International Journal of Mathematical Archive-11(12), 2020, 14-20
Available online through www.ijma.info ISSN 2229 – 5046

ON A COMMON FIXED POINT RESULT IN INTUITIONISTIC FUZZY METRIC SPACE

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(Received On: 30-11-20; Revised & Accepted On: 28-12-20)

ABSTRACT

In this paper, we prove a common fixed point theorem for new type of common limit range property under functionally rationalized contractive condition in intuitionistic fuzzy metric space.

2010 Mathematics Subject Classification: *47H10, 54H25.*

Keywords: *Intuitionistic fuzzy metric space (IFMS), weakly compatible mapping, new type of common limit in the range property.*

1. INTRODUCTION

Kramosil and Michalek [10] introduced the notion of fuzzy metric space using the idea of [17], which opened the way for further development of analysis. IFMS is defined by Alaca *et al.* [3] using continuous t-norms and t-conorms with the help of intuitionistic fuzzy set as a generalization of fuzzy metric space, as defined by Kramosil and Michalek [10]. The consequence of fixed point theory is marked from the fact that it has its applications in different disciplines of Science, Engineering, and Economics in dealing with problems arising in: Approximation theory, potential theory, game theory, mathematical economics, etc. Intuitionistic fuzzy fixed point theory has turn into a topic of enormous attention for expert in fixed point theory for the reason that this branch of mathematics has cover novel possibilities for fixed point theorists. Some recent important results using IFMS are [5, 8, 14].

There is a diversity of different mappings used by different authors to take a range of fixed point theorems in various fuzzy spaces. Aamri and Moutawakil [1] gave the idea of property E.A. for a pair of self mappings which contains the class of non-compatible mappings. On the other hand, the notion of common limit range property is given by Sintunavarat and Kumam [13] as an improvement of property E.A., which relaxes the condition of closedness of the underlying subspace. Recently, Popa and Patriciu [11] introduced a new type of common limit in the range property in metric space and proved some fixed point results. Some recent important results using common limit in the range property are [4, 5, 6, 7, 15, 16].

The aim of this paper is to prove a common fixed point result in IFMS under functionlly rantioallized contrctive conditon using new type of common limit range property motivated by [11].

2. PRELIMINARIES

In this section, we have recalled some definitions and useful results which are already in the literature.

Definition 2.1 [12]: A binary operation $*: [0,1] \times [0,1] \rightarrow [0,1]$ is called a continuous t-norm if $([0,1],*)$ is an abelian topological monoid with the unit 1 such that $a * b \leq c * d$, whenever $a \leq c$ and $b \leq d$, $\forall a,b,c,d \in [0,1]$.

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Definition 2.2 [12]: A binary operation ◊ : [0, 1] ×[0, 1] → [0, 1] is continuous t-conorm if ◊ satisfies the following conditions: (i) ◊ is commutative and associative; (ii) ◊ is continuous; (iii) a◊0 = a for all a ∈ [0, 1]; (iv) a◊b ≤ c◊d whenever a ≤ c and b ≤ d for all a, b, c, d∈[0, 1].
Alaca et al. [3] defined the notion of IFMS as follows:

Definition 2.3 [3]: A 5-tuple (X, M, N, *, ◊) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous t-norm, ◊ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times [0, \infty)$ satisfying

(i) M(x, y, t) + N(x, y, t) ≤ 1 ∀ x, y ∈ X and t > 0;
(ii) M(x, y, 0) = 0 ∀ x, y ∈ X;
(iii) M(x, y, t) = 1 ∀ x, y ∈ X and t > 0 if and only if x = y;
(iv) M(x, y, t) = M(y, x, t) for all x, y ∈ X and t > 0;
(v) M(x, y, t) *M(y, z, s) ≤ M(x, z, t + s) ∀ x, y, z ∈ X and s, t > 0;
(vi) ∀ x, y ∈ X, M(x, y, ·) : [0,∞) → [0, 1] is left continuous;
(vii) $\lim_{t\to\infty}$ M(x, y, t) = 1 ∀ x, y ∈ X and t > 0;
(viii) N(x, y, 0) = 1 ∀ x, y ∈ X;
(ix) N(x, y, t) = 0 ∀ x, y ∈ X and t > 0 if and only if x = y;
(x) N(x, y, t) = N(y, x, t) for all x, y ∈ X and t > 0;
(xi) N(x, y, t) ◊ N(y, z, s) ≥ N(x, z, t + s) ∀ x, y, z ∈ X and s, t > 0;
(xii) N(x, y, ·) : [0,∞) → [0, 1] is right continuous;
(xiii) $\lim_{t\to\infty}$ N(x, y, t) = 0 ∀ x, y ∈ X

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of nonnearness between x and y with respect to t, respectively.

