8)>}. Consider a IFS W={<x,(0.2,0.2),(0.8,0.8)>}.
Then W is IFg*sCS but not IFG α CS since *Iαcl* (W)=$V^c \not\subseteq$ V.

Example 3.30: Let X={a,b}. Let τ = {$\tilde{\mathbf{0}}$, V,$\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X,
where V = {<x,(0.5,0.6) ,(0.4,0.3)>}. Consider a IFS W={<x,(0.6,0.7),(0.3,0.2)>}.Then W is IFGαCS . Take an IFg-open set Q={<x,(0.7,0.8),(0.3,0.2)>}. Then *IScl* (W)=$\tilde{\mathbf{1}}\not\subseteq$Therefore W is not IFg*s-closed.

Theorem 3.31: Union of two IFg*s closed sets is again an IFg*s closed set.

Proof: Let A and B be two IFg*s closed sets in IFTS (X,). And let Q be an IFgOS in X, Such that A U B⊆ Q . Since A and B are IFg*sCSs we have, *IScl*(A) ⊆ Q &*IScl*(B) ⊆ Q. Therefore *IScl*(A) U *IScl*(B) ⊆*IScl*(A U B) ⊆ Q. Hence A U B is An IFg*s closed set.

Theorem 3.32: Intersection of two IFg*s closed sets is again an IFg*s closed set

Proof: Let A and B be two IFg*s closed sets in IFTS (X, τ) .And let A⊆Q and B ⊆ Q ,Q is an IFgOS in X, such that (A∩ B) ⊆ Q. Since A and B are IFg*sCSs we have, *IScl*(A) ⊆ Q &*IScl*(B) ⊆ Q.
Therefore *IScl*(A) ∩*IScl*(B) ⊆ *IScl*(A ∩B) ⊆ Q . Hence A ∩ B is An IFg*s closed set.

Lemma 3.33: If A is IFg-open and IFg*s-closed set in X, then A is Intuitionistic Fuzzy semi-closed set.

Proof: Since A is IFg-open and IFg*s-closed, then *IScl*(A) ⊆ A, we know that A ⊆*IScl*(A).
Therefore *IScl*(A)=A. Hence, A is IF semi closed set.

Theorem 3.34: Let A be intuitionistic fuzzy g*s-closed set in an intuitionistic fuzzy topological space (X, τ) and A ⊆ B ⊆*IScl* (A) .Then B is intuitionistic fuzzy g*s-closed in X.

Proof: Let V be an intuitionistic fuzzy g-open set in X such that B ⊆ V. Then A ⊆V and since A is intuitionistic fuzzy g*s-closed, *IScl*(A) ⊆ V. By hypothesis B ⊆*IScl* (A) then *IScl* (B) ⊆*IScl*(*IScl* (A)) .Thus *IScl* (B) ⊆*IScl*(A) This implies *IScl* (B) ⊆ V. Hence B is intuitionistic fuzzy g*s-closed set.

Theorem 3.35: Let (X, τ) be an IFTS. Then IFC(X) = IF g*sC(X) if every IFS in (X, τ) is an IFgOS in X, where IFC(X) denotes the collection of IFCSs of an IFTS (X, τ).

Proof: Suppose that every IFS in (X, τ) is an IFgOS in X. Let A∈IF g*sC(X). Then *IScl*(A) ⊆ U whenever A⊆U and U is an IFgOS in X. Since every IFS is an IFgOS, A is also an IFgOS and A ⊆A. Therefore *IScl*(A)⊆A. Hence *IScl*(A)= A. Therefore A ∈ IFC(X). Hence IF g*sC(X) ⊆ IFC(X) → (1) Let A ∈IFC(X). Then by Theorem 3.5, A∈IF g*sC(X). Hence IFC(X) ⊆ IFg*sC(X) → (2). From (1) and (2), we have IFC(X) = IFg*sC(X).

Theorem 3.36:
A subset V of X isIF g*s-closed set in X iff *IScl*(V)-V contains no non empty g-closed set in X.

Proof: Suppose that Q is a non-empty IFg-closed subset of *IScl*(A) - V. Now Q⊆*IScl*(V)-V.
Then Q ⊆*IScl*(V) ⋂V^c .Therefore Q ⊆*IScl*(V) & Q ⊆V^c .Since Q^c is IFg-open set and V is IFg*s-closed,
IScl (V) ⊆Q^c .That is Q ⊆[*IScl*(V)]^c. Hence Q ⊆*IScl*(V) ⋂ [*IScl*(V)]^c = ∅ That is Q = ∅.Thus *IScl*(V)-V contains no non empty IFg-closed set.

