

DISCRETE MARSHALL–OLKIN LOMAX DISTRIBUTION APPLICATION OF COVID-19

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ABSTRACT

This research aims to manage the risks of spreading Corona-Virus over the world, by specifying the optimal statistical modeling to analyze the daily count of new cases of the COVID-19, therefore discrete distributions were needed. A new three-parameter discrete distribution has been improved named as a Discrete Marshall–Olkin Lomax (DMOL) distribution. Probability mass function and hazard rate are discussed. Based on the maximum likelihood estimation (MLE) for the DMOL distribution parameters are discussed. A numerical study is done using the daily count of new cases in Argentina and Uganda. Monte Carlo Simulation has been performed to evaluate the restricted sample properties of the proposed distribution.

Keywords: COVID-19; Hazard Rate; Discrete Distributions; Survival Discretization; Maximum Likelihood Estimation; Marshall–Olkin Lomax.

1. INTRODUCTION

In December 2019, Corona-Virus "COVID-19" was started in Wuhan, China, On March 11, 2020, World Health Organization (WHO) described COVID-19 as a pandemic. See Figure 1 and 2. Therefore, countries around the world have been increased their measures trying to decrease the spread rate of the COVID-19.

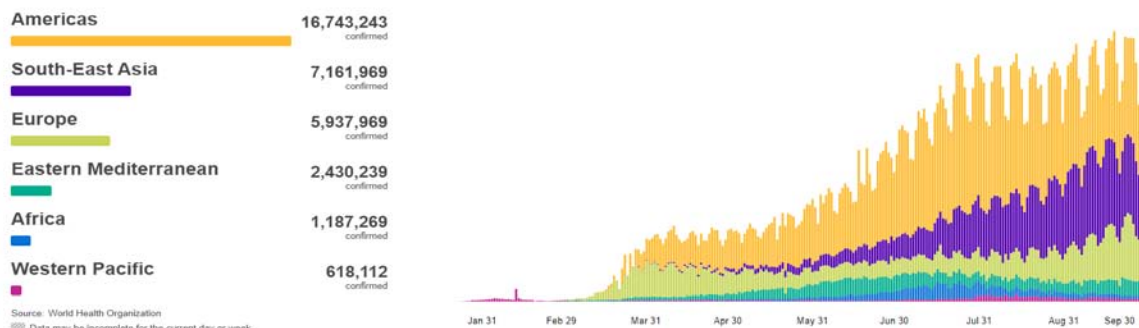


Figure 1: The situation for the daily new cases over the world by the WHO Region.

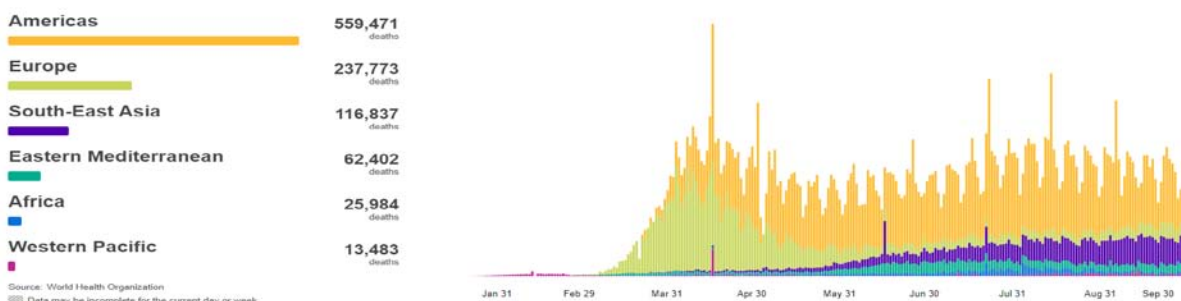


Figure-2: The situation for the daily new deaths over the world by the WHO Region.

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To model daily cases and deaths in the world, a natural discrete Lindley distribution has been introduced by Al-Babtain *et al.* (2020). A discrete Marshall–Olkin generalized exponential distribution has been introduced by Almetwally *et al.* (2020) to discussed the daily new cases of Egypt. The study was carried out by Hasab *et al.* (2020) where they used the Susceptible Infected Recovered (SIR) epidemic dynamics of the COVID-19 pandemic for modeling of the novel Coronavirus epidemic in Egypt. El-Morshedy *et al.* (2020) studied a new discrete distribution, called discrete generalized Lindley, to analyze the counts of the daily coronavirus cases in Hong Kong and daily new deaths in Iran. An autoregressive time series model based on the two-piece scale mixture normal distribution has been used by Maleki *et al.* (2020) to forecast the recovered and confirmed COVID-19 cases. The daily new COVID-19 cases in China have been predicted by Nesteruk (2020) and Batista (2020_b) by using the mathematical model, called susceptible, infected, and recovered (SIR). Batista (2020_a) used the logistic growth regression model is used for the estimation of the final size and its peak time of the coronavirus epidemic. And etc. as and Almetwally and Ibrahim (2020) and Ibrahim and Almetwally (2021).