Remark 2.4 [3]. Every fuzzy metric space (X, M, *) is an IFMS of the form (X, M, 1 − M, *, ◊) such that t-norm * and t-conorm ◊ are associated as x◊y = 1−((1−x)*(1−y)) ∀ x, y ∈X.

Remark 2.5 [3]. In an IFMS (X, M, N, *, ◊), M(x, y, ·) is non-decreasing and N(x, y, ·) is non-increasing ∀ x, y ∈ X.

Definition 2.6 [9]: Two self-maps P and Q on set X are said to be weakly compatible if they commute at their coincident point.

Definition 2.7 [1]: A pair of self mapping (f, g) on X said to satisfy the property (E.A.) if there exist a sequence x_n such that

$$\lim_{n\to\infty} f x_n = \lim_{n\to\infty} g x_n = t,$$

for some $t \in X$.

Definition 2.8 [13]: A pair (f, g) of self-mappings of a fuzzy metric space $(X, M, *)$ is said to satisfy the common limit in the range property with respect to mapping g (briefly CLR_g) property, if there exists a sequence x_n in X such that

$$\lim_{n\to\infty} f x_n = \lim_{n\to\infty} g x_n = gz, \text{ for some } z \in X.$$

Recently, Popa and Patriciu [11] introduced a new type of common limit range property for self mappings in metric space.

Definition 2.9 [11]: Let A, S and T be self mappings of a metric space (X, d). The pair (A, S) is said to satisfy common limit range property with respect to T (shortly $CLR_{(A,S)T}$ property), if there exists a sequence x_n in X such that

$$\lim_{n\to\infty} A x_n = \lim_{n\to\infty} S x_n = t, \text{ for some } t \in S(X) \cap T(X).$$

Lemma 2.10 [2]: Let (X, M, N, ∗, ◊) be an IFMS and if for a number k ε (0, 1),

$$M(x, y, kt) \geq M(x, y, t) \text{ and } N(x, y, kt) \leq N(x, y, t)$$

for all x, y ε X, t > 0, then x = y.

Lemma 2.11 [11]: Let f, g be two weakly compatible self mappings of a nonempty set X. If f and g have a unique point of coincidence $w = fx = gx$ for some $x \in X$, then w is the unique common fixed point of f and g.

We use the following relations in our results:

(δ) $\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2 : [0,1] \to [0,1]$ are continuous functions such that $\alpha_1 \& \alpha_2, \beta_1 \& \beta_2$ and $\gamma_1 \& \gamma_2$ cannot be simultaneously 0, and for $h \in [0,1]$

$$\alpha_1(h) + \alpha_2(h) = 1,\ \beta_1(h) + \beta_2(h) = 1, \gamma_1(h) + \gamma_2(h) = 1.$$

(δ^*) Let Φ be the set of all real valued continuous function $\phi, \phi' : [0, \infty)^5 \to [0, \infty)$ such that

($\delta_{2.1}$) ϕ is non increasing and ϕ' is non decreasing,

$(\delta_{2.2})$ for all s, $s' \geq 0$,

$$\phi(1,1,s,1,s) \geq s, \quad \phi'(1,1,s',1,s') \leq s' \text{ and}$$
$$\phi(1,s,1,s,1) \geq s, \quad \phi'(1,s',1,s',1) \leq s' \text{ and}$$
$$\phi(s,1,1,s,s) \geq s, \quad \phi'(s',1,1,s',s') \leq s'.$$

Motivated form Definition 2.9, we can have following definition in intuitionistic fuzzy metric space:

Definition 2.12: Let A, S and T be self mappings of an intuitionistic fuzzy metric space ()IMFS) $(X, M, N, *, \Diamond)$. The pair (A, S) is said to satisfy common limit range property with respect to T (shortly $CLR_{(A,S)T}$ property), if there exists a sequence x_n in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x,$$

i.e.,

$$\lim_{n\to\infty} M(Ax_n, x, t) = \lim_{n\to\infty} M(Sx_n, x, t) = 1$$

and

$$\lim_{n\to\infty} N(Ax_n, x, t) = \lim_{n\to\infty} N(Sx_n, x, t) = 0,$$

for some $x \in S(X) \cap T(X)$.