Conversely,
Assume that *IScl*(V)-V contains no non empty IFg-closed set. Let V ⊆K, K is IFg-open. Suppose that *IScl*(V) is not contained in K. Then *IScl*(V) ⋂K^c is a non-emptyIFg-closed set and contained in *IScl*(V)-V. Which is a contradiction. Therefore *IScl* (V) ⊆ K, and hence V is g*s-closed set.Hence proved.

Theorem3.37: In an intuitionistic fuzzy topological space (X, τ), every IFg-open set is Intuitionistic Fuzzy semi closed iff every subset of X is IFg*s- closed set.

Proof:
Necessity: Suppose that every IFg-open set is semi closed. Let A be a subset of X such that A⊆ U whenever U is IFg-open. But *IScl*(A) ⊆*IScl*(U)=U. Therefore A is IFg*s closed set.

Sufficiency: Suppose that every subset of X is IFg*s closed. Let U be IFg-open. Since U is g*s-closed, we have *IScl*(U) ⊆ U. Therefore, *IScl*(U) = U. Hence the proof.

Theorem3.38: Let A be a g*s-closed set of a topological space (X, τ). Then

(1) *ISint*(A) is g*s-closed.
(2) *Ipcl*(A) is g*s-closed.
(3) If A is regular open, then *pint(A)* and *IScl*(A) are also ĝ-closed sets.

Proof:

(1) First we note that for a subset A of (X, τ), *IScl*(A) = A $\cup$*Iint*(cl(A)) and
Ipcl(A) = A $\cup$*Icl*(*Iint*(A)). Moreover *ISint*(A) = A $\cap$*Icl*(*Iint*(A)) and *Ipint*(A) = A $\cap$*Iint*(*Icl*(A)).
Since *Icl*(*Iint*(A)) is a closedset, then A and *Icl*(*Iint*(A)) are g*s-closed sets. By the Theorem 3.32, A $\cap$*Icl*(*Iint*(A)) is also a g*s-closed set. That is *ISint*(A) is g*s closed set.

(2) *Ipcl*(A) is the union of two g*s-closed sets A and *Icl*(*Iint*(A)).
Again by the Theorem 3.31,*Ipc*l(A) is g*s-closed.

(3) Since A is regular open, then A = *Iint*(cl(A)). Then *IScl*(A) = A $\cup$*Iint*(cl(A)) = A.
Thus *IScl*(A) is g*s-closed. Similarly *Ipint*(A) is also a g*s-closed set

IV. INTUITIONISTIC FUZZY g*s -OPEN SET

In this section we introduce Intuitionistic fuzzy g*s open set and studied some of its properties.

Definition 4.1: An IFS A is said to be an Intuitionistic fuzzy g*s open set (IFg*sOS in short) in (X,τ) if the complement A^c is an IFg*sCS in X. The family of all IFg*sOSs of IFTS (X,τ)is denoted by IFg*sO(X)

Example4.2: Let X = {a, b} and intuitionistic fuzzy set M is defined as followsM = { < x, (0.5,0.6) ,(0.5,0.4) >}
Let $\tau = \{\tilde{\mathbf{0}}, M, \tilde{\mathbf{1}}\}$be an intuitionistic fuzzy topology on X.
Then P = {< x, (0.7, 0.6) , (0.3, 0.4) >} is intuitionistic fuzzy g*s open set.

Theorem 4.3: For any IFTS (X,), we have the following:
Every IFSOS is an IFg*sOS
Every IFOS is an IFg*sOS
Every IFαOS is an IFg*sOS
Every IFROS is an IFg*sOS.
Every IFg*OS is an IFg*sOS

Proof: Straight forward.

Corollary 4.4: Every IFg*sOS need not be an IFSOS in (X,τ). It is shown in the following example.

Example 4.5: Let X = {a, b} and intuitionistic fuzzy set P is defined as follows P = < x, (0.3, 0.4),(0.7, 0.6) >
Let $\tau = \{\tilde{\mathbf{0}}, P, \tilde{\mathbf{1}}\}$ be an intuitionistic fuzzy topology on X.Then the intuitionistic fuzzy set
Q = <x, (0.7, 0.8), (0.3, 0.2) > is intuitionistic fuzzy g*s open set. But *Icl*(*Iint*(Q)) = $A^c \not\subseteq$Q .
Therefore Q is not a intuitionistic fuzzy semi open set.

