

STUDY OF VARIOUS NEW MATHEMATICAL CONSTANTS
 BY APPLITION OF TWO DEFINED BASE 10 POSITIONAL NUMBERS

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ABSTRACT

Various mathematical constants have been investigated in mathematics, in this paper it is shown that if $A = N_1 + 10N_2 + 100N_3 + 1000N_4 + \dots$, & $B_n = N_n + 10N(n+1) + 100N(n+2) + 1000N(n+3) + \dots$, are two positive integer of base 10 positional numeral system then ratio of A & B_n in a defined manner create many Constants, where $N_1, N_2, N_3, N_4 \dots$ are decimal digits (0 to 9) B_n is Nth order numbers & n is order of numbers.

Keywords: Base 10 numeral system, Decimal digit, Euler number, Positive integer.

1. INTRODUCTION

Between two numbers, $A = N_1 + 10N_2 + 100N_3 + 1000N_4 + \dots$, and $B_n = N_n + 10N(n+1) + 100N(n+2) + 1000N(n+3) + \dots$, B_n depends upon A, and is less than A.

Suppose,

$$\lim_{A \rightarrow \infty} \left(1 + \frac{A}{B_n}\right)^{B_n/A}, \lim_{A \rightarrow \infty} \left(1 + \frac{B_n}{A}\right)^{A/B_n}$$

$$\lim_{A \rightarrow \infty} \left(1 - \frac{A}{B_n}\right)^{B_n/A}, \lim_{A \rightarrow \infty} \left(1 - \frac{B_n}{A}\right)^{A/B_n}$$

are some equations in which limit of A is infinity, as soon as the value of A taken very large, A/B_n reaches toward $10^{(n-1)}$. For second order numbers ($n = 2$), A/B_2 will be 10. $A = 10$ & $B_2 = 1$ give correct value of second order constant up to infinite decimal places. For onward orders 3^{rd} , 4^{th} , 5^{th} A/B_3 , A/B_4 , A/B_5 (common ratio) for each order will be 100, 1000, 10000 respectively. Equation for different order numbers is $B_n = N_n + 10N(n+1) + 100N(n+2) + 1000N(n+3) + \dots$ if $n = 2$, $B_2 = N_2 + 10N_3 + 100N_4 + 1000N_5 + \dots$, $n = 3$, $B_3 = N_3 + 10N_4 + 100N_5 + 1000N_6 + \dots$

1.1. Definition

Base 10 numeral system In math, 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 are base ten numerals. We can only count to nine without the need for two numerals or digits. All numbers in the number system are made by combining these 10 numerals or digits

Decimal digits For writing numbers, the decimal system uses ten decimal digits, a decimal mark, and, for negative numbers, a minus sign "-". The decimal digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Positive integers The positive integers are the numbers 1, 2, 3, ...(OEIS A000027), sometimes called the counting numbers or natural numbers.

2. EVALUATION OF CONSTANTS

If A and B_n are two positive integer of base 10 positional numeral system in term of $A = N_1 + 10N_2 + 100N_3 + 1000N_4 + \dots$, & $B_n = N_n + 10N(n+1) + 100N(n+2) + 1000N(n+3) + \dots$, where $N_1, N_2, N_3, N_4 \dots$, are decimal digits (0 to 9) and B_n is Nth order numbers & n is order of numbers then

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Case- 1:

$$\lim_{A \rightarrow \infty} \left(1 + \frac{A}{B_n}\right)^{B_n/A} = \lim_{A \rightarrow \infty} \left(1 + \frac{10^{\wedge n}}{10}\right)^{10/10^{\wedge n}} = C_n \text{ (Cn is Nth order Constant for case1)}$$

Case-2:

$$\lim_{A \rightarrow \infty} \left(1 + \frac{B_n}{A}\right)^{A/B_n} = \left(1 + \frac{10}{10^{\wedge n}}\right)^{10/10^{\wedge n}} = C'_n \text{ (C'n is Nth order Constant for case2)}$$

Case-3:

$$\lim_{A \rightarrow \infty} \left(1 - \frac{A}{B_n}\right)^{B_n/A} = \lim_{A \rightarrow \infty} \left(1 - \frac{10^{\wedge n}}{10}\right)^{10/10^{\wedge n}} = C''_n \text{ (C''n is Nth order Constant for case3)}$$

