

GENERALIZATION OF REGULAR CONTINUOUS FUNCTION  
IN FUZZY TOPOLOGICAL SPACES

ARUN KUMAR GALI\*

Department of Mathematics,  
Government First Grade College Sector no.-43,  
Navanagar Bagalkote-587103, Karnataka State, INDIA.

(Received On: 10-11-20; Revised & Accepted On: 01-12-20)

---

ABTRACT

*In this paper, we introduce the concept of fuzzy generalized regular -continuous maps and fuzzy generalized regular-irresolute maps in fuzzy topological spaces. We prove that the composition of two fuzzy generalized regular -continuous maps need not be fuzzy generalized regular -continuous and study some of their properties.*

**2010 Mathematics Subject Classification:** 54A40.

**Key words and phrases:** Fuzzy rg-continuous maps, fuzzy rg-irresolute maps.

---

INTRODUCTION

The concept of a fuzzy subset was introduced and studied by L.A.Zadeh in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. In the year 1965, L.A.Zadeh [1] introduced the concept of fuzzy subset as a generalization of that of an ordinary subset. The introduction of fuzzy subsets paved the way for rapid research work in many areas of mathematical science. In the year 1968, C.L.Chang [2] introduced the concept of fuzzy topological spaces as an application of fuzzy sets to topological spaces. Subsequently several researchers contributed to the development of the theory and applications of fuzzy topology. The theory of fuzzy topological spaces can be regarded as a generalization theory of topological spaces. An ordinary subset  $A$  or a set  $X$  can be characterized by a Function called characteristic Function  $\mu_A : X \rightarrow [0,1]$  of  $A$ , defined by

$$\mu_A(x) = \begin{cases} 1, & \text{if } x \in A. \\ 0, & \text{if } x \notin A \end{cases}$$

Thus an element  $x \in X$  is in  $A$  if  $\mu_A(x) = 1$  and is not in  $A$  if  $\mu_A(x) = 0$ . In general if  $X$  is a set and  $A$  is a subset of  $X$  then  $A$  has the following representation.  $A = \{(x, \mu_A(x)) : x \in X\}$ , here  $\mu_A(x)$  may be regarded as the degree of belongingness of  $x$  to  $A$ , which is either 0 or 1. Hence  $A$  is the class of objects with degree of belongingness either 0 or 1 only. Prof. L.A.Zadeh [1] introduced a class of objects with continuous grades of belongingness ranging between 0 and 1; he called such a class as fuzzy subset. A fuzzy subset  $A$  in  $X$  is characterized as a membership Function  $\mu_A : X \rightarrow [0,1]$ , which associates with each point in  $x$  a real number  $\mu_A(x)$  between 0 and 1 which represents the degree or grade membership of belongingness of  $x$  to  $A$ .

The purpose of this paper is to introduce a new class of fuzzy sets called fuzzy rg-closed sets in fuzzy topological spaces and investigate certain basic properties of these fuzzy sets. Among many other results it is observed that every fuzzy closed set is fuzzy rg-closed but not

conversely. Also we introduce fuzzy rg-open sets in fuzzy topological spaces and study some of their properties.

---

**Corresponding Author: Arun Kumar Gali\***  
**Department of Mathematics, Government First Grade College Sector no-43,**  
**Navanagar Bagalkote-587103, Karnataka State, INDIA.**

## 1. PRELIMINARIES

**1.1 Definition: [1]** A fuzzy subset  $A$  in a set  $X$  is a Function  $A: X \rightarrow [0, 1]$ . A fuzzy subset in  $X$  is empty iff its membership Function is identically 0 on  $X$  and is denoted by  $0$  or  $\mu_\phi$ . The set  $X$  can be considered as a fuzzy subset of  $X$  whose membership Function is identically 1 on  $X$  and is denoted by  $\mu_x$  or  $I_x$ . In fact every subset of  $X$  is a fuzzy subset of  $X$  but not conversely. Hence the concept of a fuzzy subset is a generalization of the concept of a subset.

