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GENERAL MULTIPLICATIVE REDUCED SECOND ZAGREB INDEX OF CERTAIN NETWORKS

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#### Abstract

The methods of graph index computation can help to find out of chemical, biological information of drugs in Chemical science. In this paper, we introduce the multiplicative reduced modified second Zagreb index, multiplicative reduced product connectivity index, multiplicative reduced atom bond connectivity index, multiplicative reduced geometricarithmetic index, multiplicative reduced $F_{1}$-Index of a graph and compute exact formulas for carbon nanocone networks, armchair polyhex nanotubes and zigzag polyhex nanotubes.


Key words: Multiplicative reduced connectivity indices, multiplicative reduced $F_{1}$-index, nanocones, nanotubes.
Mathematics Subject Classification: 05C05, 05C07, 05C12, 05C90.

## 1. INTRODUCTION

A molecular graph or a chemical graph such that its vertices correspond to the atoms and the edges to the bonds. The whole structure of a graph is characterized by numeric quantity called a graph index or a topological index. Several graph indices [1] have been considered in Theoretical chemistry have found some applications, especially in QSPR/QSAR study, see [2, 3]. Let $G$ be a finite connected in directed graphs without loops and multiple edges. Let $G$ be a such graph with vertex set $V(G)$ and edge set $E(G)$. The degree $D_{G}(v)$ of a vertex $v$ is the number of vertices adjacent to $v$. for basic notations and terminologies, we follow the book [4].

The multiplicative reduced second Zagreb index and multiplicative reduced second hyper Zagreb index were introduced by Kulli in [5] and they are defined as

$$
\begin{aligned}
& R M_{1} I I(G)=\prod_{u v \in E(G)}\left(d_{G}(u)-1\right)\left(d_{G}(v)-1\right) . \\
& R H M_{2} I I(G)=\prod_{u v \in E(G)}\left[\left(d_{G}(u)-1\right)\left(d_{G}(v)-1\right)\right]^{2} .
\end{aligned}
$$

Recently some reduced Zagreb indices were studied in $[6,7,8]$.
We introduce the multiplicative reduced modified second Zagreb index, multiplicative reduced product connectivity index, multiplicative reduced reciprocal connectivity index, multiplicative reduced $F_{1}$ - index of a graph as follows:

The multiplicative reduced modified second Zagreb index of a graph $G$ is defined as

$$
{ }^{m} R M_{2} I I(G)=\prod_{u v \in E(G)} \frac{1}{\left(d_{G}(u)-1\right)\left(d_{G}(v)-1\right)} .
$$

The multiplicative reduced product connectivity index of a graph $G$ is defined as

$$
\operatorname{RPII}(G)=\prod_{u v \in E(G)} \frac{1}{\sqrt{\left(d_{G}(u)-1\right)\left(d_{G}(v)-1\right)}} .
$$

The multiplicative reduced reciprocal product connectivity index of a graph $G$ is defined as

$$
\operatorname{RRII}(G)=\prod_{u V \in E(G)} \sqrt{\left(d_{G}(u)-1\right)\left(d_{G}(v)-1\right)}
$$

The multiplicative reduced $F_{1}$ Index of a graph $G$ is defined as

$$
R F_{1} I I(G)=\prod_{u v \in E(G)}\left[\left(d_{G}(u)-1\right)^{2}+\left(d_{G}(v)-1\right)^{2}\right] .
$$

We also propose the multiplicative reduced atom bond connectivity index, multiplicative reduced geometric arithmetic index, and multiplicative reduced arithmetic -geometric index of a graph as follows.

