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THE MULTIPLICATIVE (*a, b*)-KULLI-BASAVA INDICES OF GRAPHS

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ABSTRACT

Recently, Kulli-Basava indices were introduced and studied their mathematical and chemical properties which have good response with mean isomer degeneracy. In this paper, we introduce the multiplicative symmetric division Kulli-Basava index, first and second multiplicative Kulli-Gourava indices, multiplicative F_1-Kulli-Basava index, multiplicative (a, b)-Kulli-Basava index of a graph. We compute these indices for regular, wheel, gear and helm graphs.

Keywords: *Multiplicative Gourava-Kulli indices, multiplicative F_1-Kulli-Basava index, multiplicative (a, b)-Kulli-Basava index, graph.*

Mathematics Subject Classification: *05C05, 05C07, 05C12.*

1. INTRODUCTION

Let $G(V(G), E(G))$ be a finite, simple connected graph. The degree $d_G(u)$ of a vertex u is the number vertices adjacent to u. The degree of an edge $e=uv$ in G in defined by $d_G(e)= d_G(u) + d_G(v) – 2$. Let $S_e(u)$ denote the sum of the degrees of all edges incident to u. We refer [1] for undefined term and notation.

A graph index or a topological index is a numerical parameter mathematically derived from the graph structure. Graph indices have been found to be useful in chemical documentation, isomer discrimination, QSPR/QSAR study. There has been considerable interest in the general problem of determining graph indices. Recently some new multiplicative graph indices were studied, for example, in [2, 3, 4, 5, 6, 7, 9].

In [10], Kulli introduced the first multiplicative Kulli-Basava index of a graph and it is defined as

$$KB_1II(G) = \prod_{uv\in E(G)} \left[S_e(u) + S_e(v)\right].$$

Recently, some Kulli-Basava indices were studied in [11, 12, 13, 14, 15, 16, 17].

We propose the multiplicative F_1-Kulli-Basava index of a graph, defined as

$$F_1KBII(G) = \prod_{uv\in E(G)} \left[S_e(u)^2 + S_e(v)^2\right].$$

We define the general Kulli-Basava index of a graph G as

$$KB_aII(G) = \prod_{uv\in E(G)} \left[S_e(u)^a + S_e(v)^a\right].$$

We now propose the multiplicative symmetric division Kulli-Basava index of a graph G, defined it as

$$SDKBII(G) = \prod_{uv\in E(G)} \left(\frac{S_e(u)}{S_e(v)} + \frac{S_e(v)}{S_e(u)}\right).$$

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We also introduce the first and second multiplicative Kulli-Gourava indices of a graph G, defined as

$$KGO_1II(G) = \prod_{uv\in E(G)} \left[S_e(u) + S_e(v) + S_e(u)S_e(v)\right].$$

$$KGO_2II(G) = \prod_{uv\in E(G)} \left[S_e(u) + S_e(v)\right]S_e(u)S_e(v).$$

Recently some Gourava indices were studied in [18].

Motivated by the work on multiplicative Kulli-Basava indices, we define the multiplicative (a, b)- Kulli-Basava index of a graph G as

$$N_{a,b}II(G) = \prod_{uv\in E(G)} \left[S_e(u)^a S_e(v)^b + S_e(u)^b S_e(v)^a\right].$$

where a, b are real numbers.

Recently, some (a, b)-indices were studied in [19, 20, 21].

In this paper, the multiplicative F_1-Kulli-Basava index, multiplicative symmetric division Kulli-Basava index, first and second multiplicative Kulli-Gourava indices, multiplicative (a, b) Kulli-Basava index for regular graphs, wheel graphs, gear graphs, helm graphs are determined.

2. OBSERVATIONS

We observe the following relations between multiplicative (a, b)-Kulli-Basava indices with some other multiplicative Kulli-Basava indices.

(i) $KB_1II(G) = N_{1,0}II(G)$.

(ii) $F_1KBII(G) = N_{2,0}II(G)$.

(iii) $KBII_a(G) = N_{a,0}II(G)$.

(iv) $SDKBII(G) = N_{1,-1}II(G)$.

(v) $KG_2II(G) = N_{2,1}II(G)$.

