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APPLICATIONS OF FOURIER SERIES IN DOUBLE SINE AND COSINE SERIES

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ABSTRACT

In this paper we have introduce the new concept of double sine and cosine series in Fourier series of two variable functions and implementing these result by the examples of two variable Fourier series.

Keywords: *Fourier series, double sine series, double cosine series.*

Subject Classification 2010: *42A16, 42B05.*

1. INTRODUCTION

A Fourier series is an expansion of a periodic function f(x) in terms of an infinite sum of sines and cosines. Fourier series make use of the orthogonality relationships of the sine and cosine functions.
"Any functions f(x) can be expressed as a Fourier series

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n cosnx + \sum_{n=1}^{\infty} b_n sinnx$$

In the interval (0, 2π) or ($-\pi$, π), $where\ a_0, a_n, b_n$ are constant.
(x) is periodic, single valued and finite
(x) has a finite number of finite discontinuities in any one period
(x) has a finite number of maxima and minima

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)dx$$

Even function $a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\ \cos(nx)dx$

Odd function $b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\ \sin(nx)dx$

2. MAIN RESULT

Fourier series of two variable functions
Any functions f(x, y) can be expressed as a Fourier series

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n cosnx\ cosny + \sum_{n=1}^{\infty} b_n sinnx\ sinny$$

In the interval (0, 2π) or ($-\pi$,), w$here\ a_0, a_n, b_n$ are constant.

$$a_0 = \frac{1}{\pi}\int_0^{\pi}\int_0^{\pi} f(x,y)\,dx\,dy$$

Even function $a_n = \frac{2}{\pi}\int_0^{\pi}\int_0^{\pi} f(x,y)\ \cos(nx)\cos(ny)\,dx\,dy$

Odd function $b_n = \frac{2}{\pi}\int_0^{\pi}\int_0^{\pi} f(x,y)\ \sin(nx)\sin(ny)\,dx$

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2.1 Examples: Find the Fourier series of the function $f(x,y) = x^2 + y^2$ in $-\pi < x < \pi$

Solution: Let the Fourier series of the function $f(x,y) = x^2 + y^2$ be

$$f(x,y) = x^2 + y^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n cosnx\, cosny + b_n sinnx\, sinny)$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} f(x,y)\, dxdy$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} (x^2 + y^2)\ \ dxdy$$

$$= \frac{1}{\pi}\left[\frac{x^3}{3}\right]_{-\pi}^{\pi} + \frac{1}{\pi}\left[\frac{y^3}{3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{\pi^3}{3} + \frac{\pi^3}{3}\right] = \frac{1}{\pi}\frac{4\pi^3}{3} = \frac{4\pi^2}{3}$$

$$a_0 = \frac{4\pi^2}{3}$$

$$a_n = \frac{2}{\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} f(x,y)\ \cos(nx)\cos(ny)\, dx\, dy$$

$$a_n = \frac{2}{\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} (x^2 + y^2)\cos(nx)\cos(ny)\, dx\, dy$$

$$\frac{2}{\pi}\int_{-\pi}^{\pi} x^2 cosnx\ \ dx + \int_{-\pi}^{\pi} y^2 cosny\, dy$$

$$\int_{-a}^{a}\int_{-a}^{a} f(x,y)\, dxdy = \begin{cases} 2\int_0^a f(x)dx, if\ f(x) is\ an\ even\ function\ of\ x \\ 0, \qquad if\ f(x) is\ an\ odd\ function\ of\ x \end{cases}$$

$$= \frac{2}{\pi}\left[x^2\frac{\sin nx}{n} - 2x\left(-\frac{\cos nx}{n^2}\right) + 2(-\frac{sinnx}{n^3})\right]_0^{\pi} +$$

$$\frac{2}{\pi}\left[y^2\frac{\sin ny}{n} - 2y\left(-\frac{\cos ny}{n^2}\right) + 2\left(-\frac{\sin ny}{n^3}\right)\right]_0^{\pi}$$

$$\frac{4(-1)^n}{n^2} + \frac{4(-1)^n}{n^2} = \frac{8(-1)^n}{n^2} \qquad [\ sinn\pi = 0\ and\ cosn\pi = (-1)^n]$$

$$\begin{cases} \frac{-8}{n^2}, when\ n\ is\ odd \\ \frac{8}{n^2}, when\ n\ is\ even \end{cases}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} f(x,y) sinnx\ sinny\ dx\ dy$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} (x^2 + y^2)\ \ sinnx\ sinny\ dx\ dy = 0$$

[$x^2$is an even function and $sinnx\ \ sinny$ is an odd function

so that $x^2 sinnx$ and $y^2 sinny$ is an odd function]