The multiplicative reduced atom bond connectivity index of a graph $G$ is defined as

$$
\operatorname{RABCII}(G)=\prod_{u v \in E(G)} \sqrt{\frac{\left(d_{G}(u)-1\right)+\left(d_{G}(v)-1\right)-2}{\left(d_{g_{j}}^{(u)-1)\left(d_{G}(v)-1\right)}\right.}}
$$

The multiplicative reduced geometric index of a graph $G$ is defined as

$$
\operatorname{RGAII}(G)=\prod_{u v \in E(G)} \frac{2 \sqrt{\left(d_{G}(u)-1\right)\left(d_{G}(v)-1\right)}}{\left(d_{G}(u)-\sqrt{1}\right)+\left(d_{G}(v)-1\right)}
$$

The multiplicative reduced arithmetic -geometric index of a graph $G$ is defined as

$$
\operatorname{RAGII}(G)=\prod_{u v \in E(G)} \frac{\left(d_{G}(u)-1\right)+\left(d_{G}(v)-1\right)}{2 \sqrt{\left(d_{G}(u)-1\right)\left(d_{G}(v)-1\right)}}
$$

In [5], Kulli introduced the general multiplicative reduced second Zagreb index of a graph, defined as

$$
R M_{2}^{a} I I(G)=\prod_{u v \in E(G)}\left[\left(d_{G}(u)-1\right)\left(d_{G}(v)-1\right)\right]^{a}
$$

Recently, some other multiplicative indices, were studied, for example in $[9,10,11,12,13,14,15,16,17,18,19]$
In this paper, multiplicative reduced $F_{1}$ index, general multiplicative reduced second Zagreb index for some chemical networks such as nanocones, nanotubes.

## 2. RESULTS FOR CARBON NANOCONE NETWORKS

The family of nanocones, denoted by $C N C_{5}[n]$ is called one pentagonal nanocomes, where $n$ is the number of hexagons layers and 5 shows the sides of polygon which acts as the core of nanocones. A 6 -dimensional one pentagonal nanocone network is shown in Figure 1.


Figure-1
Let $G$ be the graph of one pentagonal nanocone $C N C_{5}[n]$. Then $G$ has $5(n+1)^{2}$ vertices and $\frac{15}{2} n^{2}+\frac{25}{2} n+5$ edges. In $G$, there are three types of edges based on degrees of end vertices of each edge. The edge based on degrees of end vertices of each edge. The edge partition of $G$ is given in Table 1.

| $D_{G}(u), D_{G}(v) \backslash u v \in E(G)$ | $(2,2)$ | $(2,3)$ | $(3,3)$ |
| :---: | :---: | :---: | :---: |
| Number of edges | 5 | $10 n$ | $\frac{15}{2} n^{2}+\frac{5}{2} n$ |

Table-1: Edge partition of $G$
Theorem 1: The multiplicative reduced second Zagreb index of $C N C_{5}[n]$ is

$$
\begin{equation*}
R M_{2}^{a} I I\left(C N C_{5}[n]\right)=2^{15 a n(n+1)} \tag{i}
\end{equation*}
$$

Proof: Let $G$ be the graph $C N C_{5}[n]$. By using equation and Table 1, we deduce

$$
\begin{aligned}
R M_{2}^{a} I I\left(C N C_{5}[n]\right) & =\prod_{u v \in E(G)}\left[\left(d_{G}(u)-1\right)\left(d_{G}(v)-1\right)\right]^{a} \\
& =[(2-1)(2-1)]^{a 5} \times[(2-1)(3-1)]^{a 10 n} \times[(3-1)(3-1)]^{a\left(\frac{15}{2} n^{2}+\frac{5}{2} n\right)} \\
& =2^{15 a n(n+1)} .
\end{aligned}
$$

We establish the following results by using Theorem 1 .
Corollary 1.1: The multiplicative reduced second Zagreb index of $\mathrm{CNC}_{5}[n]$ is
$R M_{2} I I\left(C N C_{5}[n]\right)=2^{15 n(n+1)}$.
Corollary 1.2: The multiplicative reduced second hyper Zagreb index of $C N C_{5}[n]$ is
$R H_{2} I I\left(\right.$ CNC $\left._{5}[n]\right)=2^{30 n(n+1)}$.
Corollary 1.3: The multiplicative reduced modified second Zagreb index of $\mathrm{CNC}_{5}[n]$ is
${ }^{m} R M_{2} I I\left(C N C_{5}[n]\right)=\left(\frac{1}{2}\right)^{15 n(n+1)}$.
Corollary 1.4: The multiplicative reduced product connectivity index of $\mathrm{CNC}_{5}[n]$ is
$\operatorname{RPII}\left(C N C_{5}[n]\right)=\left(\frac{1}{\sqrt{2}}\right)^{15 n(n+1)}$.
Corollary 1.5: The multiplicative reduced reciprocal product connectivity index of $\mathrm{CNC}_{5}[n]$ is
$\operatorname{RRPII}\left(\mathrm{CNC}_{5}[n]\right)=(\sqrt{2})^{15 n(n+1)}$.
Proof: Put a = 1, 2, $-1,-1 / 2,1 / 2$ in equation (1), we get the desired results.
Theorem 2: The multiplicative reduced $F_{1}$-index of $C N C_{5}[n]$ is given by