**1.2 Definition: [1]** If  $A$  and  $B$  are any two fuzzy subsets of a set  $X$ , then  $A$  is said to be included in  $B$  or  $A$  is contained in  $B$  iff  $A(x) \leq B(x)$  for all  $x$  in  $X$ . Equivalently,  $A \leq B$  iff  $A(x) \leq B(x)$  for all  $x$  in  $X$ .

**1.3 Definition: [1]** Two fuzzy subsets  $A$  and  $B$  are said to be equal if  $A(x) = B(x)$  for every  $x$  in  $X$ .

Equivalently  $A = B$  if  $A(x) = B(x)$  for every  $x$  in  $X$ .

**1.4 Definition: [1]** The complement of a fuzzy subset  $A$  in a set  $X$ , denoted by  $A' = 1 - A$ , is the fuzzy subset of  $X$  defined by  $A'(x) = 1 - A(x)$  for all  $x$  in  $X$ . Note that  $(A')' = A$ .

**1.5 Definition: [1]** The union of two fuzzy subsets  $A$  and  $B$  in  $X$ , denoted by  $A \vee B$ , is a fuzzy subset in  $X$  defined by  $(A \vee B)(x) = \text{Max}\{\mu_A(x), \mu_B(x)\}$  for all  $x$  in  $X$ .

**1.6 Definition: [1]** The intersection of two fuzzy subsets  $A$  and  $B$  in  $X$ , denoted by  $A \wedge B$ , is a fuzzy subset in  $X$  defined by  $(A \wedge B)(x) = \text{Min}\{A(x), B(x)\}$  for all  $x$  in  $X$ .

**1.7 Definition: [1]** A fuzzy set on  $X$  is 'Crisp' if it takes only the values 0 and 1 on  $X$ .

**1.8 Definition: [2]** Let  $X$  be a set and  $\tau$  be a family of fuzzy subsets of  $(X, \tau)$  is called a fuzzy topology on  $X$  iff  $\tau$  satisfies the following conditions.

(i)  $\mu_\phi, \mu_x \in \tau$ : That is 0 and 1  $\in \tau$

(ii) If  $G_i \in \tau$  for  $i \in I$  then  $\bigvee_{i \in I} G_i \in \tau$

(iii) If  $G, H \in \tau$  then  $G \wedge H \in \tau$

The pair  $(X, \tau)$  is called a fuzzy topological space (abbreviated as fts). The members of  $\tau$  are called fuzzy open sets and a fuzzy set  $A$  in  $X$  is said to be closed iff  $1 - A$  is a fuzzy open set in  $X$ .

**1.9 Remark: [2]** Every topological space is a fuzzy topological space but not conversely.

**1.10 Definition: [2]** Let  $X$  be a fts and  $A$  be a fuzzy subset in  $X$ . Then  $\bigwedge \{B: B \text{ is a closed fuzzy set in } X \text{ and } B \geq A\}$  is called the closure of  $A$  and is denoted by  $A$  or  $\text{cl}(A)$ .

**1.11 Definition: [2]** Let  $A$  and  $B$  be two fuzzy sets in a fuzzy topological space  $(X, \tau)$  and let  $A \geq B$ . Then  $B$  is called an interior fuzzy set of  $A$  if there exists  $G \in \tau$  such that  $A \geq G \geq B$ , the least upper bound of all interior fuzzy sets of  $A$  is called the interior of  $A$  and is denoted by  $A^0$ .

**1.12 Definition: [3]** A fuzzy set  $A$  in a fts  $X$  is said to be fuzzy semiopen if and only if there exists a fuzzy open set  $V$  in  $X$  such that  $V \leq A \leq \text{cl}(V)$ .