In Lomax distribution: It is an important model for lifetime analysis, medical and business failure data, moreover it has been widely applied in a variety of contexts, it known as Lomax or Pareto type II distribution. Many authors have studied more applications by using extended Lomax distribution. Rao *et al.* (2009) studied and introduced the Marshall–Olkin extended Lomax (MOL) distribution. The CDF, PDF and hazard rate function of MOL distribution with parameters γ, δ and ϑ are given respectively, as

$$F(x; \gamma, \delta, \vartheta) = \frac{\left(1 + \frac{x}{\vartheta}\right)^\delta - 1}{\left(1 + \frac{x}{\vartheta}\right)^\delta - (1 - \gamma)}; \quad x > 0, \gamma, \delta, \vartheta > 0, \tag{1.1}$$

$$f(x; \gamma, \delta, \vartheta) = \frac{\frac{\gamma\delta}{\vartheta} \left(1 + \frac{x}{\vartheta}\right)^{\delta-1}}{\left[\left(1 + \frac{x}{\vartheta}\right)^\delta - (1 - \gamma)\right]^2}, \tag{1.2}$$

and

$$h(x; \gamma, \delta, \vartheta) = \frac{\frac{\delta}{\vartheta} \left(1 + \frac{x}{\vartheta}\right)^{\delta-1}}{\left(1 + \frac{x}{\vartheta}\right)^\delta - (1 - \gamma)}. \tag{1.3}$$

Figure 3 show the PDF and hazard rate function of the MOL distribution for different parameter. The MOL distribution is a flexible model which provides left-skewed, symmetrical, and reversed-J shaped densities. Its hazard rate function (HRF) can provide decreasing, constant, increasing and decreasing.

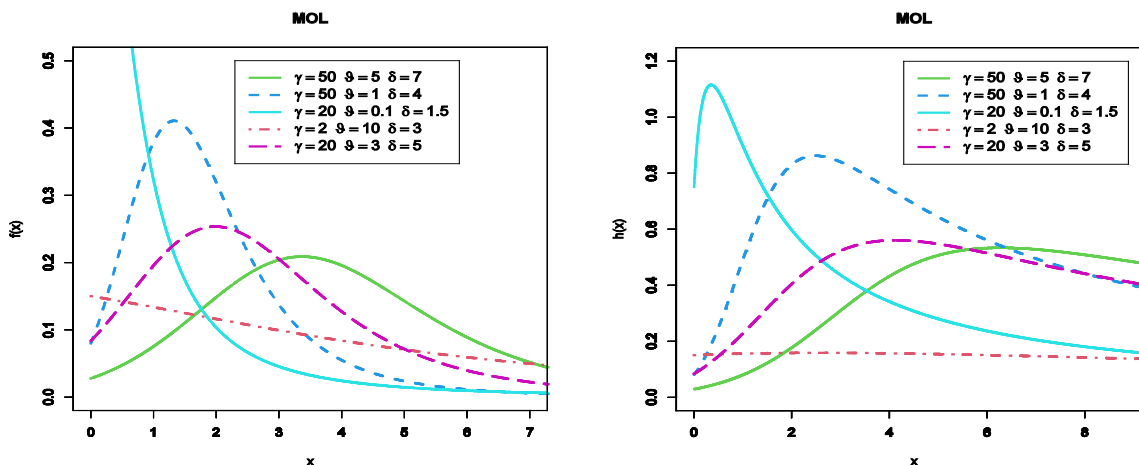


Figure-3: Plot of the PDF and HRF of the MOL distribution.

Discrete Burr type XII and discrete Lomax distributions have been introduced by Para and Jan (2016). Discrete Lomax (DL) distribution is helpful in modeling discrete data which exhibits heavy tails and can be useful in medical science and other fields. The PMF and CDF of DL distribution with parameters P and ϑ are given respectively, as

$$P(x; P, \vartheta) = P^{\ln\left(1 + \frac{x}{\vartheta}\right)} - P^{\ln\left(1 + \frac{x+1}{\vartheta}\right)}; \quad x = \mathbb{N}_0, \vartheta > 0, 0 < P < 1, \tag{1.4}$$

$$F(x; P, \vartheta) = 1 - P^{\ln\left(1 + \frac{x}{\vartheta}\right)}. \tag{1.5}$$

In order to ensure members of the Argentina and Uganda society from the risks arising from the spread of Corona-Virus in Argentina and Uganda, this study aims to model the daily new cases and deaths of the COVID-19 employing a new statistical tool. An aspect of the importance of research is the necessity of mathematical and statistical modeling of the extent and spread of the Coronavirus. The discrete Marshall–Olkin Lomax distribution will be introduced that can be denoted as DMOL distribution. In sampling tanique’s see Almongy and Almetwaly (2018) and Saleh *et al.* (2020).

The rest of the paper is organized as follows. In Section 2, the survival discretization method. Discrete Marshall–Olkin Lomax distribution and maximum likelihood estimation of DMOL are introduced in Section 3. Section 4 Simulation study by using bias and MSE to inference MLE estimator. Daily new cases of COVID-19 in the case of Argentina and Uganda are used to validate the use of models in fitting lifetime count data are presented in Section 5. Finally, conclusions are provided in Section 6.

2. SURVIVAL DISCRETIZATION METHOD

In the statistics literature, sundry methods are available to obtain a discrete distribution from a continuous one. The most commonly used technique to generate discrete distribution is called a survival discretization method, it requires the existence of cumulative distribution function (CDF), survival function should be continuous and non-negative and times are divided into unit intervals. The PMF of discrete distribution is defined in Roy (2003, 2004) as follows

$$P(X = x) = P(x \leq X \leq x + 1) = S(x) - S(x + 1); x = 0, 1, 2, \dots \tag{2.1}$$

Where $S(x) = P(X \geq x) = F(x; \Phi)$, where $F(x; \Phi)$ is a CDF of continuous distribution and Φ is a vector of parameters. The random variable X is said to have the discrete distribution if its CDF is given by

$$P(X < x) = F(x + 1; \Phi).$$

The hazard rate is given by $hr(x) = \frac{P(X=x)}{S(x)}$. The reversed failure rate of discrete distribution is given as $rfr(x) = \frac{P(X=x)}{1-S(x)}$.