$$
R F_{1} I I\left(C N C_{5}[n]\right)=32 \times 5^{10 a} \times 2^{\frac{45}{2} n^{2}+\frac{15}{2} n}
$$

Proof: Let $G=C N C_{5}[n]$ By using equation and Table 1, we deduce

$$
\begin{aligned}
R F_{1} I I\left(C N C_{5}[n]\right) & =\prod_{u v \in E(G)}\left[\left(d_{G}(u)-1\right)^{2}\left(d_{G}(v)-1\right)^{2}\right] \\
& =\left[(2-1)^{2}(2-1)^{2}\right]^{5} \times\left[(2-1)^{2}(3-1)^{2}\right]^{10 n} \times\left[(3-1)^{2}(3-1)^{2}\right]^{\frac{15}{2} n^{2}+\frac{5}{2} n} \\
& =32 \times 5^{10 n} \times 2^{\frac{45}{2} n^{2}+\frac{15}{2} n} .
\end{aligned}
$$

Theorem 3: The multiplicative reduced ABC index of $\mathrm{CNC}_{5}[n]$ is
$\operatorname{RABCII}\left(C N C_{5}[n]\right)=\left(\frac{1}{2}\right)^{\frac{15}{4} n^{2}+\frac{25}{4} n}$.

Proof: Let $G=C N C_{5}[n]$. By using equation and Table 1, we derive

$$
\begin{aligned}
\operatorname{RABCII}(G) & =\prod_{u v \in E(G)} \sqrt{\frac{\left(d_{G}(u)-1\right)+\left(d_{G}(v)-1\right)-2}{\left(d_{G}(u)-1\right)\left(d_{G}(v)-1\right)}} \\
& =\left(\frac{2-1+2-1-2}{(1-1)(1-1)}\right)^{\frac{5}{2}} \times\left(\frac{2-1+3-1-2}{(1-1)(3-1)}\right)^{\frac{10 n}{2}} \times\left(\frac{3-1+3-1-2}{(3-1)(3-1)}\right)^{\frac{15}{4} n^{2}+\frac{5}{4} n} \\
& =\left(\frac{1}{2}\right)^{\frac{15}{4} n^{2}+\frac{25}{4} n}
\end{aligned}
$$

Theorem 4: The multiplicative reduced geometric arithmetic index of $\mathrm{CNC}_{5}[n]$ is

$$
\operatorname{RGAII}\left(C N C_{5}[n]\right)=\left(\frac{2 \sqrt{2}}{3}\right)^{10 n}
$$

Proof: Let $G=C N C_{5}[n]$. By using equation and Table 1, we obtain

$$
\begin{aligned}
\operatorname{RGAII}(G) & =\prod_{u v \in E(G)} \frac{2 \sqrt{\left(d_{G}(u)-1\right)\left(d_{G}(v)-1\right)}}{\left(d_{G}(u)-1\right)+\left(d_{G}(v)-1\right)} \\
& =\left(\frac{2 \sqrt{(2-1)(2-1)}}{2-1+2-1}\right)^{5} \times\left(\frac{2 \sqrt{(2-1)(3-1)}}{2-1+3-1}\right)^{10 n} \times\left(\frac{2 \sqrt{(3-1)(3-1)}}{3-1+3-1}\right)^{\frac{15}{2} n+\frac{5}{2} n} \\
& =\left(\frac{2 \sqrt{2}}{3}\right)^{10 n} .
\end{aligned}
$$