Case-4

$$\lim_{A \rightarrow \infty} \left(1 - \frac{B_n}{A}\right)^{A/B_n} = \left(1 - \frac{10}{10^{\wedge n}}\right)^{10/10^{\wedge n}} = C'''_n \text{ (C_n''' is Nth order Constant for case4)}$$

Proof:

Example of A & Bn for different order numbers

For second order numbers (n = 2)

A = N1 + 10N2 + 100N3 + 1000N4 + , N1, N2, N3, N4..... (Decimal digits 0 to 9)

Bn = Nn + 10N(n+1)+ 100N(n+2) + 1000N(n+3)..... , ,

B2 = N2 + 10N3 + 100N4 + 1000N5 , ,

Let N1, N2, N3, N4, N5= 1,2,3,4,5

A= 1 + 10*2 + 100*3 + 1000*4 + 10000*5

A=54321

B2 = 2 + 10*3 + 100*4 + 1000*5

B2 = 5432

A/B2 = 54321/5432=10.00184....

(1)

Let N1,N2,N3,N4,N5,N6,N7,N8,N9= 9,8,7,6,5,4,3,2,1,

A = 9+10*8+100*7+1000*6+10000*5+100000*4+1000000*3+10000000*2+100000000*1

A =123456789

B2 = N2 + 10N3 + 100N4 + 1000N5 ,

B2 = 8+10*7+100*6+1000*5+10000*4+100000*3+1000000*2+10000000*1

B2 = 12345678

A/B2 = 123456789/12345678 = 10.000000729.....

(2)

Let N1, N2, N3, N4, N5, N6, N7, N8, N9, N10, N11= 4, 5, 6, 7, 8, 9, 1, 2, 3, 4, 5

A=

4+10*5+100*6+1000*7+10000*8+100000*9+1000000*1+10000000*2+100000000*3+1000000000*4+10000000000

*5

A = 54321987654

B2 = 5+10*6+100*7+1000*8+10000*9+100000*1+1000000*2+10000000*3+100000000*4+1000000000*5

B2 = 5432198765

A/B2 = 54321987654/5432198765 = 10.0000000073....

(3)

Based on eqn. no. (1) ,(2) & (3) for second order numbers ,common ratio A/B2 will reach towards 10

As limit of A reaches infinity

A/B2 = 10

Example of A & Bn For third order numbers (n = 3)

$A = N_1 + 10N_2 + 100N_3 + 1000N_4 + \dots$, ($N_1, N_2, N_3, N_4, \dots$ (Decimal digits 0 to 9))
 $B_n = N_n + 10N_{(n+1)} + 100N_{(n+2)} + 1000N_{(n+3)} \dots$, ,

$B_3 = N_3 + 10N_4 + 100N_5 + 1000N_6 \dots$, ,

Let $N_1, N_2, N_3, N_4, N_5 = 1, 2, 3, 4, 5$
 $A = 1 + 10*2 + 100*3 + 1000*4 + 10000*5$
 $A = 54321$

$B_3 = 3 + 10*4 + 100*5$

$B_3 = 543$

$A/B_3 = 54321/543 = 100.038\dots$ (4)

Let $N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9 = 9, 8, 7, 6, 5, 4, 3, 2, 1$,
 $A = 9 + 10*8 + 100*7 + 1000*6 + 10000*5 + 100000*4 + 1000000*3 + 10000000*2 + 100000000*1$
 $A = 123456789$

$B_3 = N_3 + 10N_4 + 100N_5 + 1000N_6 + \dots$, .

$B_3 = 7 + 10*6 + 100*5 + 1000*4 + 10000*3 + 100000*2 + 1000000*1$

$B_3 = 1234567$

$A/B_3 = 123456789/1234567 = 100.000072\dots$ (5)

Let $N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9, N_{10}, N_{11} = 4, 5, 6, 7, 8, 9, 1, 2, 3, 4, 5$

$A =$

$4 + 10*5 + 100*6 + 1000*7 + 10000*8 + 100000*9 + 1000000*1 + 10000000*2 + 100000000*3 + 1000000000*4 + 10000000000*5$

$A = 54321987654$

$B_3 = 6 + 10*7 + 100*8 + 1000*9 + 10000*1 + 100000*2 + 1000000*3 + 10000000*4 + 100000000*5$

$B_3 = 543219876$

$A/B_3 = 54321987654/543219876 = 100.00000099\dots$ (6)