3. DISCRETE MARSHALL-OLKIN LOMAX DISTRIBUTION

In this Section, we introduce a new flexible discrete model, can be denoted as discrete Marshall–Olkin Lomax (DMOL) distribution. Parameter estimation of DMOL distribution are discussed by using MLE.

1.1 The DMOL Distribution

The continuous Marshall–Olkin Lomax (MOL) distribution is introduced by Rao *et al.* (2009). The survival function, of the MOL distribution, is given by

$$S(x; \gamma, \delta, \vartheta) = \frac{\gamma}{\left(1 + \frac{x}{\vartheta}\right)^\delta - (1 - \gamma)}; x > 0, \gamma, \delta, \vartheta > 0,$$

Using the survival discretization method and survival function of MOL distribution, we define the PMF of the DMOL distribution as given below

$$P(x; \gamma, \delta, \vartheta) = \frac{\gamma}{\left(1 + \frac{x}{\vartheta}\right)^\delta - (1 - \gamma)} - \frac{\gamma}{\left(1 + \frac{x+1}{\vartheta}\right)^\delta - (1 - \gamma)}; x = 0, 1, 2, \dots$$

Let $\rho = e^{-\delta}$ then $0 < \rho < 1$, the PMF can be rewritten as following

$$P(x; \gamma, \vartheta, \rho) = \frac{\gamma}{\rho^{-\ln\left(1+\frac{x}{\vartheta}\right)} - 1 + \gamma} - \frac{\gamma}{\rho^{-\ln\left(1+\frac{x+1}{\vartheta}\right)} - 1 + \gamma}.$$

Figure 4 shows the PMF plots for different values of the model parameters. From Figure 4, the PMF of the DMOL distribution is unimodal and right-skewed.

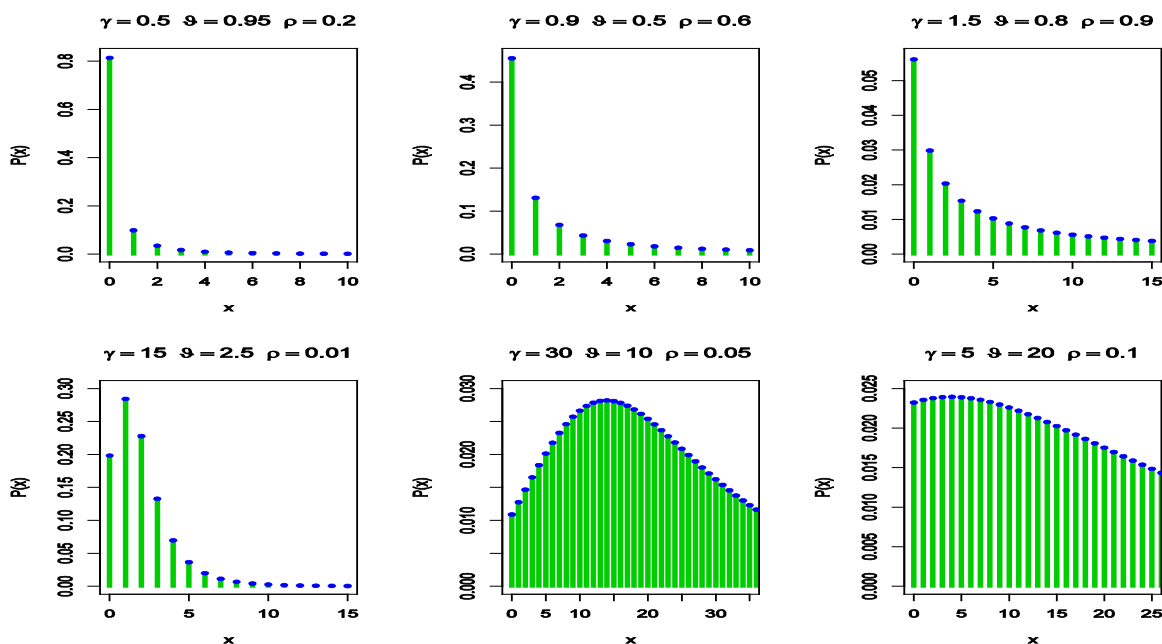


Figure-4: The PMF plots of the DMOL distribution.

The CDF of the DMOL distribution is given as below

$$F(x; \gamma, \vartheta, \rho) = \frac{\rho^{-\ln\left(1+\frac{x+1}{\vartheta}\right)} - 1}{\rho^{-\ln\left(1+\frac{x+1}{\vartheta}\right)} - (1 - \gamma)}, \quad x \in \mathbb{N}_0,$$

Where $\mathbb{N}_0 = \{0, 1, 2, \dots\}$. Survival function of the DMOL distribution is given by

$$S(x; \gamma, \vartheta, \rho) = \frac{\gamma}{\rho^{-\ln\left(1+\frac{x+1}{\vartheta}\right)} - (1 - \gamma)}$$

The *hr* function of the DMOL distribution is given by

$$hr(x; \gamma, \vartheta, \rho) = \frac{\rho^{-\ln\left(1+\frac{x+1}{\vartheta}\right)} - \rho^{-\ln\left(1+\frac{x}{\vartheta}\right)}}{\rho^{-\ln\left(1+\frac{x}{\vartheta}\right)} - 1 + \gamma}, \quad x \in \mathbb{N}_0.$$

Figure 5 shows the *HRF* plots of the DMOL distribution. It is noted that the shape of the *HRF*s increasing, left-skewed and decreasing.