Theorem 5: The multiplicative reduced arithmetic geometric index of $C N C_{5}[n]$ is

$$
\operatorname{RAGII}\left({C N C_{5}}[n]\right)=\left(\frac{3}{2 \sqrt{2}}\right)^{10 n}
$$

Proof: Let $G=C N C_{5}[n]$ By using equation and Table 1, we deduce

$$
\begin{aligned}
\operatorname{RAGII}(G) & =\prod_{u v \in E(G)} \frac{\left(d_{G}(u)-1\right)+\left(d_{G}(v)-1\right)}{2 \sqrt{\left(d_{G}(u)-1\right)\left(d_{G}(v)-1\right)}} \\
& =\left(\frac{2-1+2-1}{2 \sqrt{(2-1)(2-1)}}\right)^{5} \times\left(\frac{2-1+3-1}{2 \sqrt{(2-1)(3-1)}}\right)^{10 n} \times\left(\frac{3-1+3-1}{2 \sqrt{(3-1)(3-1)}}\right)^{\frac{15}{2} n+\frac{5}{2} n} \\
& =\left(\frac{3}{2 \sqrt{2}}\right)^{10 n} .
\end{aligned}
$$

## 3. RESULTS FOR ARMCHAIR POLYHEX NANOTUBES

Carbon polyhex nanotubes exist in nature with remarkable stability and posses very interesting thermal, electrical and mechanical properties. Cylindrical surface of these nanotubes is made up of entirely hexagons. We consider the family of armchair polyhex nanotubes which is denoted by $T U A C_{6}[p, q]$. A 2-dimensional network $T U A C_{6}[p, q]$ is shown in Figure 2.


Figure-2
Let $G$ be the graph $T U A C_{6}[p, q]$. By calculation, $G$ has $2 p(q+1)$ vertices and $3 p q+2 p$ edges. In $G$, there are three types of edges based on degrees of end vertices of each edge. The edge partition of $G$ is given in Table 2.

| $d_{G}(u), d_{G}(v) \backslash u v \in E(G)$ | $(2,2)$ | $(2,3)$ | $(3,3)$ |
| :--- | :---: | :---: | :---: |
| Number of edges | $p$ | $2 p$ | $3 p q-p$ |

Table-2: Edge partition of $T U A C_{6}[p, q]$.
Theorem 6: The general multiplicative reduced second Zagreb index of $T U A C_{6}[p, q]$. is

$$
\begin{equation*}
R M_{2}^{a} I I\left(T U A C_{6}[p, q]\right)=2^{6 a p q} \tag{2}
\end{equation*}
$$

Proof: Let $G=T U A C_{6}[p, q]$. By using definition and Table 2, we derive

$$
\begin{aligned}
R M_{2}^{a} I I\left(\operatorname{TUAC}_{6}[p, q]\right) & =\prod_{u v \in E(G)}\left[\left(d_{G}(u)-1\right)\left(d_{G}(v)-1\right)\right]^{a} \\
& =[(2-1)(2-1)]^{a p} \times[(2-1)(3-1)]^{2 a p} \times[(3-1)(3-1)]^{a(3 p q-p)} \\
& =2^{6 a p q} .
\end{aligned}
$$

Corollary 6.1: The multiplicative reduced second Zagreb index of $T U A C_{6}[p, q]$ is

$$
R M_{2} I I\left(T U A C_{6}[p, q]\right)=2^{6 p q}
$$

Corollary 6.2: The multiplicative reduced second hyper Zagreb index of $T U A C_{6}[p, q]$ is
$R H M_{2} I I\left(T U A C_{6}[p, q]\right)=2^{12 p q}$.
Corollary 6.3: The multiplicative reduced modified second Zagreb index of $T U A C_{6}[p, q]$ is
${ }^{m} R M_{2} I I\left(T U A C_{6}[p, q]\right)=\left(\frac{1}{2}\right)^{6 p q}$.
Corollary 6.4: The multiplicative reduced product connectivity index of $T U A C_{6}[p, q]$ is
$\operatorname{RPII}\left(T U A C_{6}[p, q]\right)=\left(\frac{1}{2}\right)^{3 p q}$.
Corollary 6.5: The multiplicative reduced reciprocal product connectivity index of $T U A C_{6}[p, q]$ is $\operatorname{RRPII}\left(\right.$ TUAC $\left._{6}[p, q]\right)=2^{3 p q}$.