Substituting the above values in (1), we obtain the required Fourier series of the function $f(x,y) = (x^2 + y^2)$ as given by

$$x^2 + y^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty}\left(\frac{4(-1)^n}{n^2}cosnx + \frac{4(-1)^n}{n^2}cosny\right) + \sum_{n=1}^{\infty} 0. sinnx\ sinny$$

$$x^2 + y^2 = \frac{\pi^2}{3} + 4\left[-cosx + \frac{1}{4}cos2x - \frac{1}{9}cos3x + \cdots..\right] + 4\left[-cosy + \frac{1}{4}cos2y - \frac{1}{9}cos3y + \cdots..\right]$$

Putting $x = 0, y = 0,$

$$0 = \frac{\pi^2}{3} + 4\left[-1 + \frac{1}{4} - \frac{1}{9} + \frac{1}{16} -\right] + 4\left[-1 + \frac{1}{4} - \frac{1}{9} + \frac{1}{16} -\right]$$

$$= \frac{\pi^2}{3} + 8\left[-2 + \frac{1}{2} - \frac{2}{9} + \frac{1}{8} - \cdots.\right]$$

$$\frac{\pi^2}{24} = 2 - \frac{1}{2} + \frac{2}{9} - \frac{1}{8} + \cdots$$

2.2 Examples: Obtain the Fourier series representing the function $f(x,y) = x + y$in the interval $0 < x < 2\pi \; and \; 0 < y < 2\pi$.

Solution: Let the Fourier series representing the functions

$$f(x,y) = x + y$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n cosnx + b_n sinnx)(1)$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi}\int_0^{2\pi} f(x,y)\ dx = \frac{1}{\pi}\int_0^{2\pi}\int_0^{2\pi}(x+y)\ \ dx\,dy$$

$$\frac{1}{\pi}\left[\frac{x^2}{2} + \frac{y^2}{2}\right]_0^{2\pi} = 2\pi + 2\pi = 4\pi$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi}\int_0^{2\pi} f(x,y) cosnx\ cosny\ dx\,dy$$

$$= \frac{1}{\pi}\int_0^{2\pi}\int_0^{2\pi}(x+y) cosnx\ cosny\ dx\,dy$$

$$= \frac{1}{\pi}\int_0^{2\pi} x\ cosnx\ dx + \frac{1}{\pi}\int_0^{2\pi} y\ cosnydy$$

$$= \frac{1}{\pi}\left[x.\frac{sinnx}{n} - 1.\left(\frac{-cosnx}{n^2}\right)\right]_0^{2\pi} + \frac{1}{\pi}\left[\left[y.\frac{sinny}{n} - 1.\left(\frac{-cosny}{n^2}\right)\right]\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{\cos 2n\pi}{n^2} - \frac{1}{n^2}\right] + \frac{1}{\pi}\left[\frac{\cos 2n\pi}{n^2} - \frac{1}{n^2}\right]$$

$$= \frac{1}{n^2\pi}(1-1) + \frac{1}{n^2\pi}(1-1) = 0$$

$$a_n = 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi}\int_0^{2\pi} f(x,y)\ sinnx\ \ sinny\ dx\,dy$$

$$= \frac{1}{\pi}\int_0^{2\pi} x\ sinnx\ dx + \frac{1}{\pi}\int_0^{2\pi} y\ sinny\ dy$$

$$= \frac{1}{\pi}\left[x\ \left(-\frac{\cos nx}{n}\right) - 1.\left(-\frac{sinnx}{n^2}\right)\right]_0^{2\pi} + \frac{1}{\pi}\left[y\ \left(-\frac{\cos ny}{n}\right) - 1.\left(-\frac{sinny}{n^2}\right)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[-\frac{2\pi\cos 2n\pi}{n}\right] + \frac{1}{\pi}\left[-\frac{2\pi\cos 2n\pi}{n}\right] = -\frac{4}{n}$$

Putting the value's of $a's\ and\ b's$ in (1), we get

$$x = \pi - 2\left[sinx + \frac{1}{2}\sin 2x + \frac{1}{3}sin3x + \cdots\right]$$

$$y = \pi - 2\left[siny + \frac{1}{2}\sin 2y + \frac{1}{3}sin3y + \cdots\right]$$

3. RESULT

To solve the examples of two variable Fourier series we see that the result are obtained as similar in two variable Fourier series.

4. CONCLUSIONS

The necessity of dealing with Fourier series by double sine and cosine series in different applications for Engineering together with the difficulty of computing them in double valued function. In this paper to be self-contained, where the basic theoretical concepts on double sine and cosine series are solved by Fourier series. This theoretical concepts have been used to compute Fourier series of double valued functions.

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