3. RESULTS FOR REGULAR GRAPHS

Theorem 1: The multiplicative (a, b)-Kulli-Basava index of an r-regular graph G with n vertices is

$$N_{a,b}II(G) = \left[2\left[2r(r-1)\right]^{a+b}\right]^{\frac{nr}{2}}. \qquad (1)$$

Proof: Suppose G is an r-regular graph with n vertices, Then G has $\frac{nr}{2}$ edges. For any vertex u in G, $S_e(u) = 2r(r-1)$. Thus

$$N_{a,b}II(G) = \prod_{uv\in E(G)} \left[S_e(u)^a S_e(v)^b + S_e(u)^b S_e(v)^a\right]$$

$$= \prod_{uv\in E(G)} \left[\{2r(r-1)\}^a \{2r(r-1)\}^b + \{2r(r-1)\}^b \{2r(r-1)\}^a\right]$$

$$= \left[2\{2r(r-1)\}^{a+b}\right]^{\frac{nr}{2}}.$$

We obtain the following results from Theorem 1.

Corollary 1.1: Let G be an r-regular graph with n vertices and $\frac{nr}{2}$ edges. Then

(i) $KB_1II(G) = N_{1,0}II(G) = \left[4r(r-1)\right]^{\frac{nr}{2}}$.

(ii) $F_2KBII(G) = N_{2,0}II(G) = \left[8r^2(r-1)^2\right]^{\frac{nr}{2}}$.

(iii) $KBII_a(G) = N_{a,0}II(G) = \left[2\{2r(r-1)\}^a\right]^{\frac{nr}{2}}$.

(iv) $SDKBII(G) = N_{1,-1}II(G) = 2^{\frac{nr}{2}}$.

(v) $KGO_2II(G) = N_{2,1}II(G) = \left[16r^3(r-1)^3\right]^{\frac{nr}{2}}$.

Corollary 1.2: Let K_n be a complete graph with n vertices. Then

$$N_{a,b}II(K_n) = \left[2\{2(n-1)(n-2)\}^{a+b}\right]^{\frac{n(n-1)}{2}}.$$

Proof: Put $r = n - 1$ in equation (1), we obtain the desired result.

Note 1: By using observations and corollary 1.2, We obtain the values of (i) $KB_1II(K_n)$, (ii) $F_1KBII(K_n)$, (iii)$KBII_a(K_n)$, (iv) $SDKBII(K_n)$ (v) $KGO_2II(K_n)$.

Corollary 1.3: Let C_n be a cycle with n vertices. Then

$$N_{a,,b}\, II\,(C_n) = [2\times 4^{a+b}]^n.$$

Proof: Put $r = 2$ in equation (1), we obtain the desired result.

Note 2: By using observations and corollary 1.3, we establish the values of (i) $KB_1II(C_n)$, (ii) $F_1KBII(C_n)$, (iii) $KBII_a(C_n)$, (iv) $SDKBII(C_n)$, (v) $KGO_2II(C_n)$.

Theorem 2: The first multiplicative Kulli-Gourava index of an r-regular graph G is

$$KGO_1II(G) = \left[4r(r-1)(r^2-r+1)\right]^{\frac{nr}{2}}. \tag{2}$$

Proof: Let G be an r-regular graph with n vertices and $\frac{nr}{2}$ edges. For any vertex u in G, $S_e(u) = 2r(r - 1)$. Thus

$$KGO_1(G) = \prod_{uv\in E(G)} \left[S_e(u) + S_e(v) + S_e(u)S_e(v)\right]$$

$$= \left[2r(r-1) + 2r(r-1) + 2r(r-1)2r(r-1)\right]^{\frac{nr}{2}}$$

$$= \left[4r(r-1)(r^2-r+1)\right]^{\frac{nr}{2}}.$$

We establish the following results from Theorem 2.

Corollary 2.1: The first multiplicative Kulli-Gourava index of a complete graph K_n is

$$KGO_1II(K_n) = \left[4(n-1)(n-2)(n^2-3n+3)\right]^{\frac{n(n-1)}{2}}.$$

Proof: Put $r = n - 1$ in equation (2), we get the desired result.

Corollary 2.2: The first multiplicative Kulli-Gourava index of a cycle C_n is

$$KGO_1II(C_n) = 24^n.$$

Proof: Put $r = 2$ in equation (2), we obtain the desired result.

4. RESULTS FOR WHEEL GRAPHS

A wheel graph W_n is the join of K_1 and C_n. A wheel graph W_5 is shown in Figure 1.