Now, we prove our main result.

3. MAIN RESULT

Theorem 3.1: Let $(X, M, N, *, \Diamond)$ be a IFMS with continuous t- norm and t-conorm. Let A, B, S and T be self-mappings from $(X, M, N, *, \Diamond)$ into itself satisfying:

(3.1) for all $x, y \in X$, $t > 0$ and for a number $k \in (0,1)$,

$$M(Ax, By, kt) \geq \phi\left(\begin{array}{c} \dfrac{\alpha_2(h)M(Ty,Ax,t) + \alpha_1(h)M(Ty,Sx,t)}{\alpha_1(h) + \alpha_2(h)M(Ax,Sx,t)}, \\ M(Ax,Sx,t), M(By,Ty,t), \\ \dfrac{\beta_2(h)M(Ax,By,t) + \beta_1(h)M(Ax,Ty,t)}{\beta_1(h) + \beta_2(h)M(By,Ty,t)}, \\ \dfrac{\gamma_2(h)M(By,Ax,t) + \gamma_1(h)M(By,Sx,t)}{\gamma_1(h) + \gamma_2(h)M(Ax,Sx,t)} \end{array}\right),$$

and

$$N(Ax, By, kt) \leq \phi'\left(\begin{array}{c} \dfrac{\alpha_2(h)N(Ty,Ax,t) + \alpha_1(h)N(Ty,Sx,t)}{\alpha_1(h) + \alpha_2(h)N(Ax,Sx,t)}, \\ N(Ax,Sx,t), N(By,Ty,t), \\ \dfrac{\beta_2(h)M(Ax,By,t) + \beta_1(h)N(Ax,Ty,t)}{\beta_1(h) + \beta_2(h)N(By,Ty,t)}, \\ \dfrac{\gamma_2(h)N(By,Ax,t) + \gamma_1(h)N(By,Sx,t)}{\gamma_1(h) + \gamma_2(h)N(Ax,Sx,t)} \end{array}\right),$$

where $\phi, \phi' \in \Phi$ and $\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2$ are defined as above and if (A, S) and T enjoys the $CLR_{(A,S)T}$ property, then $C(A,S) \neq \emptyset$ and $C(B,T) \neq \emptyset$.

Moreover, if (A, S) and (B, T) are weakly compatible then A, B, S and T have a unique common fixed point in X.

Proof: Since (A, S) and T enjoys $CLR_{(A,S)T}$ property, then there exists a sequence x_n in X such that

$$lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = z, \text{ for some } z \in S(X) \cap T(X).$$

Since $z \in T(X)$, there exists $u \in X$ such that $z = Tu$.

Using (3.1), we have

$$M(Ax_n, Bu, kt) \geq \phi\left(\begin{array}{c} \dfrac{\alpha_2(h)M(Tu,Ax_n,t) + \alpha_1(h)M(Tu,Sx_n,t)}{\alpha_1(h) + \alpha_2(h)M(Ax_n,Sx_n,t)}, \\ M(Ax_n,Sx_n,t), M(Bu,Tu,t), \\ \dfrac{\beta_2(h)M(Ax_n,Bu,t) + \beta_1(h)M(Ax_n,Tu,t)}{\beta_1(h) + \beta_2(h)M(Bu,Tu,t)}, \\ \dfrac{\gamma_2(h)M(Bu,Ax_n,t) + \gamma_1(h)M(Bu,Sx_n,t)}{\gamma_1(h) + \gamma_2(h)M(Ax_n,Sx_n,t)} \end{array}\right).$$