Proof: Put $a=1,2,-1,-1 / 2,1 / 2$ in equation (2) we obtain the desired results.
Theorem 7: The multiplicative reduced $F_{1}$ index of $T U A C_{6}[p, q]$ is

$$
R F_{1} I I\left(T U A C_{6}[p, q]\right)=2^{2 p} \times 2^{9 p q-2 p}
$$

Proof: Let $G=T U A C_{6}[p, q]$. By using definition and Table 2 we obtain

$$
\begin{aligned}
\operatorname{RF}_{1} I I\left(T U A C_{6}[p, q]\right) & =\prod_{u v \in E(G)}\left[\left(d_{G}(u)-1\right)^{2}\left(d_{G}(v)-1\right)^{2}\right] \\
& =\left[(2-1)^{2}+(2-1)^{2}\right]^{p} \times\left[(2-1)^{2}+(3-1)^{2}\right]^{2 p} \times\left[(3-1)^{2}+(3-1)^{2}\right]^{3 p q-p} \\
& =5^{2 p} \times 2^{9 p q-2 p} .
\end{aligned}
$$

Theorem 8: The multiplicative reduced ABC index of $\mathrm{TUAC}_{6}[p, q]$ is

$$
\operatorname{RABCII}\left(T U A C_{6}[p, q]\right)=\left(\frac{1}{2}\right)^{\frac{1}{2}(3 p q+p)}
$$

Proof: Let $G=T U A C_{6}[p, q]$. By using definition and Table 2, we derive

$$
\begin{aligned}
\operatorname{RABCII}\left(\text { TUAC }_{6}[p, q]\right) & =\prod_{u v \in E(G)} \sqrt{\frac{\left(d_{G}(u)-1\right)+\left(d_{G}(v)-1\right)-2}{\left(d_{G}(u)-1\right)\left(d_{G}(v)-1\right)}} \\
& =\left(\frac{2-1+2-1-2}{(2-1)(2-1)}\right)^{\frac{p}{2}} \times\left(\frac{2-1+3-1-2}{(2-1)(3-1)}\right)^{\frac{2 p}{2}} \times\left(\frac{3-1+3-1-2}{(3-1)(3-1)}\right)^{\frac{3 p q-p}{2}} \\
& =\left(\frac{1}{2}\right)^{\frac{1}{2}(3 p q+p)}
\end{aligned}
$$

Theorem 9: The multiplicative reduced geometric-arithmetic index of $T U A C_{6}[p, q]$ is

$$
\operatorname{RGAII}\left(T U A C_{6}[p, q]\right)=\left(\frac{2 \sqrt{2}}{3}\right)^{2 p}
$$

Proof: Let $G T U A C_{6}[p, q]$. By using definition and Table 2, we deduce

$$
\begin{aligned}
\operatorname{RGAII}\left(\text { TUAC }_{6}[p, q]\right) & =\prod_{u v \in E(G)} \frac{2 \sqrt{\left(d_{G}(u)-1\right)\left(d_{G}(v)-1\right)}}{d_{G}(u)-1+d_{G}(v)-1} \\
& =\left(\frac{2 \sqrt{(2-1)(2-1)}}{2-1+2-1}\right)^{p} \times\left(\frac{2 \sqrt{(2-1)(3-1)}}{2-1+3-1}\right)^{2 p} \times\left(\frac{2 \sqrt{(3-1)(3-1)}}{3-1+3-1}\right)^{3 p q-p} \\
& =\left(\frac{2 \sqrt{2}}{3}\right)^{2 p}
\end{aligned}
$$