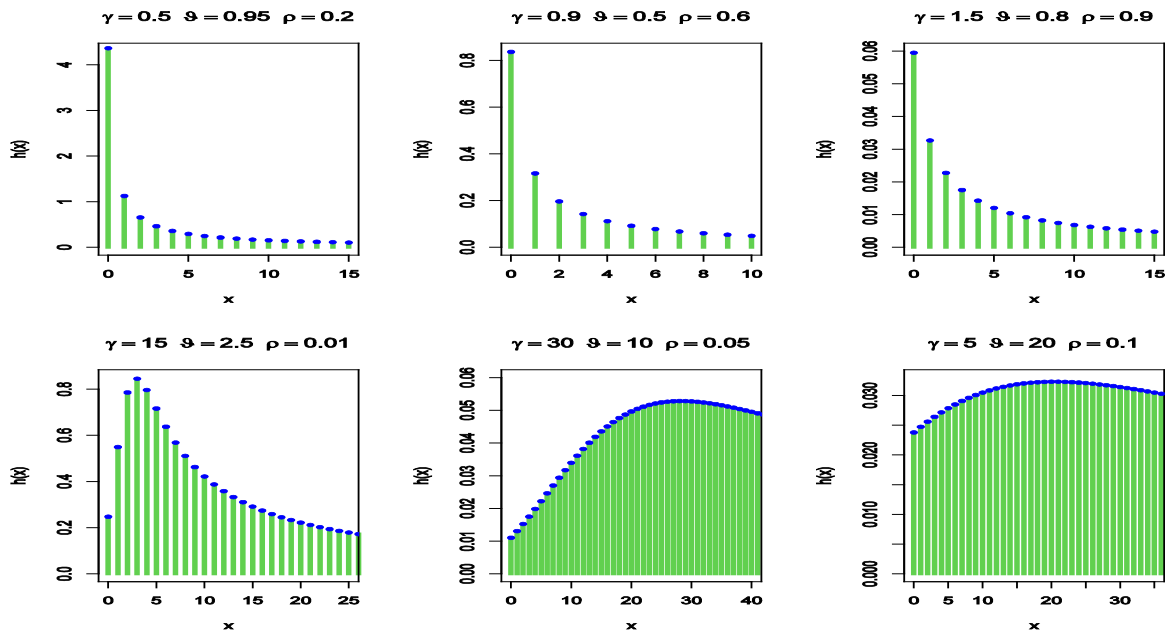


Figure-5: The *HRF* plots of the DMOL distribution.

1.2. Parameter Estimation

The unknown parameters of the DMOL distribution are obtained by the maximum likelihood estimation (MLE) method. This method is based on the maximization of the log-likelihood for a given data set, assume that $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$ is a random sample of size n from a DMOL (γ, P, ϑ) distribution. The log-likelihood function becomes

$$l(\gamma, \vartheta, \rho) = n \ln(\gamma) + \sum_{x=1}^n \ln \left[\rho^{-\ln\left(1+\frac{x+1}{\vartheta}\right)} - \rho^{-\ln\left(1+\frac{x}{\vartheta}\right)} \right] - \sum_{x=1}^n \ln \left[\rho^{-\ln\left(1+\frac{x+1}{\vartheta}\right)} - 1 + \gamma \right] - \sum_{x=1}^n \ln \left[\rho^{-\ln\left(1+\frac{x}{\vartheta}\right)} - 1 + \gamma \right]$$

Hence, the likelihood equations are

$$\begin{aligned} \frac{\partial l(\gamma, \vartheta, \rho)}{\partial \gamma} &= \frac{n}{\gamma} - \sum_{i=1}^n \frac{1}{\rho^{-\ln\left(1+\frac{x+1}{\vartheta}\right)} - 1 + \gamma} - \sum_{i=1}^n \frac{1}{\rho^{-\ln\left(1+\frac{x}{\vartheta}\right)} - 1 + \gamma} \\ \frac{\partial l(\gamma, \vartheta, \rho)}{\partial \rho} &= \sum_{x=1}^n \frac{-\ln\left(1+\frac{x+1}{\vartheta}\right) \rho^{-\ln\left(1+\frac{x+1}{\vartheta}\right)-1} + \ln\left(1+\frac{x}{\vartheta}\right) \rho^{-\ln\left(1+\frac{x}{\vartheta}\right)-1}}{\rho^{-\ln\left(1+\frac{x+1}{\vartheta}\right)} - \rho^{-\ln\left(1+\frac{x}{\vartheta}\right)}} \\ &\quad + \sum_{x=1}^n \frac{\ln\left(1+\frac{x+1}{\vartheta}\right) \rho^{-\ln\left(1+\frac{x+1}{\vartheta}\right)-1}}{\rho^{-\ln\left(1+\frac{x+1}{\vartheta}\right)} - 1 + \gamma} + \sum_{x=1}^n \frac{\ln\left(1+\frac{x}{\vartheta}\right) \rho^{-\ln\left(1+\frac{x}{\vartheta}\right)-1}}{\rho^{-\ln\left(1+\frac{x}{\vartheta}\right)} - 1 + \gamma}, \end{aligned}$$