**1.13 Definition: [3]** A fuzzy set  $A$  in a fts  $X$  is said to be fuzzy semi-closed if and only if there exists a fuzzy closed set  $V$  in  $X$  such that  $\text{int}(V) \leq A \leq V$ . It is seen that a fuzzy set  $A$  is fuzzy semiopen if and only if  $1 - A$  is a fuzzy semi-closed.

**1.14 Theorem: [3]** The following are equivalent:

- (a)  $\mu$  is a fuzzy semi-closed set,
- (b)  $\mu^c$  is a fuzzy semiopen set,
- (c)  $\text{int}(\text{cl}(\mu)) \leq \mu$ .
- (d)  $\text{int}(\text{cl}(\mu)) \geq \mu^c$

**1.15 Theorem: [3]** Any union of fuzzy semiopen sets is a fuzzy semiopen set and (b) any intersection of fuzzy semi-closed sets is a fuzzy semi-closed.

**1.16 Remark: [3]**

- (i) Every fuzzy open set is a fuzzy semiopen but not conversely.
- (ii) Every fuzzy closed set is a fuzzy semi-closed set but not conversely.
- (iii) The closure of a fuzzy open set is fuzzy semiopen set
- (iv) The interior of a fuzzy closed set is fuzzy semi-closed set

**1.17 Definition: [3]** A fuzzy set  $\mu$  of a fts  $X$  is called a fuzzy regular open set of  $X$  if  $\text{int}(\text{cl}(\mu)) = \mu$ .

**1.18 Definition: [3]** A fuzzy set  $\mu$  of fts  $X$  is called a fuzzy regular closed set of  $X$  if  $\text{cl}(\text{int}(\mu)) = \mu$ .

**1.19 Theorem: [3]** A fuzzy set  $\mu$  of a fts  $X$  is a fuzzy regular open if and only if  $\mu^c$  fuzzy regular closed set.

**1.20 Remark: [3]**

- (i) Every fuzzy regular open set is a fuzzy open set but not conversely.
- (ii) Every fuzzy regular closed set is a fuzzy closed set but not conversely.

**1.21 Theorem: [3]**

- (i) The closure of a fuzzy open set is a fuzzy regular closed.
- (ii) The interior of a fuzzy closed set is a fuzzy regular open set.

**1.22 Definition: [4]** A fuzzy set  $\alpha$  in fts  $X$  is called fuzzy rwclosed if  $\text{cl}(\alpha) \leq \mu$  whenever  $\alpha \leq \mu$  and  $\mu$  is regular semi-open in  $X$ .

**1.23 Definition: [5]** A fuzzy set  $\alpha$  in fts  $X$  is called fuzzy pgprw closed if  $\text{p-cl}(\alpha) \leq \mu$  whenever  $\alpha \leq \mu$  and  $\mu$  is  $\text{rg}\alpha$ -open set in  $X$ .

**1.24 Defintion: [5]** A fuzzy set  $\alpha$  of a fts  $X$  is fuzzy pgprw-open set, if it's complement  $\alpha^c$  is a fuzzy pgprw-closed in fts  $X$ .

**1.25 Definition: [2]** Let  $X$  and  $Y$  be fts. A map  $f: X \rightarrow Y$  is said to be a fuzzy continuous mapping if  $f^{-1}(\mu)$  is fuzzy open in  $X$  for each fuzzy open set  $\mu$  in  $Y$ .

**1.26 Definition: [6]** Let  $X$  and  $Y$  be fts. A map  $f: X \rightarrow Y$  is said to be a fuzzy almost continuous mapping if  $f^{-1}(\mu)$  is fuzzy open in  $X$  for each fuzzy regular open set  $\mu$  in  $Y$ .

**1.27 Definition: [6]** Let  $X$  and  $Y$  be fts. A map  $f: X \rightarrow Y$  is said to be a fuzzy irresolute if  $f^{-1}(\mu)$  is fuzzy semi-open set in  $X$  for each fuzzy semi-open set  $\mu$  in  $Y$ .