Theorem 10: The multiplicative reduced arithmetic geometric index of $T U A C_{6}[p, q]$ is

$$
\operatorname{RAGII}\left(T U A C_{6}[p, q]\right)=\left(\frac{3}{2 \sqrt{2}}\right)^{2 p} .
$$

Proof: Let $G=T U A C_{6}[p, q]$. By using definition and Table 2, we derive

$$
\begin{aligned}
& \operatorname{RAGII}\left(\text { TUAC }_{6}[p, q]\right)=\prod_{u v \in E(G)} \frac{d_{G}(u)-1+d_{G}(v)-1}{2 \sqrt{\left(d_{G}(u)-1\right)\left(d_{G}(v)-1\right)}} \\
& \quad=\left(\frac{2-1+2-1}{2 \sqrt{(2-1)(2-1)}}\right)^{p} \times\left(\frac{2-1+3-1}{2 \sqrt{(2-1)(3-1)}}\right)^{2 p} \times\left(\frac{3-1+3-1}{2 \sqrt{(3-1)(3-1)}}\right)^{3 p q-p} \\
& \quad=\left(\frac{3}{2 \sqrt{2}}\right)^{10 n} .
\end{aligned}
$$

## 4. RESULTS FOR ZIGZAG POLYHEX NANOTUBES

In this section, we consider the family of zigzag polyhex nanotubes which is symbolized by $T U A C_{6}[p, q]$, where p is the number of hexagons in a row whereas $q$ is the number of hexagons in a column. A 2-dimensional network of $T U A C_{6}[p, q]$ is presented in Figure 3.


Figure-3
Let $G$ be the graph of $T U Z C_{6}[p, q]$. By calculation $G$ has $2 p(q+1)$ vertices and $3 p q+2 p$ edges. In $G$, there are two types of edges based on degrees of end vertices of each edge. The edge partition of $G$ is given in Table 3.

| $D_{G}(u), D_{G}(v) \backslash u v \in E(G)$ | $(2,3)$ | $(3,3)$ |
| :--- | :---: | :---: |
| Number of edges | $4 p$ | $3 p q-2 p$ |

Table-3: Edge Partition of $T U Z C_{6}[p, q]$
Theorem 11: The general multiplicative reduced second Zagreb index of $T_{U Z C_{6}}[p, q]$ is

$$
R M_{2}^{a} I I\left(T U Z C_{6}[p, q]\right)=2^{6 a p q}
$$

Proof: Let $G=T U Z C_{6}[p, q]$. By using definition and Table 3, we derive

$$
\begin{aligned}
\operatorname{RM}_{2}^{a} I I\left(\operatorname{TUZC}_{6}[p, q]\right) & =\prod_{u v \in E(G)}\left[\left(d_{G}(u)-1\right)\left(d_{G}(v)-1\right)\right]^{a} \\
& =[(2-1)(3-1)]^{4 a p} \times[(3-1)(3-1)]^{a(3 p q-2 p)} \\
& =2^{6 a p q} .
\end{aligned}
$$

From Theorem 6, and by using definitions, we obtain the following results.

1) $R M_{2} I I\left(\operatorname{TUZC}_{6}[p, q]\right)=2^{6 p q}$.
2) $R H M_{2} I I\left(T U Z C_{6}[p, q]\right)=2^{12 p q}$.
3) ${ }^{m} R M_{2} I I\left(T U Z C_{6}[p, q]\right)=\left(\frac{1}{2}\right)^{6 p q}$.
4) $\operatorname{RPII}\left(\right.$ TUZC $\left._{6}[p, q]\right)=2^{-3 p q}$.
5) $\quad \operatorname{RRPII}\left(T U Z C_{6}[p, q]\right)=2^{3 p q}$.