and

$$\begin{aligned} \frac{\partial l(\gamma, \vartheta, \rho)}{\partial \vartheta} &= -\frac{\ln(\rho)}{\vartheta} \sum_{x=1}^n \frac{\frac{(x+1)}{\vartheta+x+1} \rho^{-\ln\left(1+\frac{x+1}{\vartheta}\right)} - \frac{x}{\vartheta+x} \rho^{-\ln\left(1+\frac{x}{\vartheta}\right)}}{\rho^{-\ln\left(1+\frac{x+1}{\vartheta}\right)} - \rho^{-\ln\left(1+\frac{x}{\vartheta}\right)}} \\ &\quad + \frac{\ln(\rho)}{\vartheta} \left[\sum_{x=1}^n \frac{\frac{(x+1)}{\vartheta+x+1} \rho^{-\ln\left(1+\frac{x+1}{\vartheta}\right)}}{\rho^{-\ln\left(1+\frac{x+1}{\vartheta}\right)} - 1 + \gamma} + \sum_{x=1}^n \frac{\frac{x}{\vartheta+x} \rho^{-\ln\left(1+\frac{x}{\vartheta}\right)}}{\rho^{-\ln\left(1+\frac{x}{\vartheta}\right)} - 1 + \gamma} \right] \end{aligned}$$

The estimate of the parameter by using MLE, which can be obtained by a numerical analysis such as the Newton–Raphson algorithm.

4. SIMULATION STUDY

A simulation study is assumed to evaluate and compare the behavior of the estimates with respect to their bias and mean square error (MSE). We generate 10000 random sample x_1, x_2, \dots, x_n of sizes $n = 50, 70, 130$ and 200 from DMOL distribution. Different sets of parameters are obtained.

The MLE of γ, P and ϑ are computed. Then, the bias and MSE of the estimates of the unknown parameters are computed. Simulated outcomes are listed in Tables 1-2 and the following observations are detected.

- The bias and MSE decrease as sample sizes increase for all estimates (see Tables 1-2).
- The bias and MSE of MLE for ρ estimate is increasing with increase γ and fixed ρ, ϑ .
- The bias and MSE of MLE for ϑ, γ estimates are decreasing with increase γ and fixed ρ, ϑ .
- The bias and MSE of MLE for ρ estimate is decreasing with increase ρ and fixed γ, ϑ .
- The bias and MSE of MLE for ϑ, γ estimates are decreasing with increase ϑ and fixed ρ, γ .

Table-1: Bias and MSE of parameters of DMOL distribution when $\vartheta = 0.5$

| γ | n | P | 0.3 | | 0.5 | | 0.7 | |
|----------|-----|-------------|---------|--------|---------|--------|---------|--------|
| | | | Bias | MSE | Bias | MSE | Bias | MSE |
| 0.25 | 50 | γ | -0.1515 | 0.0414 | -0.1537 | 0.0505 | -0.0544 | 0.0192 |
| | | P | 0.4360 | 0.3316 | 0.2949 | 0.2211 | 0.0506 | 0.1091 |
| | | ϑ | 0.6030 | 0.5218 | 0.4880 | 0.3437 | 0.5262 | 0.4554 |
| | 70 | γ | -0.1520 | 0.0387 | -0.1494 | 0.0431 | -0.0401 | 0.0127 |
| | | P | 0.4175 | 0.2767 | 0.2814 | 0.1759 | -0.0082 | 0.0878 |
| | | ϑ | 0.5568 | 0.4347 | 0.4397 | 0.2590 | 0.5172 | 0.4259 |
| | 130 | γ | -0.1504 | 0.0318 | -0.1383 | 0.0320 | -0.0254 | 0.0060 |
| | | P | 0.4043 | 0.2293 | 0.2230 | 0.1238 | -0.1014 | 0.0452 |
| | | ϑ | 0.5012 | 0.3338 | 0.4267 | 0.2167 | 0.4918 | 0.3872 |
| | 200 | γ | -0.1532 | 0.0301 | -0.1304 | 0.0265 | -0.0335 | 0.0050 |
| | | P | 0.3997 | 0.1936 | 0.1807 | 0.1049 | -0.0360 | 0.0232 |
| | | ϑ | 0.4660 | 0.2775 | 0.4070 | 0.1986 | 0.4640 | 0.2890 |
| 0.85 | 50 | γ | -0.1399 | 0.0297 | -0.1395 | 0.0362 | -0.0573 | 0.0150 |
| | | P | 0.7902 | 0.8076 | 0.5351 | 0.4654 | 0.1970 | 0.1701 |
| | | ϑ | 0.5136 | 0.3677 | 0.4561 | 0.2842 | 0.6172 | 0.6072 |
| | 70 | γ | -0.1407 | 0.0275 | -0.1388 | 0.0331 | -0.0460 | 0.0102 |
| | | P | 0.7992 | 0.7841 | 0.5484 | 0.4771 | 0.0941 | 0.0945 |
| | | ϑ | 0.4599 | 0.3057 | 0.4209 | 0.2311 | 0.5877 | 0.5549 |
| | 130 | γ | -0.1406 | 0.0243 | -0.1370 | 0.0267 | -0.0393 | 0.0052 |
| | | P | 0.7855 | 0.6964 | 0.5211 | 0.3931 | 0.0446 | 0.0284 |
| | | ϑ | 0.4078 | 0.2312 | 0.3875 | 0.1721 | 0.4721 | 0.5189 |
| | 200 | γ | -0.1426 | 0.0237 | -0.1312 | 0.0230 | -0.0345 | 0.0045 |
| | | P | 0.7531 | 0.6301 | 0.4389 | 0.2636 | 0.0447 | 0.0150 |
| | | ϑ | 0.4330 | 0.2257 | 0.4556 | 0.2332 | 0.4565 | 0.2673 |
| 2 | 50 | γ | -0.1224 | 0.0236 | -0.1137 | 0.0263 | -0.0507 | 0.0121 |
| | | P | 0.9215 | 1.1521 | 0.5627 | 0.5216 | 0.2652 | 0.2720 |
| | | ϑ | 0.2527 | 0.1464 | 0.2473 | 0.1477 | 0.4834 | 0.7259 |
| | 70 | γ | -0.1227 | 0.0217 | -0.1137 | 0.0249 | -0.0429 | 0.0085 |
| | | P | 0.9049 | 1.0316 | 0.5072 | 0.5043 | 0.1689 | 0.1142 |
| | | ϑ | 0.1820 | 0.1157 | 0.2744 | 0.1018 | 0.4517 | 0.6803 |
| | 130 | γ | -0.1242 | 0.0190 | -0.1148 | 0.0198 | -0.0403 | 0.0050 |
| | | P | 0.8821 | 0.9022 | 0.5274 | 0.3906 | 0.1640 | 0.0648 |
| | | ϑ | 0.1333 | 0.0928 | 0.2541 | 0.0991 | 0.4448 | 0.3768 |
| | 200 | γ | -0.1230 | 0.0179 | -0.1087 | 0.0164 | -0.0369 | 0.0039 |
| | | P | 0.8241 | 0.7946 | 0.4676 | 0.2843 | 0.1847 | 0.0608 |
| | | ϑ | 0.1851 | 0.0653 | 0.2261 | 0.0695 | 0.2709 | 0.1531 |