**1.28 Definition: [2]** Let  $X$  and  $Y$  be fts. A map  $f: X \rightarrow Y$  is said to be a fuzzy semi-continuous if  $f^{-1}(\mu)$  is fuzzy semi-open set in  $X$  for each fuzzy open set  $\mu$  in  $Y$ .

**1.29 Definition: [4]** Let  $X$  and  $Y$  be fts. A map  $f: X \rightarrow Y$  is said to be a fuzzy rw-continuous if  $f^{-1}(\mu)$  is fuzzy rw-open set in  $X$  for each fuzzy open set  $\mu$  in  $Y$ .

**1.30 Definition: [7]** Let  $X$  and  $Y$  be fts. A map  $f: X \rightarrow Y$  is said to be a fuzzy completely semi-continuous if  $f^{-1}(\mu)$  is fuzzy regular semi-open set in  $X$  for each fuzzy open set  $\mu$  in  $Y$ .

**2. Fuzzy rg-continuous maps and fuzzy rg-irresolute maps in fuzzy topological spaces**

**Definition 2.1:** Let  $X$  and  $Y$  be fts. A map  $f: X \rightarrow Y$  is said to be fuzzy regular – generalized continuous map (fuzzy rg-continuous map) if the inverse image of every fuzzy open set in  $Y$  is fuzzy generalized regular-open in  $X$ . and Let  $X$  and  $Y$  be fts. A map  $f: X \rightarrow Y$  is said to be a fuzzy rg-irresolute if  $f^{-1}(\mu)$  is fuzzy open set in  $X$  for each fuzzy open set  $\mu$  in  $Y$ .

**Theorem 2.2:** If a map  $f: (X,T) \rightarrow (Y,T)$  is fuzzy continuous, then  $f$  is fuzzy rg-continuous map.

**Proof:** Let  $\mu$  be a fuzzy open set in afts  $Y$ . Since  $f$  is fuzzy continuous map  $f^{-1}(\mu)$  is a fuzzy open set in fts  $x$ , as every open set is fuzzy rg-open, we have  $f^{-1}(\mu)$  is fuzzy a rg-open set in fts  $X$ . Therefore  $f$  is fuzzy continuous map.

The converse of the above theorem need not be true in general as seen from the following example

**Example 2.3:** Let  $X = Y = \{a, b, c, d\}$  and the Functions  $\alpha, \beta, \gamma, \delta: X \rightarrow [0, 1]$  be defined as:

$$\begin{aligned} \alpha(x) &= \begin{cases} 1, & \text{if } x = a \\ 0, & \text{otherwise} \end{cases} \\ \beta(x) &= \begin{cases} 1, & \text{if } x = b \\ 0, & \text{otherwise} \end{cases} \\ \gamma(x) &= \begin{cases} 1, & \text{if } x = a, b \\ 0, & \text{otherwise} \end{cases} \\ \delta(x) &= \begin{cases} 1, & \text{if } x = a, b, c \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

Consider  $T = \{1, 0, \alpha, \beta, \gamma, \delta\}, \sigma = \{1, 0, \alpha, \beta, \gamma, \delta\}$ . Now  $(X, T)$  and  $(Y, \sigma)$  are the fts. Define a map  $f: (X, T) \rightarrow (Y, \sigma)$  by  $f(a)=c, f(b)=a, f(c)=b, f(d)=d$ , then  $f$  is fuzzy rg-continuous map but not fuzzy continuous map as the inverse image of the fuzzy set  $\gamma$  in  $(Y, \sigma)$  is  $\mu: X \rightarrow [0, 1]$  defined as

$$\mu(x) = \begin{cases} 1, & \text{if } x = b, c \\ 0, & \text{otherwise} \end{cases}$$

This is not a fuzzy open set in  $(X, T)$ .

**Theorem 2.4:** A map  $f: (X, T) \rightarrow (Y, \sigma)$  is fuzzy rg-continuous map iff the inverse image of every fuzzy closed set in a fts  $Y$  is a fuzzy rg-closed set in fts  $X$ .