Theorem 12: The multiplicative reduced $\mathrm{F}_{1}$-index of $\operatorname{TUZC}_{6}[p, q]$ is

$$
R F_{1} I I\left(T U A C_{6}[p, q]\right)=2^{2 p} \times 2^{9 p q-2 p}
$$

Proof: Let $G=T U Z C_{6}[p, q]$. By using definition and Table 3, we obtain

$$
\begin{aligned}
R F_{1} I I\left(T U Z C_{6}[p, q]\right) & =\prod_{u v \in E(G)}\left[\left(d_{G}(u)-1\right)^{2}+\left(d_{G}(v)-1\right)^{2}\right] \\
& =\left[(2-1)^{2}+(3-1)^{2}\right]^{4 p} \times\left[(3-1)^{2}+(3-1)^{2}\right]^{3 p q-2 p} \\
& =5^{4 p} \times 2^{9 p q-6 p} .
\end{aligned}
$$

Theorem 13: The multiplicative reduced ABC index of $T U Z C_{6}[p, q]$ is

$$
\operatorname{RABCII}\left(\text { TUZC }_{6}[p, q]\right)=\left(\frac{1}{2}\right)^{\frac{3}{2} p q+p}
$$

Proof: Let $G=T U Z C_{6}[p, q]$. By using definition and Table 3 we have

$$
\begin{aligned}
\operatorname{RABCII}\left(\text { TUZC }_{6}[p, q]\right) & =\prod_{u v \in E(G)} \sqrt{\frac{\left(d_{G}(u)-1\right)+\left(d_{G}(v)-1\right)-2}{\left(d_{G}(u)-1\right)\left(d_{G}(v)-1\right)}} \\
& =\left(\frac{2-1+3-1-2}{(2-1)(3-1)}\right)^{\frac{4 p}{2}} \times\left(\frac{b^{-1+3-1-2}}{(3-1)(3-1)}\right)^{\frac{3 p q-2 p}{2}} \times\left(\frac{1}{2}\right)^{\frac{3}{2} \frac{p q-p}{}} \\
& =\left(\frac{1}{2}\right)^{\frac{3}{2} \frac{p q+p}{}} .
\end{aligned}
$$

Theorem 14: The multiplicative reduced geometric arithmetic index of $\operatorname{TUZC}_{6}[p, q]$ is

$$
\operatorname{RGAII}\left({T U A C_{6}}[p, q]\right)=\left(\frac{2 \sqrt{2}}{3}\right)^{4 p}
$$

Proof: Let $G=T U Z C_{6}[p, q]$ by using definition and Table 3, we derive

$$
\begin{aligned}
\operatorname{RGAII}\left(\text { TUZC }_{6}[p, q]\right) & =\prod_{u v \in E(G)} \frac{2 \sqrt{\left(d_{G}(u)-1\right)\left(d_{G}(v)-1\right)}}{d_{G}(u)-1+d_{d}(v)-1} \\
& =\left(\frac{2 \sqrt{(2-1)(3-1)}}{2-1+\sqrt{3-1}}\right)^{4 p} \times\left(\frac{2 \sqrt{(3-1)(3-1)}}{3-1+\sqrt{3-1}}\right)^{3 p q-2 p} \\
& =\left(\frac{2 \sqrt{2}}{3}\right)^{2 p} .
\end{aligned}
$$

Theorem 15: The multiplicative reduced arithmetic arithmetic index of $\operatorname{TUZC}_{6}[p, q]$ is

$$
\operatorname{RAGII}\left(\text { TUZC }_{6}[p, q]\right)=\left(\frac{3}{2 \sqrt{2}}\right)^{4 p} .
$$

Proof: Let $G=T U Z C_{6}[p, q]$. From definition and by using Table 3, we deduce

$$
\begin{aligned}
\operatorname{RAGII}\left(\text { TUAC }_{6}[p, q]\right) & =\prod_{u v \in E(G)} \frac{d_{G}(u)-1+d_{G}(v)-1}{2 \sqrt{\left(d_{G}(u)-1\right)\left(d_{G}(v)-1\right)}} \\
& =\left(\frac{2-1+3-1}{2 \sqrt{(2-1)(3-1)}}\right)^{4 p} \times\left(\frac{3-1+3-1}{2 \sqrt{(3-1)(3-1)}}\right)^{3 p q-2 p} \\
& =\left(\frac{3}{2 \sqrt{2}}\right)^{4 p} .
\end{aligned}
$$

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