Table-2: Bias and MSE of parameters of DMOL distribution when $\vartheta = 1.5$

| γ | n | P | 0.3 | | 0.5 | | 0.7 | |
|----------|-----|-------------|---------|--------|---------|--------|---------|--------|
| | | | Bias | MSE | Bias | MSE | Bias | MSE |
| 0.25 | 50 | γ | -0.1481 | 0.0311 | -0.1150 | 0.0244 | -0.0212 | 0.0041 |
| | | P | 0.3245 | 0.1814 | -0.0410 | 0.2402 | -0.2424 | 0.2576 |
| | | ϑ | 0.6917 | 0.6408 | 0.5741 | 0.4258 | 0.5388 | 0.5148 |
| | 70 | γ | -0.1472 | 0.0284 | -0.1009 | 0.0204 | -0.0221 | 0.0039 |
| | | P | 0.3016 | 0.1443 | -0.0505 | 0.1114 | -0.2165 | 0.2060 |
| | | ϑ | 0.6491 | 0.5342 | 0.5020 | 0.3689 | 0.4604 | 0.3152 |
| | 130 | γ | -0.1421 | 0.0242 | -0.1019 | 0.0156 | -0.0280 | 0.0023 |
| | | P | 0.2565 | 0.0880 | 0.1333 | 0.0846 | -0.1484 | 0.1405 |
| | | ϑ | 0.5967 | 0.4252 | 0.5159 | 0.3030 | 0.3416 | 0.1810 |
| | 200 | γ | -0.1150 | 0.0206 | -0.1036 | 0.0135 | 0.0006 | 0.0018 |
| | | P | 0.2082 | 0.0710 | 0.0997 | 0.0154 | -0.0795 | 0.0571 |
| | | ϑ | 0.5263 | 0.3457 | 0.4201 | 0.1971 | 0.2369 | 0.1615 |
| 0.85 | 50 | γ | -0.1188 | 0.0205 | -0.0790 | 0.0133 | -0.0188 | 0.0038 |
| | | ρ | 0.7455 | 0.7632 | 0.3334 | 0.2504 | -0.0909 | 0.2665 |
| | | ϑ | 0.8110 | 0.8771 | 0.5854 | 0.4763 | 0.6980 | 1.0220 |
| | 70 | γ | -0.1179 | 0.0185 | -0.0677 | 0.0124 | -0.0187 | 0.0034 |
| | | P | 0.7235 | 0.6241 | 0.0528 | 0.3256 | -0.1037 | 0.1882 |
| | | ϑ | 0.7541 | 0.7209 | 0.9016 | 1.0286 | 0.6208 | 0.6183 |
| | 130 | γ | -0.1067 | 0.0132 | -0.0737 | 0.0093 | -0.0201 | 0.0017 |
| | | P | 0.5682 | 0.3464 | 0.1605 | 0.1702 | 0.0123 | 0.0633 |
| | | ϑ | 0.6776 | 0.5074 | 0.7044 | 0.5663 | 0.3796 | 0.2470 |
| | 200 | γ | -0.1017 | 0.0124 | -0.0652 | 0.0060 | -0.0032 | 0.0016 |
| | | P | 0.5278 | 0.2959 | 0.2579 | 0.0848 | -0.1134 | 0.0525 |
| | | ϑ | 0.6099 | 0.4423 | 0.4782 | 0.2582 | 0.3587 | 0.1994 |
| 2 | 50 | γ | -0.0758 | 0.0116 | -0.0462 | 0.0075 | -0.0139 | 0.0030 |
| | | P | 0.8163 | 0.9825 | 0.3933 | 0.4264 | 0.0711 | 0.2387 |
| | | ϑ | 0.5548 | 0.4890 | 0.3942 | 0.3141 | 0.3773 | 0.4650 |
| | 70 | γ | -0.0783 | 0.0104 | -0.0443 | 0.0080 | -0.0010 | 0.0014 |
| | | P | 0.8742 | 0.9933 | 0.3173 | 0.3755 | -0.0524 | 0.1580 |
| | | ϑ | 0.4859 | 0.4351 | 0.3581 | 0.2905 | 0.2501 | 0.1891 |
| | 130 | γ | -0.0715 | 0.0066 | -0.0496 | 0.0055 | -0.0136 | 0.0009 |
| | | P | 0.7120 | 0.5551 | 0.2306 | 0.2700 | 0.0860 | 0.0945 |
| | | ϑ | 0.4402 | 0.2436 | 0.2956 | 0.2015 | 0.1733 | 0.0562 |
| | 200 | γ | -0.0527 | 0.0041 | -0.0346 | 0.0024 | 0.0206 | 0.0005 |
| | | P | 0.5275 | 0.3321 | 0.2819 | 0.1090 | -0.0730 | 0.0424 |
| | | ϑ | 0.3677 | 0.1600 | 0.2702 | 0.0896 | 0.1090 | 0.0398 |