**Proof:** Let  $\delta$  be a fuzzy closed set in a fts  $Y$  then  $\delta^c$  is fuzzy open in fts  $Y$ . Since  $f$  is fuzzy rg-continuous map,  $f^{-1}(\delta^c)$  is rg-open in fts  $X$  but  $f^{-1}(\delta^c) = 1 - f^{-1}(\delta)$  and so  $f^{-1}(\delta)$  is a fuzzy rg-closed set in fts  $X$ . Conversely, Assume that the inverse image of every fuzzy closed set in  $Y$  is fuzzy rg closed in fts  $X$ . Let  $\mu$  be a fuzzy open set in fts  $Y$  then  $\mu^c$  is fuzzy closed in  $Y$ ; by hypothesis  $f^{-1}(\mu^c) = 1 - f^{-1}(\mu)$  is fuzzy rg-closed in  $X$  and so  $f^{-1}(\mu)$  is a fuzzy rg-open set in fts  $X$ . Thus  $f$  is fuzzy rg-continuous map.

**Theorem 2.5:** If a Function  $f: (X, T) \rightarrow (Y, \sigma)$  is fuzzy almost continuous map, then it is fuzzy rg-continuous map.

**Proof:** Let a Function  $f: (X, T) \rightarrow (Y, \sigma)$  be a fuzzy almost continuous map and  $\mu$  be fuzzy open set in fts  $Y$ . Then  $f^{-1}(\mu)$  is a fuzzy regular open set in fts  $X$ . Now,  $f^{-1}(\mu)$  is fuzzy rg-open in  $X$ , as every fuzzy regular open set is fuzzy rg-open. Therefore  $f$  is fuzzy rg-continuous map.

The converse of the above theorem need not be true in general as seen from the following example.

**Example 2.6:** consider the fts  $(X, T)$  and  $(Y, \sigma)$  as defined in example 2.3. define a map  $f: (X, T) \rightarrow (Y, \sigma)$  by  $f(a) = c, f(b) = a, f(c) = b, f(d) = d$ . Then  $f$  is fuzzy rg-continuous map but it is not almost continuous map.

**Example 2.7:** fuzzy semi-continuous maps and fuzzy rg-continuous maps are independent as seen from the following examples.

**Example 2.8:** Let  $X = Y = \{a, b, c, d\}$  and the Functions  $\alpha, \beta, \gamma, \delta: X \rightarrow [0, 1]$  be defined as

$$\begin{aligned} \alpha(x) &= \begin{cases} 1, & \text{if } x = a \\ 0, & \text{otherwise} \end{cases} \\ \beta(x) &= \begin{cases} 1, & \text{if } x = b \\ 0, & \text{otherwise} \end{cases} \\ \gamma(x) &= \begin{cases} 1, & \text{if } x = a, b \\ 0, & \text{otherwise} \end{cases} \\ \delta(x) &= \begin{cases} 1, & \text{if } x = a, b, c \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

Consider  $T = \{1, 0, \alpha, \beta, \gamma, \delta\}, \sigma = \{1, 0, \alpha\}$ . Let map  $f: X \rightarrow Y$  defined by  $f(a) = b, f(b) = a, f(c) = c, f(d) = c$ , then  $f$  is fuzzy rg-continuous map but it is not fuzzy semi continuous map as the inverse image of fuzzy set  $\alpha^c$  in  $(Y, \sigma)$  is

$$\mu(x) = \begin{cases} 1, & \text{if } x = a, b, d \\ 0, & \text{otherwise} \end{cases}$$

This is not a fuzzy semi closed set in fts  $X$ .