Take the limit as $n \to \infty$, we get,

$$M(z, Bu, kt) \geq \phi\begin{pmatrix} \frac{\alpha_2(h)M(z,z,t) + \alpha_1(h)M(z,z,t)}{\alpha_1(h) + \alpha_2(h)M(z,z,t)}, \\ M(z,z,t), M(Bu,z,t), \\ \frac{\beta_2(h)M(z,Bu,t) + \beta_1(h)M(z,z,t)}{\beta_1(h) + \beta_2(h)M(Bu,z,t)}, \\ \frac{\gamma_2(h)M(Bu,z,t) + \gamma_1(h)M(Bu,z,t)}{\gamma_1(h) + \gamma_2(h)M(z,z,t)} \end{pmatrix}.$$

$$M(z, Bu, kt) \geq \phi\big(1, 1, M(Bu,z,t), 1, M(Bu,z,t)\big).$$

Using $(\delta_{2.2})$, we have,

$$M(Bu, z, kt) \geq M(Bu, z, t) \tag{1}$$

and

$$N(Ax_n, Bu, kt) \leq \phi'\begin{pmatrix} \frac{\alpha_2(h)N(Tu,Ax_n,t) + \alpha_1(h)N(Tu,Sx_n,t)}{\alpha_1(h) + \alpha_2(h)N(Ax_n,Sx_n,t)}, \\ N(Ax_n,Sx_n,t), N(Bu,Tu,t), \\ \frac{\beta_2(h)N(Ax_n,Bu,t) + \beta_1(h)N(Ax_n,Tu,t)}{\beta_1(h) + \beta_2(h)N(Bu,Tu,t)}, \\ \frac{\gamma_2(h)N(Bu,Ax_n,t) + \gamma_1(h)N(Bu,Sx_n,t)}{\gamma_1(h) + \gamma_2(h)N(Ax_n,Sx_n,t)} \end{pmatrix}.$$

Take the limit as $n \to \infty$, we get,

$$N(z, Bu, kt) \leq \phi'\begin{pmatrix} \frac{\alpha_2(h)N(z,z,t) + \alpha_1(h)N(z,z,t)}{\alpha_1(h) + \alpha_2(h)N(z,z,t)}, \\ N(z,z,t), N(Bu,z,t), \\ \frac{\beta_2(h)N(z,Bu,t) + \beta_1(h)N(z,z,t)}{\beta_1(h) + \beta_2(h)M(Bu,z,t)}, \\ \frac{\gamma_2(h)N(Bu,z,t) + \gamma_1(h)N(Bu,z,t)}{\gamma_1(h) + \gamma_2(h)N(z,z,t)} \end{pmatrix}.$$

$$N(z, Bu, kt) \leq \phi'\big(1, 1, N(Bu,z,t), 1, N(Bu,z,t)\big).$$

Using $(\delta'_{2.2})$, we have,

$$N(Bu, z, kt) \leq N(Bu, z, t) \tag{2}$$

Using (1), (2) and Lemma 2.10, we have, $Bu = z = Tu$. Therefore $C(B,T) \neq \emptyset$.

Since $z \in S(X)$, there exists $v \in X$ such that $z = Sv$. Using (3.1), we have

$$M(Av, Bu, kt) \geq \phi\begin{pmatrix} \frac{\alpha_2(h)M(Tu,Av,t) + \alpha_1(h)M(Tu,Sv,t)}{\alpha_1(h) + \alpha_2(h)M(Av,Sv,t)}, \\ M(Av,Sv,t), M(Bu,Tu,t), \\ \frac{\beta_2(h)M(Av,Bu,t) + \beta_1(h)M(Av,Tu,t)}{\beta_1(h) + \beta_2(h)M(Bu,Tu,t)}, \\ \frac{\gamma_2(h)M(Bu,Av,t) + \gamma_1(h)M(Bu,Sv,t)}{\gamma_1(h) + \gamma_2(h)M(Av,Sv,t)} \end{pmatrix}.$$

$$M(Av, z, kt) \geq \phi\begin{pmatrix} \frac{\alpha_2(h)M(z,Av,t) + \alpha_1(h)M(z,z,t)}{\alpha_1(h) + \alpha_2(h)M(Av,z,t)}, \\ M(Av,z,t), M(z,z,t), \\ \frac{\beta_2(h)M(Av,z,t) + \beta_1(h)M(Av,z,t)}{\beta_1(h) + \beta_2(h)M(z,z,t)}, \\ \frac{\gamma_2(h)M(z,Av,t) + \gamma_1(h)M(z,z,t)}{\gamma_1(h) + \gamma_2(h)M(Av,z,t)} \end{pmatrix}.$$