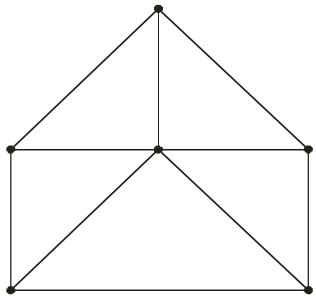

Figure-1: Wheel graph W_5

A wheel graph W_n has n+1 vertices and 2n edges. Then wheel graph W_n has two types of edges as given in Table 1.

$S_e(u), S_e(v) \setminus uv \in E(W_n)$	$(n(n+1), n+9)$	$(n+9, n+9)$
Number of edges	n	n

Table-1: Edge partition of W_n

Theorem 3: The multiplicative (a, b)-Kulli-Basava index of a wheel graph W_n is

$$N_{a,b}II(W_n) = \left[\{n(n+1)\}^a (n+9)^b + \{n(n+1)\}^b (n+9)^b\right]^n + \left[2(n+9)^{a+b}\right]^n.$$

Proof: Let W_n, be a wheel graph with n+1 vertices and 2n edges. From definition and by using Table 1, we deduce

$$N_{a,b}II(W_n) = \prod_{uv\in E(W_n)} \left[S_e(u)^a S_e(v)^b + S_e(u)^b S_e(v)^a\right]$$

$$= \left[\{n(n+1)\}^a (n+9)^b + \{n(n+1)\}^b (n+9)^a\right]^n \times \left[(n+9)^a (n+9)^b + (n+9)^b (n+9)^a\right]^n$$

$$= \left[\{n(n+1)\}^a (n+9)^b + \{n(n+1)\}^b (n+9)^a\right]^n + \left[2(n+9)^{a+b}\right]^n.$$

From Theorem 3 and by using observations, we find the following results.

Corollary 3.1: Let W_n be a wheel graph with n + 1 vertex and 2n edges. Then

(i) $KB_1II(W_n) = N_{1,0}II(W_n) = 2^n (n+9)^n (n^2+2n+9)^n$.

(ii) $F_1KBII(W_n) = N_{2,0}II(W_n) = 2^n (n+9)^{2n} \left[n^2(n+1)^2 + (n+9)^2\right]^n$.

(iii) $KBII_a(W_n) = N_{a,0}II(W_n) = \left[\{n(n+1)\}^a + (n+9)^a\right]^n \times \left[2(n+9)^a\right]^n$.

(iv) $SDKBII(W_n) = N_{1,-1}II(W_n) = 2^n \left[\frac{n(n+1)}{n+9} + \frac{n+9}{n(n+1)}\right]^n$.

(v) $KGO_2II(W_n) = N_{2,1}II(W_n) = 2^n n^n (n+1)^n (n+9)^{4n} (n^2+2n+9)^n$.

Theorem 4: The first multiplicative Kulli-Gourava index of a wheel graph W_n is

$$KGO_1II(W_n) = (n^3+11n^2+11n+9)^n (n+9)^n (n+11)^n.$$

Proof: Let W_n be a wheel graph with n+1 vertices and 2n edges. Then

$$KGO_1II(W_n) = \prod_{uv\in E(W_n)} \left[S_e(u) + S_e(v) + S_e(u)S_e(v)\right]$$

$$= \left[n(n+1) + (n+9) + n(n+1)(n+9)\right]^n \times \left[n+9+n+9+(n+9)(n+9)\right]^n$$

$$= (n^3+11n^2+11n+9)^n (n+9)^n (n+11)^n.$$

5. RESULTS FOR GEAR GRAPHS

A graph is a gear graph obtained from W_n by adding a vertex between each pair of adjacent rim vertices and it is denoted by G_n. A graph G_n is presented in Figure 2.

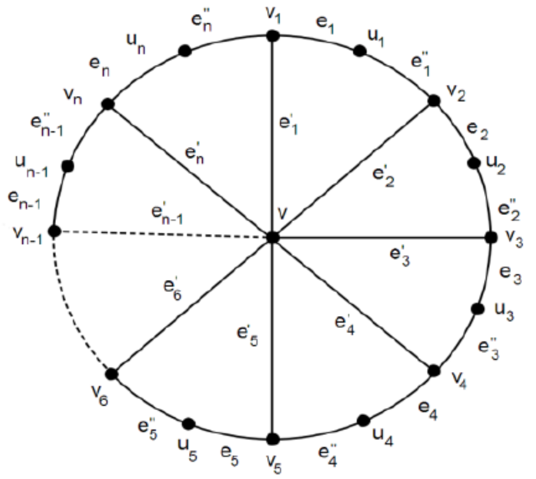

Figure-2: Gear graph G_n

A gear graph G_n has $2n+1$ vertices and $3n$ edges. In G_n, there are two types of edges as given in Table 2.