**Example 2.9:** Let  $X = Y = \{a, b, c\}$  and the Functions  $\alpha, \beta: X \rightarrow [0, 1]$  be defined as

$$\begin{aligned} \alpha(x) &= \begin{cases} 1, & \text{if } x = a \\ 0, & \text{otherwise} \end{cases} \\ \beta(x) &= \begin{cases} 1, & \text{if } x = b, c \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

Consider  $T = \{1, 0, \alpha, \beta\}, \sigma = \{1, 0, \alpha\}$ . Let map  $f: X \rightarrow Y$  defined by  $f(a) = b, f(b) = a, f(c) = c$ . then  $f$  is fuzzy semi-continuous map but  $f$  is not fuzzy rg continuous map as the inverse image of fuzzy set  $\alpha^c$  in  $(Y, \sigma)$  is

$$\mu(x) = \begin{cases} 1, & \text{if } x = a, c \\ 0, & \text{otherwise} \end{cases}$$

This is not fuzzy rg-closed set in fts  $X$ .

**Remark 3.0:** fuzzy rw-continuous maps and fuzzy rg-continuous maps are independent as seen from the following examples.

**Example 3.1:** Let  $X = \{a, b, c, d\}$ ,  $Y = \{a, b, c\}$  and the Functions  $\alpha, \beta, \gamma, \delta: X \rightarrow [0, 1]$  be defined as

$$\alpha(x) = \begin{cases} 1, & \text{if } x = a \\ 0, & \text{otherwise} \end{cases}$$

$$\beta(x) = \begin{cases} 1, & \text{if } x = b \\ 0, & \text{otherwise} \end{cases}$$

$$\gamma(x) = \begin{cases} 1, & \text{if } x = a, b \\ 0, & \text{otherwise} \end{cases}$$

$$\delta(x) = \begin{cases} 1, & \text{if } x = a, b, c \\ 0, & \text{otherwise} \end{cases}$$

Consider  $T = \{1, 0, \alpha, \beta, \gamma, \delta\}$ ,  $\sigma = \{1, 0, \alpha\}$ . Let map  $f: X \rightarrow Y$  defined by  $f(a) = b, f(b) = a, f(c) = a, f(d) = c$ , then  $f$  is fuzzy rg-continuous map but it is not fuzzy rw continuous map as the inverse image of fuzzy set  $\alpha^c$  in  $(Y, \sigma)$  is

$$\mu(x) = \begin{cases} 1, & \text{if } x = a, d \\ 0, & \text{otherwise} \end{cases}$$

This is not fuzzy rw-closed set in fts  $X$ .

**Example 3.2:** Let  $X = Y = \{a, b, c\}$  and the Functions  $\alpha, \beta: X \rightarrow [0, 1]$  be defined as

$$\alpha(x) = \begin{cases} 1, & \text{if } x = a \\ 0, & \text{otherwise} \end{cases}$$

$$\beta(x) = \begin{cases} 1, & \text{if } x = b, c \\ 0, & \text{otherwise} \end{cases}$$

Consider  $T = \{1, 0, \alpha, \beta\}$ ,  $\sigma = \{1, 0, \alpha\}$ . Let map  $f: X \rightarrow Y$  defined by  $f(a) = b, f(b) = a, f(c) = c$ . then  $f$  is fuzzy rw-continuous map but  $f$  is not fuzzy rg continuous map as the inverse image of fuzzy set  $\alpha^c$  in  $(Y, \sigma)$  is

$$\mu(x) = \begin{cases} 1, & \text{if } x = a, c \\ 0, & \text{otherwise} \end{cases}$$

This is not fuzzy rg-closed set in fts  $X$ .

**Remark 3.3:** From the above discussion and known results we have the following implication.

**Theorem 3.4:** If a Function  $f: (X, T) \rightarrow (Y, \sigma)$  is fuzzy rg-continuous map and fuzzy completely continuous map then it is fuzzy continuous map.

