International Journal of Mathematical Archive-11(11), 2020, 22-26
Available online through www.ijma.info ISSN 2229 – 5046

APPLICATION OF UNDIRECTED GRAPH IN METRIC SPACE

MANISHA BHADORIYA
Research Scholar

DR. CHITRA SINGH*
Associate Professor Mathematics
Rabindranath Tagore University Bhopal {M.P.}, India.

(Received On: 19-10-20; Revised & Accepted On: 10-11-20)

ABSTRACT

We give some graph theory in the setting of metric spaces endowed with a Undirected graphs. The presented results extend and improve several well-known results in the literature. In particular, we discuss a some example& theorems of metric space by the using of undirected graph theory.

Keywords: *Undirected graph, Metric space, Self-loop, adjacency, incidence.*

INTRODUCTION

In many problems dealing with discrete objects and binary relations, a graphical representation of the object and the binary relations on them is very convenient form of representation. This leads to naturally to a study of graph, graph theory has a very wide range of applications in engineering in physical, social and mathematical science. In this paper, we shall study with basic terminology of graphs and metric space.

Definition: An undirected graph G is defined abstractly as an ordered pair (V,E) where V is an non empty set and E is an multiples of two elements from V.

An undirected graph can be represented geometrically as a set of marked points V and set of lines E between the points.

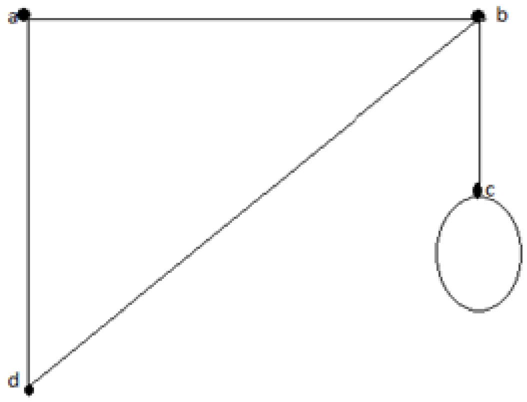

Fig.-1.1

Example: G= ({a, b, c, d}, {a, b}, {a, d}, {b, c}, {b, d}, {c, c}) is an undirected graph.

Definition: A graph $G = (V, E)$ that has neither self loop nor parallel edges is called a simple graph.

Let $\{v = v_1, v_2, v_3, v_4\}$ and $E = \{e_1, e_2, e_3, e_4\}$ where $e_1 = (v_1, v_2)$,$e_2 = (v_2, v_3)$,$e_3 = (v_1, v_3)$, and $e_4 = (v_3, v_4)$. Then $G = (V, E)$ is simple graph as shown in fig 1.2

Corresponding Author: Dr. Chitra Singh*
Associate Professor Mathematics, Rabindranath Tagore University Bhopal {M.P.}, India.

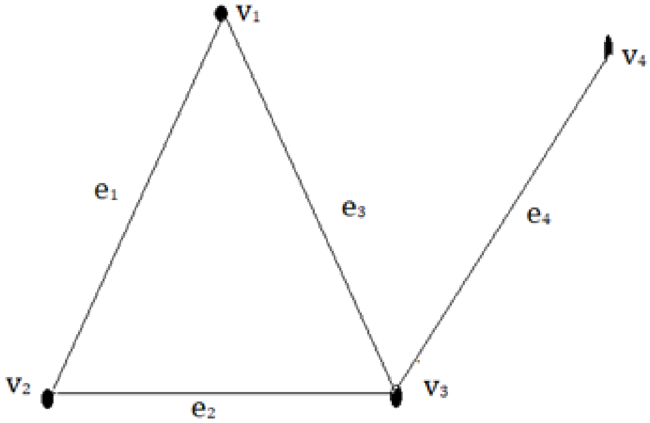

Fig.-1.2

Definition: A graph with a finite number of vertices as well as finite number of edges is called a finite graph otherwise it is an infinite.

Definition: If $G = (V, E)$ is a finite graph, then the number of vertices is denoted by $|V|$ and is called the order of the graph G. The number of edges is denoted by $|E|$

Definition: Let e_k be an edge joining two vertices $v_i and\ v_j$ of graph $G = (V, E)$. Then the edge e_k is said to be incident on each of its vertices v_i andv_j.

Example: In the graph of fig 1.2 edge e_2 is incident on vertices$v_2\ and\ v_3$.

Definition: Two vertices in a graph $G = (V, E)$ are said to be adjacent if there exists an edge joining the vertices.

Example: In graph of fig 1.2 vertices $v_1 and\ v_3$ are adjacent while vertices $v_1\ and\ v_4$ are not adjacent .

Definition: The degree of a vertex v in a graph G written as $d(v)$ is equal to the number of edges which are incident on v with self loop counted twice

Example: In graph 1.2 we have

$$d(v_1) = 2, d(v_2) = 2, d(v_3) = 3\ , d(v_4) = 1.$$

Definition: Let X be a non empty set. A metric(or distance function) on X is a mapping $d: X \times X \to R$ which satisfies the following axioms for all $x, y, z \in X$.