Corollary 4.6: Every IFg*sOS need not be an IFαOS in (X, τ). It is shown in the following example.

Example 4.7: Let X = {a, b} and intuitionistic fuzzy set A is defined as A = < x, (0.2,0.3), (0.8,0.7) >
Let $\tau = \{\tilde{\mathbf{0}}, A, \tilde{\mathbf{1}}\}$ be an intuitionistic fuzzy topology on X. Then the IFS Q = <x, (0.4,0.3) , (0.6,0.7) > is intuitionistic fuzzy g*s –open set. But *Iint*(Q) $\neq$ Q , therefore it is not intuitionistic fuzzy open set.

Corollary 4.8: Every IFg*sOS need not be an IFOS in (X,τ). It is shown in the following example.

Example 4.9: Let X = {a, b} and $\tau = \{\tilde{\mathbf{0}}, A, \tilde{\mathbf{1}}\}$ be an IFTS on X, where A= < x, (0.1,0.3), (0.6,0.5) >.
Then the IFS S= < x, (0.2, 0.3), (0.5, 0.4) > is an IF g*s-open set. But S $\not\subseteq$*Iint(Icl(Iint(S)))* =A . Therefore S is not IFα-open.

Corollary 4.10: Every IFg*sOS need not be an IFROS in (X,τ). It is shown in the following example.

Example 4.11: Let X = {a, b } and intuitionistic fuzzy set O is defined asO = < x, (0.2,0.4) , (0.3,0.6) >
Let $\tau = \{\tilde{\mathbf{0}}, O, \tilde{\mathbf{1}}\}$be an intuitionistic fuzzy topology on X. Then the IFS A = < x , (0.4,0.5) , (0.1,0.2)> is intuitionistic fuzzy g*s open set. But *Iint*(*Icl*(A)) $\neq$ A, therefore it is not intuitionistic fuzzy regular-open set.

Corollary 4.12: Every IFg*sOS need not be an IFg*OS in (X,τ). It is shown in the following example.

Example4.13: Let X = {a, b} and intuitionistic fuzzy set G is defined as G < x,(0.3,0.4), (0.7,0..5) >
Let τ = {$\tilde{\mathbf{0}}$, G, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X.
Then the intuitionistic fuzzy set S =<x, (0.6, 0.4), (0.3, 0.5) > is IF g*s –open set but not IF g*-open set.

Theorem4.14: For any IFTS (X,), we have the following:
Every IF g*s-open set IF GS-open.
Every IF g*s-open set IFGSP- open.

Corollary 4.15: Every IFGSOS need not be an IFg*sOS in (X,τ). It is shown in the following example.

Example4.16: Let X = {a, b} and intuitionistic fuzzy sets O is defined as follows O = < x,(0.5,0.6),(0.5,0.4)>.
Let τ = {$\tilde{\mathbf{0}}$, O, $\tilde{\mathbf{1}}$}be an intuitionistic fuzzy topology on X. Then the IFS P = {<x, (0.4,0.4) ,(0.6,0.6)>} is intuitionistic fuzzy GS -open but it is not intuitionistic fuzzy g*s-open.

Corollary 4.17: Every IFGSPOS need not be an IFg*sOS in (X,τ). It is shown in the following example.

Example4.18: Let X={a,b}. Let τ = {$\tilde{\mathbf{0}}$, U, V, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where
U={<x,(0.5,0.6),(0.5,0.4)>}, V= {<x,(0.6,0.7),(0.4,0.3).Then the intuitionistic fuzzy set
A = {<x,(0.4,0.4) ,(0.6,0.6)>} is IFGSP-open but it is not intuitionistic fuzzy g*s- open.

Remark4.19: IFPOS and IFg*sOS are independent.

Example 4.20: Let X={a,b}. Let τ = {$\tilde{\mathbf{0}}$, V, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where
V = {<x,(0.8,0.3) ,(0.2,0.6)>}. Consider a IFS W={<x,(0.7,0.6), (0.2,0.4)>}.Then W is IFPOS but not IFg*sOS.

Example4.21: Let X={a,b}. Let τ = {$\tilde{\mathbf{0}}$, V, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where
V = {<x,(0.2,0.3) ,(0.8,0.7)>}. Consider a IFS W={<x,(0.4,0.3) ,(0.6,0.7)>}.Then W is IFg*sOS but not IFPOS.

Remark4.22: IFGOS and IFg*sOS are independent.