**Proof:** Let a Function  $f: (X, T) \rightarrow (Y, \sigma)$  be a fuzzy rg-continuous map and fuzzy completely semi-continuous map. Let  $\mu$  be a fuzzy closed set in fts  $Y$ . Then  $f^{-1}(\mu)$  is both fuzzy closed set in fts  $Y$ . Then  $f^{-1}(\mu)$  is both fuzzy rg $\alpha$ -open and fuzzy rg-closed set in fts  $X$ . If a fuzzy set  $\alpha$  of fts  $X$  is both fuzzy rg $\alpha$  and fuzzy rg-closed then  $\alpha$  is a fuzzy closed in fts  $X$  thus  $f^{-1}(\mu)$  is a fuzzy closed set in fts  $X$ . Therefore  $f$  is fuzzy continuous map.

**Theorem 3.5:** If  $f: (X, T) \rightarrow (Y, \sigma)$  is fuzzy rg-continuous map and  $g: (Y, \sigma) \rightarrow (Z, \rho)$  is fuzzy continuous map, then their composition  $g \circ f: (X, T) \rightarrow (Z, \rho)$  is fuzzy rg-continuous map.

**Proof:** Let  $\mu$  be a fuzzy open set in fts  $Z$ . Since  $g$  is fuzzy continuous map,  $g^{-1}(\mu)$  is fuzzy open set in fts  $Y$ . Since  $f$  is fuzzy rg-continuous map,  $f^{-1}(g^{-1}(\mu))$  is a fuzzy rg-open set in fts  $X$ .

But  $(g \circ f)^{-1}(\mu) = f^{-1}(g^{-1}(\mu))$  thus  $g \circ f$  is fuzzy rg-continuous map.

**Definition 3.6:** Let  $X$  and  $Y$  be fts. A map  $f: X \rightarrow Y$  is said to be a fuzzy rg-irresolute map if the inverse image of every fuzzy rg-open in  $Y$  is a fuzzy rg-open set in  $X$ .

**Theorem 3.7:** If a map  $f: X \rightarrow Y$  is fuzzy rg-irresolute map then it is fuzzy rg-continuous map.

**Proof:** Let  $\beta$  be a fuzzy open set in  $Y$ . since every fuzzy open set is fuzzy rg-open,  $\beta$  is a fuzzy rg-open set in  $Y$ . Since  $f$  is fuzzy rg-irresolute map,  $f^{-1}(\beta)$  is fuzzy rg-open in  $X$ . Thus  $f$  is fuzzy rg-continuous map.

The converse of the above theorem need not be true in general as seen from the following example.

**Example 3.8:** Let  $X = \{a, b, c, d\}$ ,  $Y = \{a, b, c\}$  the Functions  $\alpha, \beta, \gamma, \delta: X \rightarrow [0, 1]$  be defined as

$$\alpha(x) = \begin{cases} 1, & \text{if } x = a \\ 0, & \text{otherwise} \end{cases}$$

$$\beta(x) = \begin{cases} 1, & \text{if } x = b \\ 0, & \text{otherwise} \end{cases}$$

$$\gamma(x) = \begin{cases} 1, & \text{if } x = a, b \\ 0, & \text{otherwise} \end{cases}$$

$$\delta(x) = \begin{cases} 1, & \text{if } x = a, b, c \\ 0, & \text{otherwise} \end{cases}$$

Consider  $T = \{1, 0, \alpha, \beta, \gamma, \delta\}, \sigma = \{1, 0, \mu\}$ . Let map  $f: X \rightarrow Y$  defined by  $f(a)=b, f(b)=a, f(c)=a, f(d)=c$ , then  $f$  is fuzzy rg-continuous map but it is not fuzzy rg-irresolutemap. Since for the fuzzy rg-closed set  $\mu: Y \rightarrow [0, 1]$  defined by

$$\mu(x) = \begin{cases} 1, & \text{if } x = b \\ 0, & \text{otherwise in } Y \end{cases}$$

$f^{-1}(\mu) = \alpha$  is not fuzzy rg-closed in  $(X, T)$ .