(M_1): $d(x, x) = 0$
(M_2): $d(x, y) = 0 \Rightarrow x = y$
(M_3): $d(x, y) = d(y, x)$.
(M_4): $d(x, z) \leq d(x, y) + d(y, z)$

If d is a metric on X, then the ordered pair (X, d) is called metric space.The number $d(x, y)$ is called the distance between the elements $x\ and\ y$.The elements of metric space X are sometimes also called point.

The axiom (M_1) says that the distance of a point from itself is zero.The axiom (M_2) means that if the distance is zero, the two points are same.The axiom (M_3) states that the distance does not depend on the order of the points $x\ and\ y$.The axiom (M_4) is commonly called the triangular inequality states " the sum of the length of two sides of a triangle is greater than or equal to the length of the third side.

Example: Consider the set R of all real numbers and define a mapping $d: R \times R \to R\ such\ that$

$$d(x, y) = |x - y| \qquad \forall\, x, y \in R$$

d is metric on R follows from the properties of modulus of real numbers

(i) $|x| = 0 \Leftrightarrow x = 0$
(ii) $|-x| = |x|$
(iii) $|x + y| \leq\ |x|\ +\ |y| \qquad \forall x, y\ \in R$.

Using these properties d satisfies all the postulates as required for a metric

$$(M_1) d(x,y) = 0 \Leftrightarrow |x-y| = 0 \Leftrightarrow x-y \Leftrightarrow x = y$$
$$(M_3) d(x,y) = |x-y| = |-(x-y)| = |y-x| = d(y,x)$$
$$(M_4) d(x,y) = |x-y| = |x-z+z-y|$$
$$\leq |x-z| + |z-y| = d(x,z) + d(z,y) \quad \forall\ x, y, z \in R$$

Hence d is a metric on R and is called usual metric on R. Thus (R,d) is a metric space called usual metric space.

MAIN RESULT

We use the graph theory solve the problems of metric space and determine the degree of vertices of metric space.

Example: Let (X,d) be a metric space . A mapping d_1 is defined such that $d_1(x,y) = \frac{d(x,y)}{1+d(x,y)}$ $\forall\ x, y \in X$.Then show that d_1 is a metric onX.

Solution:

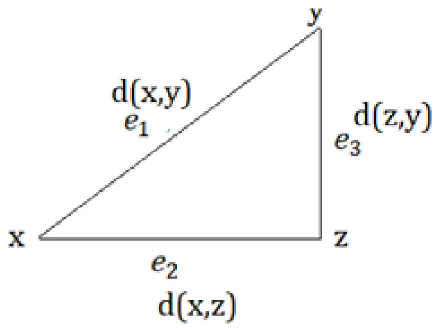

Fig.-1.3

Let $G = (V,E)$be a finite graph x, y, z be a vertices $and\ e_1, e_2, e_3$ be edges of metric space (X,d).

$$(M_1) d_1(x,y) = 0 \Leftrightarrow \frac{d(x,y)}{1+d(x,y)} = 0 \Leftrightarrow d(x,y) = 0 \Leftrightarrow x = y$$
$$[\therefore d(x,y) = 0 \Leftrightarrow x = y]$$
$$(M_3) d_1(x,y) = \frac{d(x,y)}{1+d(x,y)} = \frac{d(y,x)}{1+d(y,x)} = d_1(y,x)$$
$$[\therefore d(x,y) = d(y,x)]$$

(M_4) Let$x, y, z \in X$ be arbitrary. Then

$$\frac{d(x,z)}{1+d(x,z)+d(z,y)} \leq \frac{d(x,z)}{1+d(x,z)} = d_1(x,z)$$
$$and\ \frac{d(z,y)}{1+d(x,z)+d(z,y)} \leq \frac{d(z,y)}{1+d(z,y)} = d_1(z,y)$$

Since d is a metric

$$d(x,y) \leq d(x,z) + d(z,y)$$
$$d_1(x,y) = \frac{d(x,y)}{1+d(x,y)} \leq \frac{d(x,z)+d(z,y)}{1+d(x,z)+d(z,y)}$$
$$= \frac{d(x,z)}{1+d(x,z)+d(z,y)} + \frac{d(z,y)}{1+d(x,z)+d(z,y)}$$
$$\leq d_1(x,z) + d_1(z,y)$$

Hence $d_1\ is\ a\ metric$

Degree of vertex in a metric (X,d)

$$d(x) = 2\ , d(y) = 2, \qquad d(z) = 2$$

Incidence of Metric space by the graph G = (V, E)

x, y, z(vertices) are row matrices $e_1,\ e_2,\ e_3$ are column(edges)

$$\mathrm{I} = \begin{array}{c} \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{ccc} e_1 & e_2 & e_3 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

Hence it is an incidence of metric space of $d(x,y)$.

Theorem1: Let X be a nonempty set. Then a mapping $d: X \times X \to R$ is a metric if and only if the following condition are satisfied
$(M_1^*) d(x,y) = 0$ if and only if $x = y \; \forall \; x, y \in X$
$(M_2^*) d(x,y) \leq d(x,z) + d(y,z) \quad \forall \; x, y, z \in X$

Proof: Let the postulates of metric space $M_1 to\ M_4$ hold. Let $G = (V, E) be$ a graph, then the number of vertices of graph $(V = x, y, z)$ and number of edges be $(E = e_1, e_2, e_3, e_4, e_5)$ and e_5 be a self loop.