Based on eqn. no. (4), (5) & (6) for Third order numbers, common ratio A/B_3 will reach towards 100 as limit of A reaches infinity

$A/B_3 = 100 = 10^2$

Like the same for Fourth order numbers

Let $A = 10000999789456$

$B_4 = 10000999789$

$A/B_4 = 10000999789456 / 10000999789 = 1000.00000045\dots$

$A = 99008800789456100$

$B_4 = 99008800789456$

$A/B_4 = 99008800789456100 / 99008800789456 = 1000.0000000000101\dots$

Common ratio for Fourth order numbers will reach toward 1000

$A/B_4 = 1000 = 10^3$

Similarly Common Ratio for Fifth , Sixth, Seventh ...order numbers will be

$A/B_5 = 10000 = 10^4$

$A/B_6 = 100000 = 10^5$

$A/B_7 = 1000000 = 10^6$

$A/B_8 = 10000000 = 10^7$

Common ratio for Nth order numbers can be written as

$A/B_n = 10^{(n-1)}$

Put the value of A/B_n in case1, case 2, case 3 & case 4

Case-1:

$$\lim_{A \rightarrow \infty} \left(1 + \frac{A}{B_n}\right)^{B_n/A} = \left(1 + \frac{10^{\wedge n}}{10}\right)^{10/10^{\wedge n}} = C_n \quad (C_n \text{ is } N\text{th order Constant for case1})$$

Example of Constants for Case 1

Constant for Second order numbers (n = 2) for case 1

$$\lim_{A \rightarrow \infty} \left(1 + \frac{A}{B_2}\right)^{B_2/A} = \left(1 + \frac{10^2}{10}\right)^{10/10^2} = C_2 \quad (C_2 \text{ is Second order Constant for case 1})$$

$$C_2 = 11^{(1/10)} = 1.2709816152.....$$

Constant for Third order numbers (n = 3) for case 1

$$\lim_{A \rightarrow \infty} \left(1 + \frac{A}{B_3}\right)^{B_3/A} = \left(1 + \frac{10^3}{10}\right)^{10/10^3} = C_3 \quad (C_3 \text{ is Third order Constant for case 1})$$

$$C_3 = 101^{(1/100)} = 1.047232745989.....$$

In such a way Infinite Constant can be find for Case 1

If n is infinity

$$C_{\infty} = \lim_{A \& n \rightarrow \infty} \left(1 + \frac{A}{B_n}\right)^{B_n/A} = \lim_{n \rightarrow \infty} \left(1 + \frac{10^{\wedge n}}{10}\right)^{10/10^{\wedge n}} = 1$$

Case-2:

$$\lim_{A \rightarrow \infty} \left(1 + \frac{B^n}{A}\right)^{A/B^n} = \left(1 + \frac{10}{10^{\wedge n}}\right)^{10^{\wedge n}/10} = C'_n \quad (C'_n \text{ is } N\text{th order Constant for case 2})$$

Case 2

Constant for second order numbers (n = 2) for case 2

$$\lim_{A \rightarrow \infty} \left(1 + \frac{B^2}{A}\right)^{A/B^2} = \left(1 + \frac{10}{10^2}\right)^{10^2/10} = C'_2 \quad (C'_2 \text{ is second order Constant for case 2})$$

$$C'_2 = 1.1^{10} = 2.59374246.....$$

Constant for third order numbers (n = 3) for case2

$$\lim_{A \rightarrow \infty} \left(1 + \frac{B^3}{A}\right)^{A/B^3} = \left(1 + \frac{10}{10^3}\right)^{10^3/10} = C'_3 \quad (C'_3 \text{ is third order Constant for case2})$$

$$C'_3 = 1.01^{100} = 2.704813829.....$$

Constant for fourth order numbers (n = 4) for case 2

$$\lim_{A \rightarrow \infty} \left(1 + \frac{B^4}{A}\right)^{A/B^4} = \left(1 + \frac{10}{10^4}\right)^{10^4/10} = C'_4 \quad (C'_4 \text{ is fourth order Constant for case2})$$

$$C'_4 = 1.001^{1000} = 2.716923932.....$$

If n is infinity

$$C'_{\infty} = \lim_{A \rightarrow \infty} \left(1 + \frac{B^{\wedge n}}{A}\right)^{A/B^{\wedge n}} = \lim_{n \rightarrow \infty} \left(1 + \frac{10}{10^{\wedge n}}\right)^{10^{\wedge n}/10} = e \quad (\text{Euler Number})$$

Like the same for case 3 & case 4 for different Order numbers Different Constant can be find.