**Theorem 3.9:** Let  $X, Y, Z$  be fts. If  $f: X \rightarrow Y$  is fuzzy rg-irresolute map and  $g: Y \rightarrow Z$  is fuzzy rg-continuous map then their composition  $g \circ f: X \rightarrow Z$  is fuzzy rg-continuous map.

**Proof:** Let  $\alpha$  be any fuzzy open set in fts  $Z$ , Since  $g$  is fuzzy rg-continuous map,  $(g^{-1}(\alpha))$  is a fuzzy rg-irresolute map  $f^{-1}((g^{-1}(\alpha)))$  is a fuzzy rg-open set in fts  $X$ . But  $(g \circ f)^{-1}(\alpha) = f^{-1}(g^{-1}(\alpha))$ . Thus  $g \circ f$  is fuzzy rg-continuous map.

**Theorem 3.10:** Let  $X, Y, Z$  be fts and  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  be fuzzy rg-irresolute maps then their composition maps then their composition  $g \circ f: X \rightarrow Z$  is fuzzy rg-irresolute map.

**Proof:** Let  $\alpha$  be a fuzzy rg-open set in fts  $Z$ . since  $g$  is fuzzy rg-irresolute map,  $g^{-1}(\alpha)$  is a fuzzy rg-open set in fts  $Y$ . since  $f$  is fuzzy rg-irresolute map,  $f^{-1}(g^{-1}(\alpha))$  is a fuzzy Rg-open set in fts  $X$  But  $(g \circ f)^{-1}(\alpha) = f^{-1}(g^{-1}(\alpha))$ . Thus  $g \circ f$  is fuzzy rg-irresolute map.

## CONCLUSION

In this paper, a new class of maps called Fuzzy rg-Continuous maps and Fuzzy rg-irresolute in fuzzy topological spaces are introduced and investigated. In future the same process will be analyzed for Fuzzy rg-properties.

## REFERENCES

1. L.A.Zadeh, Fuzzy sets, Information and control, 8 (1965) 338-353.
2. C.L.Chang, Fuzzy topological spaces, JI.Math.Anal. Appl., 24(1968), 182-190.
3. K.K.Azad, On fuzzy semi continuity, fuzzy almost continuity and fuzzy weakly continuity. JI.Math.Anal. Appl. 82 No. 1 (1981), 14-32.
4. R.S.Wali, on some topics in general and fuzzy topological spaces; ph.dthesis, Karnatak university Dharwad. (2007).
5. R.S.Wali and Vivekananda Dembre, Fuzzy rg-closed sets and Fuzzy rg-open sets in Topological Spaces; journal of global research in Mathematics archives April 2016 (accepted).
6. Azad, k.k. on fuzzy semi-continuity, fuzzy almost continuity and fuzzy weak continuity, j.math.anal app 82(1981), 14-81.
7. Mukerjii, M.N. and Sinha, S.P, irresolute and almost open continuous between fts, fuzzy sets and systems, 29(1989) 381-388.
8. Anjan Mukherjee, on fuzzy completely semi-continuous & weakly completely, semi-continuity Functions, indian J pure Appl math, 29(2)(1998), 191-197.
9. Thakur S.S. and Bajpai Pandey Jyoti "Intuitionistic Fuzzy  $rg\alpha$ -closed sets", International Journal of Fuzzy system and Rough System 4(1), 67-73.
10. S.P.Arya and T.M.Nour, Characterizations of S-normal spaces, Indian J pure APP.Math 21(1990), 717-719.
11. R.S.Wali and Vivekananda Dembre, on pre generalized pre regular weakly closed sets in topological spaces, to appear in Journal of Computer and Mathematical Science, (2015).
12. S.Bhattacharya, Int j. Contemp Math Sciences, Vol 6 2011, no 3 145-152.

**Source of support: Nil, Conflict of interest: None Declared.**

**[Copy right © 2020. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]**