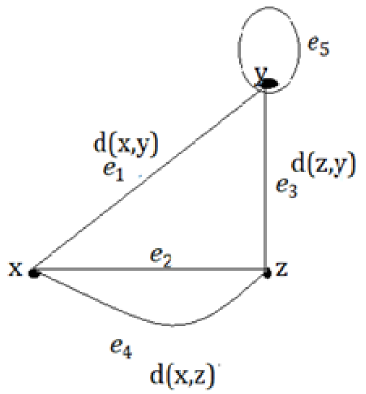

Fig.-1.4

$(M_1) d(x,x) = 0$ if and only if $x = x \quad \forall \; x \epsilon X$
$(M_2) d(x,y) = 0 \Rightarrow x = y \quad \forall \; x, y \in X$
$(M_3) d(x,y) = d(y,x) \quad \forall \; x, y \; \in X$
$(M_4) d(x,y \leq d(x,z) + d(z,y)$ (1)

Also by (M_3) ,

$$d(z,y) = d(y,z) \tag{2}$$

It follows from (1) & (2) that

$$d(x,y) \leq d(x,z) + d(y,z) \;\; \forall \; x, y, z \in X$$

which is (M_2^*) and (M_1^*) is the consequence of (M_1) $and(M_2)$

Conversely suppose that the condition $(M_1^*)$and (M_2^*) hold. Obviously (M_1) $and(M_2)$ are direct consequences of(M_1^*).

Now, let x, y be any two arbitrary points of X. Then applying
(M_2^*) for x, y, x we get

$$d(x,y) \leq d(x,x) + d(y,x)$$
$$0 + \; d(y,x) \qquad \text{by } (M_1^*)$$

Thus $\quad d(x,y) \leq d(y,x)$ (3)

Similarly applying (M_2^*) for y, x, y , $we\ get$

$$d(y,x) \leq d(y,y) + d(x,y)$$
$$0 + \; d(x,y) \qquad \text{by } (M_1^*)$$

Thus $\quad d(y,x) \leq d(x,y)$ (4)

From (3) & (4) we have

$$d(x,y) = d(y,x) \tag{5}$$

and so (M_3) is satisfied.

Finally for any$x, y, z \in X$, by (M_2^*)

$$d(x,y) \leq d(x,z) + d(y,z)$$
$$= d(x,z) + d(z,y) \quad \text{by(5)}$$

Which is (M_4).

By use the graph theory determine the degree of vertex of graph 1.4

$$d(x) = 3, d(y) = 4, d(z) = 3$$

Incidence of Matrix of graph 1.4

$$\begin{array}{cc} & \begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 \end{matrix} \\ I = \begin{matrix} x \\ y \\ z \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Adjacency matrix of graph 1.4

$$A = \begin{matrix} & x & y & z \\ x & 0 & 1 & 1 \\ y & 1 & 0 & 1 \\ z & 1 & 1 & 0 \end{matrix}$$

RESULTS

Hence prove that in a metric space has also a vertices and edges of metric $d(x, y)$ and satisfy the condition of undirected graph theory.

REFERENCES

1. Biggs, N.; Lloyd, E.; Wilson, R. (1986). Graph Theory, 1736–1936. Oxford University Press.
2. Bondy, J. A.; Murty, U. S. R. (2008). Graph Theory. Springer. ISBN 978-1-84628-969-9.
3. Bollobás, Béla; Riordan, O. M. (2003). Mathematical results on scale-free random graphs in "Handbook of Graphs and Networks" (S. Bornholdt and H.G. Schuster (eds)) (1st ed.). Weinheim: Wiley VCH.
4. Chartrand, Gary (1985). Introductory Graph Theory. Dover. ISBN 0-486-24775-9.
5. Deo, Narsingh (1974). Graph Theory with Applications to Engineering and Computer Science (PDF). Englewood, New Jersey: Prentice-Hall. ISBN 0-13-363473-6.
6. Harary, Frank (1969). Graph Theory. Reading, Massachusetts: Addison-Wesley.
7. Mahadev, N. V. R.; Peled, Uri N. (1995). Threshold Graphs and Related Topics. North-Holland.
8. Newman, Mark (2010). Networks: An Introduction. Oxford University Press.
9. Victor Bryant, Metric Spaces: Iteration and Application, Cambridge University Press, 1985, ISBN 0-521-31897-1.
10. Dmitri Burago, Yu D Burago, Sergei Ivanov, A Course in Metric Geometry, American Mathematical Society, 2001, ISBN 0-8218-2129-6.
11. Mícheál Ó Searcóid, Metric Spaces, Springer Undergraduate Mathematics Series, 2006, ISBN 1-84628-369-8.
12. Lawvere, F. William, "Metric spaces, generalized logic, and closed categories", [Rend. Sem. Mat. Fis. Milano 43 (1973), 135—166 (1974); (Italian summary).