For case 1, case 2, case 3 & case 4, for different order numbers, different Constants are as under

Case 1		Case 2		
Order(n)	Constant (Cn)	Constant (C'n)	Constant (C'n)	
	$\lim_{A \rightarrow \infty} \left(1 + \frac{A}{B^{A^n}}\right)^{B^{A^n}/A} = \left(1 + \frac{10^{A^n}}{10}\right)^{10/10}$		$\lim_{A \rightarrow \infty} \left(1 + \frac{B^{A^n}}{A}\right)^{A/B^{A^n}} = \left(1 + \frac{10}{10^{A^n}}\right)^{10^{A^n}/10}$	
2	C2	11^(1/10) = 1.2709816152...	C'2	1.1^10 = 2.5937424601
3	C3	101^(1/100) = 1.047232745...	C'3	1.01^100 = 2.70481382942152....
4	C4	1001^(1/1000) = 1.006932675....	C'4	1.001^1000 = 2.7169239322358924...
5	C5	10001^(1/10000) = 1.000921468328	C'5	1.0001^10000 = 2.718145926825224864...
6	C6	100001^(1/100000) = 1.000115135982	C'6	1.00001^100000 = 2.7182682371744896680...
7	C7	1000001^(1/1000000) = 1.000013815606....	C'7	1.000001^1000000 = 2.71828046931937...
∞	C∞	1	C'∞	e

Case 3		Case 4		
Order(n)	Constant (C''n)	Constant (C''n)	Constant (C''n)	
	$\lim_{A \rightarrow \infty} \left(1 - \frac{A}{B^{A^n}}\right)^{B^{A^n}/A} = \left(1 - \frac{10^{A^n}}{10}\right)^{10/10}$		$\lim_{A \rightarrow \infty} \left(1 - \frac{B^{A^n}}{A}\right)^{A/B^{A^n}} = \left(1 - \frac{10}{10^{A^n}}\right)^{10^{A^n}/10}$	
2	C''2	(-9)^(1/10)	C''2	0.9^10 = 0.3486784401....
3	C''3	(-99)^(1/100)	C''3	0.99^100 = 0.36603234127323....
4	C''4	(-999)^(1/1000)	C''4	0.999^1000 = 0.36769542477096...
5	C''5	(-9999)^(1/10000)	C''5	0.9999^10000 = 0.36786104643293....
6	C''6	(-99999)^(1/100000)	C''6	0.99999^100000 = 0.36787760176657....
7	C''7	(-999999)^(1/1000000)	C''7	0.999999^1000000 = 0.36787925723165...
∞	C''∞	1	C''∞	(1/e)

3. CONCLUSION

For Case 1 Nth order Constant

$$C_n = \lim_{A \rightarrow \infty} \left(1 + \frac{A}{B_n^{A^n}}\right)^{B_n^{A^n}/A} = \left(1 + \frac{10^{A^n}}{10}\right)^{10/10^{A^n}}$$

For Case 2 Nth order Constant

$$C'_n = \lim_{A \rightarrow \infty} \left(1 + \frac{B_n^{A^n}}{A}\right)^{A/B_n^{A^n}} = \left(1 + \frac{10}{10^{A^n}}\right)^{10^{A^n}/10}$$

For Case 3 Nth order Constant

$$C''_n = \lim_{A \rightarrow \infty} \left(1 - \frac{A}{B_n^{A^n}}\right)^{B_n^{A^n}/A} = \left(1 - \frac{10^{A^n}}{10}\right)^{10/10^{A^n}}$$

For Case 4 Nth order Constant

$$C''_n = \lim_{A \rightarrow \infty} \left(1 - \frac{B_n^{A^n}}{A}\right)^{A/B_n^{A^n}} = \left(1 - \frac{10}{10^{A^n}}\right)^{10^{A^n}/10}$$

4. REFERENCES

1. www.splashlearn.com
2. https://en.wikipedia.org
3. https://mathworld.wolfram.com/PositiveInteger.html

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