5. APPLICATION ANALYSIS

In this section, the DMOL distribution is fitted to more famous fields of survival times of Covid-19 with different country including as Argentina and Uganda. We compare the fits of the discrete Marshall-Olkin generalized exponential (DMOGEx) [Almetwally *et al.* (2020)] model, discrete generalized exponential (DGEx) [Nekoukhou *et al.* (2013)], natural discrete Lindley (NDL) [Al-Babtain *et al.* (2020)], discrete Gompertzexponential (DGzEx) [El-Morshedy *et al.* (2020)], discrete Burr (DB) [Krishna and Pundir (2009)], discrete Lindley (DLi) [Gómez-Déniz and Calderín-Ojeda (2011)], discrete Lomax (DL_o) [Para and Jan (2016)] and Geometric models in Tables 3, 4.

Tables 3, 4 provide values of Cramér-von Mises (W*), Anderson-Darling (A*) and Kolmogorov- Smirnov (KS) statistic along with its P-value for the all models fitted based on Two real data sets. In addition, these tables contain the MLE and standard errors (SEs) of the parameters for the considered models. The fitted DMOL, PMF, CDF, PP-plot and QQ-plot of the four data sets are displayed in Figures 6, 7, respectively. These figures indicate that the DMOL distribution get the lowest values of W*, A*, KS and largest P-value, among all fitted models.

Firstly: This is a COVID-19 data belong to Argentina of 80 days, that is recorded from 12 march to 30 May 2020. This data formed of daily new cases. The data are as follows: 2, 12, 14, 0, 11, 9, 32, 0, 27, 30, 0, 67, 76, 0, 201, 0, 87, 101, 55, 75, 146, 88, 79, 132, 88, 98, 103, 74, 87, 80, 134, 46, 0, 277, 84, 141, 121, 96, 90, 55, 121, 113, 124, 143, 139, 222, 137, 54, 127, 182, 103, 172, 56, 149, 118, 123, 154, 229, 225, 150, 244, 110, 244, 285, 410, 161, 345, 326, 263, 303, 438, 474, 648, 718, 704, 723, 552, 600, 705, 769.

Table-3: ML estimates, K-S, P-values, W* and A* for COVID-19 data in Argentina data

| Argentina | | Estimate | SE | KS | P-Value | W* | A* |
|-----------------|-------------|----------|----------|--------|---------|--------|--------|
| DMOL | γ | 7.9364 | 1.6938 | 0.0852 | 0.6066 | 0.2099 | 1.3987 |
| | ϑ | 67.7191 | 29.1580 | | | | |
| | ρ | 0.1154 | 0.0663 | | | | |
| DB | γ | 0.9540 | 0.2332 | 0.4032 | 0.0000 | 1.3864 | 7.9660 |
| | ϑ | 0.7927 | 0.0486 | | | | |
| DLi | D | 0.9892 | 0.0008 | 0.1725 | 0.0171 | 0.2132 | 1.4183 |
| DGE | γ | 0.9957 | 0.0006 | 0.1568 | 0.0392 | 0.2440 | 1.4638 |
| | ϑ | 0.6938 | 0.0992 | | | | |
| DMOGE | α | 0.2110 | 0.2130 | 0.1196 | 0.2027 | 0.2958 | 1.7068 |
| | γ | 8.0661 | 1.8739 | | | | |
| | ϑ | 0.0066 | 0.0011 | | | | |
| Geometric | ρ | 0.0055 | 0.0006 | 0.1024 | 0.3715 | 0.2409 | 1.4504 |
| DL _o | γ | 563.0061 | 269.7579 | 0.1336 | 0.1151 | 0.2991 | 1.7895 |
| | ϑ | 0.0199 | 0.0131 | | | | |
| NDL | ρ | 0.0107 | 0.0008 | 0.1779 | 0.0127 | 0.2134 | 1.2840 |