$$M(Av, z, kt) \geq \phi(1, M(Av,z,t), 1, M(Av,z,t), 1).$$

Using $(\delta_{2.2})$, we have,

$$M(Av,z,kt) \geq M(Av,z,t) \tag{3}$$

and

$$N(Av,Bu,kt) \leq \phi'\left(\begin{array}{c}\dfrac{\alpha_2(h)N(Tu,Av,t)+\alpha_1(h)N(Tu,Sv,t)}{\alpha_1(h)+\alpha_2(h)N(Av,Sv,t)},\\ N(Av,Sv,t),N(Bu,Tu,t),\\ \dfrac{\beta_2(h)N(Av,Bu,t)+\beta_1(h)N(Av,Tu,t)}{\beta_1(h)+\beta_2(h)N(Bu,Tu,t)},\\ \dfrac{\gamma_2(h)N(Bu,Av,t)+\gamma_1(h)N(Bu,Sv,t)}{\gamma_1(h)+\gamma_2(h)N(Av,Sv,t)}\end{array}\right).$$

$$N(Av,z,kt) \leq \phi'\left(\begin{array}{c}\dfrac{\alpha_2(h)N(z,Av,t)+\alpha_1(h)N(z,z,t)}{\alpha_1(h)+\alpha_2(h)N(Av,z,t)},\\ N(Av,z,t),N(z,z,t),\\ \dfrac{\beta_2(h)N(Av,z,t)+\beta_1(h)N(Av,z,t)}{\beta_1(h)+\beta_2(h)N(z,z,t)},\\ \dfrac{\gamma_2(h)N(z,Av,t)+\gamma_1(h)N(z,z,t)}{\gamma_1(h)+\gamma_2(h)N(Av,z,t)}\end{array}\right).$$

$$N(Av,z,kt) \leq \phi'(1,N(Av,z,t),1,N(Av,z,t),1).$$

Using $(\delta'_{2.2})$, we have,

$$N(Av,z,kt) \leq N(Av,z,t) \tag{4}$$

Using (3), (4) and Lemma 2.10, we have, $Av = z = Sv$. Therefore $C(A,S) \neq \emptyset$.

Hence, $z = Av = Sv = Bu = Tu$ and z is a point of coincidence of (A,S) and of (B,T).

We show that z is the unique point of coincidence of (A,S) and (B,T).

Let p be another point of coincidence of (A,S), i.e., $p = Aw = Sw$. Using (3.1), we get

$$M(Aw,Bu,kt) \geq \phi\left(\begin{array}{c}\dfrac{\alpha_2(h)M(Tu,Aw,t)+\alpha_1(h)M(Tu,Sw,t)}{\alpha_1(h)+\alpha_2(h)M(Aw,Sw,t)},\\ M(Aw,Sw,t),M(Bu,Tu,t),\\ \dfrac{\beta_2(h)M(Aw,Bu,t)+\beta_1(h)M(Aw,Tu,t)}{\beta_1(h)+\beta_2(h)M(Bu,Tu,t)},\\ \dfrac{\gamma_2(h)M(Bu,Aw,t)+\gamma_1(h)M(Bu,Sw,t)}{\gamma_1(h)+\gamma_2(h)M(Aw,Sw,t)}\end{array}\right),$$

$$M(p,z,kt) \geq \phi\left(\begin{array}{c}\dfrac{\alpha_2(h)M(z,p,t)+\alpha_1(h)M(z,p,t)}{\alpha_1(h)+\alpha_2(h)M(p,p,t)},\\ M(p,p,t),M(p,p,t),\\ \dfrac{\beta_2(h)M(p,z,t)+\beta_1(h)M(p,z,t)}{\beta_1(h)+\beta_2(h)M(z,z,t)},\\ \dfrac{\gamma_2(h)M(z,p,t)+\gamma_1(h)M(z,p,t)}{\gamma_1(h)+\gamma_2(h)M(p,p,t)}\end{array}\right),$$