Example4.23: Let X={a,b}. Let τ = {$\tilde{\mathbf{0}}$, V, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where
V = {<x,(0.5,0.6),(05,0.4)>}. Consider a IFS W={<x,(0.4,0.4) ,(0.6,0.6)>}.Then W is IFGOS but not IFg*sOS

Example4.24: Let X={a,b}. Let τ = {$\tilde{\mathbf{0}}$, V, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where
V = {<x,(0.2,0.4) ,(0.3,0.6)>}. Consider a IFS W={<x,(0.4,0.5) ,(0.1,0.2)>}.Then W is IFg*sOS but not IFGOS.

Remark4.25: IFGαOS and IFg*sOS are independent.

Example4.26: Let X={a,b}. Let τ = $\tilde{\mathbf{0}}$, V, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where
V = {<x,(0.2,0.2) ,(0.8,0.8)>}. Consider a IFS W={<x,(0.8,0.8) ,(0.2,0.2)>}.Then W is IFg*sOS but not IFGαOS.

Example4.27: Let X={a,b}. Let τ = {$\tilde{\mathbf{0}}$, V, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where
V = {<x,(0.5,0.6) ,(0.4,0.3)>}. Consider a IFS W={<x,(0.3,0.2) ,(0.6,0.7)>}.Then W is IFGαOS . Take an IFg-open set Q={<x,(0.7,0.8),(0.3,0.2)>}. Then *IScl* (W^c)=$\tilde{\mathbf{1}}\nsubseteq$ Q.Therefore W^c is not IFg*s-closed. Which implies W is not IFg*OS.

Theorem4.28: If A and B are IFg*s-open sets in an IFTS (X, τ), (A$\cap$B) and (A$\cup$B) also IFg*s-open sets in (X, τ).

Proof: Let A and B be IFg*s-open sets in IFTS (X, τ). Therefore A^c and B^c are IFg*S-closed sets in (X, τ).
By Theorem 3.31 and 3.32, ($A^c \cup B^c$) and ($A^c \cap B^c$) are IFg*S-closed sets in (X, τ).
Since ($A^c \cup B^c$) = $(A\cap B)^c$&($A^c \cap B^c$)= $(A \cup B)^c$, therefore (A$\cap$B)and (A $\cup$ B) are IFg*s open sets in (X, τ).

Theorem 4.29: A IFsubset B of a IFTS (X ,τ) is IFg*s- open if and only if K⊆*ISint*(B)
whenever K is IFg-closed and K⊆B.

Proof: Suppose that B is IFg*s - open in X, K is IFg-closed and K⊆B. Then K^c is IFg-open and
$B^c \subseteq K^c$. Since, B^c is IFg*s- closed, then*IScl*(B^c) ⊆K^c. But, *IScl*(B^c) =[*ISint*(B)]^c⊆K^c. Hence K⊆*ISint*(B).

Conversely, Suppose that K⊆*ISint*(B) whenever K⊆B and K is IFg-closed. If H is an IF g-open set in X containing B^c, then H^c is a IFg-closed setcontained in B. Hence by hypothesis, H^c⊆*ISint*(B), then by taking the complements, we have, *IScl*(B^c) ⊆H. Therefore $B^c$is IFg*s − closed in X and hence B is IFg*s-open in X.

Corollary 4.30: If B is IFg*s -open in IFTS (X,τ), then H=X, whenever H is IFg-open and *ISint*(B) U (B^c) ⊆H .

Proof: Assume that H is IFg-open and *ISint*(B) ∪(B^c) ⊆H. Hence H^c⊆*IScl*(B^c) ∩B = *IScl*(B^c) - (B^c).
Since, H^c is IFg-closed and B^c is IFg*s −closed, then by Theorem 3.35, H^c= ∅and hence, H=X.

Lemma 4.31: If B is IFg*s -closed, then *IScl*(B)-B is IFg*s-open.

Proof: Suppose that B is IFg*s -closed. Then by Theorem.3.36, *IScl*(B)−B does not contain any non-empty IFg-closed set. Therefore, *IScl*(B)−B is IFg*s-open.

Theorem4.32: Let (X,τ) be an IFTS. Then IFO(X) = IFg*sO(X) if every IFS in (X, τ) is an IFg-open in X, where IFO(X) denotes the collection of IFOSs of an IFTS(X, τ).

Proof: Suppose that every IFS in (X, τ) is an IFg-open in X. Then by theorem 3.45, we have IFC(X) = IFg*SC(X). Therefore IFO(X) = IFg*sO(X).

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