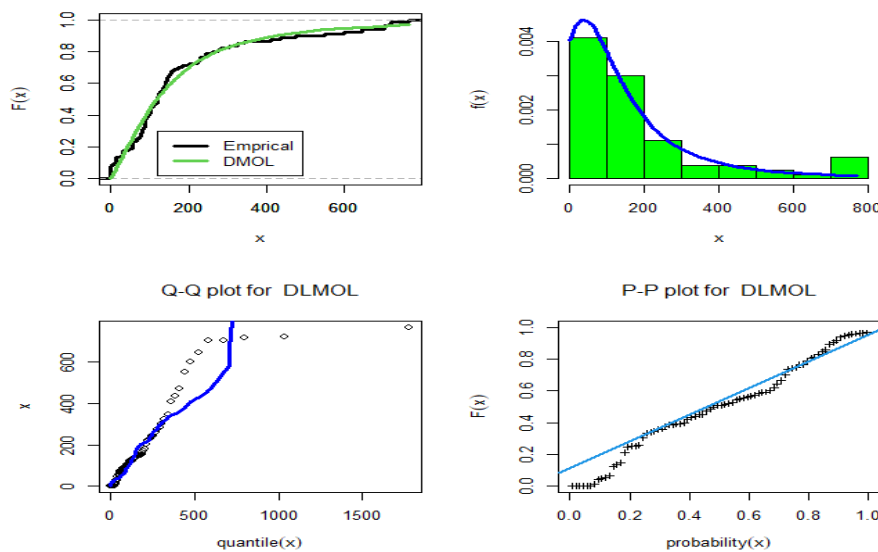


Figure-6: Estimated PMF, CDF, PP-plot and QQ-plot of DMOL for COVID-19 data in Argentina data

Secondly: This is a COVID-19 data belong to Uganda of 37 days, that is recorded from 17 August to 22 September 2020. This data formed of daily new cases. The data are as follows: 66, 60, 43, 53, 94, 0, 416, 97, 99, 64, 98, 0, 232, 0, 132, 84, 65, 75, 176, 65, 186, 128, 109, 124, 201, 190, 86, 326, 96, 179, 145, 143, 114, 210, 427, 270 and 181.

Table-4: ML estimates, K-S, P-values, W* and A* for COVID-19 data in Uganda data

| Uganda | | Estimate | SE | KS | P-Value | W* | A* |
|-----------|---|----------|----------|--------|---------|--------|--------|
| DMOL | 1 | 28.0782 | 30.3923 | 0.0794 | 0.9739 | 0.0975 | 0.8405 |
| | 2 | 79.0892 | 66.5930 | | | | |
| | 3 | 0.0234 | 0.0301 | | | | |
| DB | 1 | 0.8884 | 0.2903 | 0.4924 | 0.0000 | 1.0596 | 5.9993 |
| | 2 | 0.7781 | 0.0700 | | | | |
| DLi | D | 0.9856 | 0.0017 | 0.0932 | 0.9049 | 0.1896 | 1.3911 |
| DGE | 1 | 0.9927 | 0.0015 | 0.2240 | 0.0488 | 0.2321 | 1.6519 |
| | 2 | 1.0057 | 0.2245 | | | | |
| DMOGE | 1 | 0.0919 | 0.1045 | 0.1233 | 0.6272 | 0.0990 | 0.8603 |
| | 2 | 60.9921 | 71.9193 | | | | |
| | 3 | 0.0153 | 0.0027 | | | | |
| Geometric | D | 0.0073 | 0.0012 | 0.2253 | 0.0467 | 0.2320 | 1.6513 |
| DLo | 1 | 563.0447 | 266.5164 | 0.2503 | 0.0194 | 0.3590 | 1.7250 |
| | 2 | 0.0082 | 0.0168 | | | | |
| NDL | D | 0.0144 | 0.0017 | 0.0895 | 0.9281 | 0.1901 | 1.3944 |

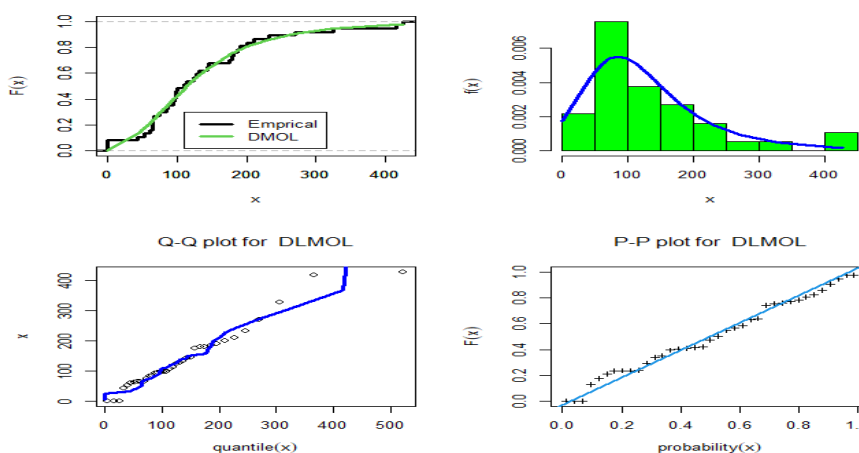


Figure-7: Estimated PMF, CDF, PP-plot and QQ-plot of DMOL for COVID-19 data in Uganda data

6. CONCLUDING REMARKS

In this article, with the aim of managing the risk of spreading Coronavirus in Argentina and Uganda, we proposed and studied the discrete Marshall–Olkin Lomax distribution. The maximum likelihood estimation method is discussed to estimate the parameter of DMOL distribution. Monte Carlo Simulations are obtained to evaluate the restricted sample properties of the DMOL distribution. We prove empirically that the DMOL model reveals its superiority over other competitive models as Marshall-Olkin generalized exponential, discrete generalized exponential, natural discrete Lindley, discrete Gompertz exponential, discrete Burr, discrete Lindley, discrete Lomax and Geometric for analysis daily new cases of the COVID-19 in the case of Argentina and Uganda. These figures indicate that the DMOL distribution gets the lowest values of W*, A*, KS, and the largest P-value, among all fitted models.

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