$$M(p,z,kt) \geq \phi\big(M(z,p,t),1,1,M(p,z,t),M(p,z,t)\big),$$

Using $(\delta_{2.2})$, we have,

$$M(p,z,kt) \geq M(p,z,t) \tag{5}$$

and

$$N(Aw,Bu,kt) \leq \phi'\left(\begin{array}{c}\dfrac{\alpha_2(h)N(Tu,Aw,t)+\alpha_1(h)N(Tu,Sw,t)}{\alpha_1(h)+\alpha_2(h)N(Aw,Sw,t)},\\ N(Aw,Sw,t),N(Bu,Tu,t),\\ \dfrac{\beta_2(h)N(Aw,Bu,t)+\beta_1(h)N(Aw,Tu,t)}{\beta_1(h)+\beta_2(h)N(Bu,Tu,t)},\\ \dfrac{\gamma_2(h)N(Bu,Aw,t)+\gamma_1(h)N(Bu,Sw,t)}{\gamma_1(h)+\gamma_2(h)N(Aw,Sw,t)}\end{array}\right),$$

$$N(p,z,kt) \leq \phi'\left(\begin{array}{c}\dfrac{\alpha_2(h)N(z,p,t)+\alpha_1(h)N(z,p,t)}{\alpha_1(h)+\alpha_2(h)N(p,p,t)},\\ N(p,p,t),N(p,p,t),\\ \dfrac{\beta_2(h)N(p,z,t)+\beta_1(h)N(p,z,t)}{\beta_1(h)+\beta_2(h)N(z,z,t)},\\ \dfrac{\gamma_2(h)N(z,p,t)+\gamma_1(h)N(z,p,t)}{\gamma_1(h)+\gamma_2(h)N(p,p,t)}\end{array}\right),$$

$$N(p,z,kt) \leq \phi'\big(N(z,p,t),1,1,N(p,z,t),N(p,z,t)\big),$$

Using $(\delta'_{2.2})$, we have,

$$N(p,z,kt) \leq N(p,z,t) \tag{6}$$

Using (5), (6) and Lemma 2.10, we have, $p = z$. Thus, z is the unique point of coincidence of (A,S). Similarly, using (3.1) it is easy to see that z is the unique point of coincidence of (B,T).

Further, if (A,S) and (B,T) are weakly compatible, then by Lemma 2.11, z is the unique common fixed point of A,B,S and T. This completes the proof of the theorem.

If we take $\alpha_2(h) = \beta_2(h) = \gamma_2(h) = 0$, for all $h \in [0,1]$ in Theorem 3.1, then we have following:

Corollary 3.2: Let $(X,M,N,*,\Diamond)$ be a IFMS with continuous t- norm and t-conorm. Let A, B, S and T be self-mappings from $(X,M,N,*,\Diamond)$ into itself satisfying:
(3.2) for all $x,y \in X$, $t > 0$ and for a number $k \in (0,1)$,

$$M(Ax,By,kt) \geq \phi\left(\begin{array}{c}M(Ty,Sx,t),M(Ax,Sx,t),\\ M(By,Ty,t),M(Ax,Ty,t),M(By,Sx,t)\end{array}\right),$$

and

$$N(Ax,By,kt) \geq \phi'\left(\begin{array}{c}N(Ty,Sx,t),N(Ax,Sx,t),\\ N(By,T\,y,t),N(Ax,Ty,t),N(By,Sx,t)\end{array}\right),$$

where ϕ, $\phi' \in \Phi$ and $\alpha_1,\alpha_2,\beta_1,\beta_2,\gamma_1,\gamma_2$ are defined above and if (A,S) and T enjoys the $CLR_{(A,S)T}$ property, then $C(A,S) \neq \emptyset$ and $C(B,T) \neq \emptyset$.

Moreover, if (A,S) and (B,T) are weakly compatible then A, B, S and T have a unique common fixed point in X.

Proof: Proof follows form Theorem 3.1 by taking $\alpha_2(h) = \beta_2(h) = \gamma_2(h) = 0$, for all $h \in [0,1]$.

CONCLUSION

In this paper, we proved a common fixed point theorem in intuitionistic fuzzy metric space using new type of common limit in the range property under functionally rationalized contractive condition for four mappings.

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Source of support: Nil, Conflict of interest: None Declared.