$S_e(u), S_e(v) \setminus uv \in E(G_n)$	$(n(n+1), n+7)$	$(n+7, 6)$
Number of edges	n	$2n$

Table-2: Edge partition of G_n

Theorem 5: The multiplicative (a, b)-Kulli-Basava index of a gear graph G_n is

$$N_{a,b}II(G_n) = \left[\{n(n+1)\}^a (n+7)^b + \{n(n+1)\}^b (n+7)^a\right]^n \times \left[(n+7)^a 6^b + (n+7)^b 6^a\right]^{2n}.$$

Proof: Let G_n be a gear graph with $2n+1$ vertices and $2n$ edges. From definition and by using Table 2, we derive

$$N_{a,b}II(G_n) = \prod_{uv \in E(G_n)} \left[S_e(u)^a S_e(v)^b + S_e(u)^b S_e(v)^a\right]$$

$$= \left[\{n(n+1)\}^a (n+7)^b + \{n(n+1)\}^b (n+7)^a\right]^n \times \left[(n+7)^a 6^b + (n+7)^b 6^a\right]^{2n}.$$

By using Theorem 5 and observations, we obtain the following results.

Corollary 5.1: Let G_n be a gear graph with $2n+1$ vertices and $3n$ edges. Then

(i) $KB_1II(G_n) = N_{1,0}II(G_n) = (n^2 + 2n + 7)^n \times (n+13)^{2n}$.

(ii) $F_1KBII(G_n) = N_{2,0}II(G_n) = (n^4 + 2n^3 + 2n^2 + 14n + 49)^n \times (n^2 + 14n + 85)^{2n}$.

(iii) $KBII_a(G_n) = N_{a,0}II(G_n) = \left[\{n(n+1)\}^a + (n+7)^a\right]^n \times \left[(n+7)^a + 6^a\right]^{2n}$.

(iv) $SDKBII(G_n) = N_{1,-1}II(G_n) = \left[\frac{n(n+1)}{n+7} + \frac{n+7}{n(n+1)}\right]^n \times \left(\frac{n+7}{6} + \frac{6}{n+7}\right)^{2n}$.

(v) $KGO_2II(G_n) = N_{2,1}II(G_n) = \left[n(n+1)(n+7)(n^2+2n+7)\right]^n \left[6(n+7)(n+1\ \ )\right]^{2n}$

Theorem 6: The first multiplicative Kulli-Gourava index of a gear graph G_n is

$$KGO_1II(G_n) = (n^3 + 9n^2 + 9n + 7)^n \times (7n+55)^{2n}.$$

Proof: Let G_n be a gear graph with $2n+1$ vertices and $3n$ edges. Then

$$KGO_1II(G_n) = \prod_{uv \in E(G_n)} \left[S_e(u) + S_e(v) + S_e(u)S_e(v)\right]$$

$$= \left[n(n+1) + n + 7 + n(n+1)(n+7)\right]^n \times \left[n + 7 + 6 + (n+7)6\right]^{2n}$$

$$= (n^3 + 9n^2 + 9n + 7)^n \times (7n+55)^{2n}.$$

6. RESULTS FOR HELM GRAPHS

A helm graph H_n is a graph obtained from wheel graph W_n by attaching an end edge to each rim vertex. A helm graph H_n is depicted in Figure 3.

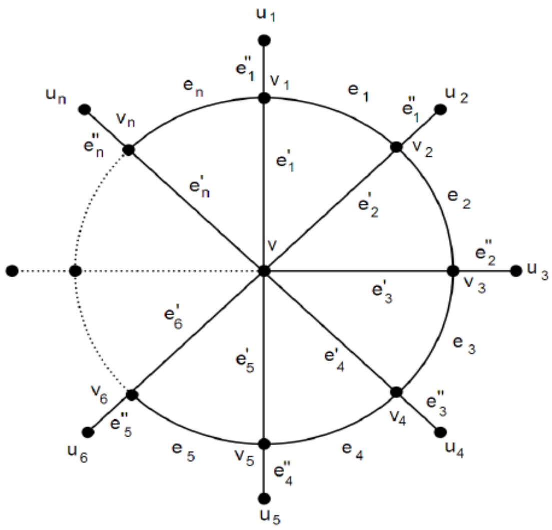

Figure-3: Helm graph H_n

Clearly, a helm graph H_n has $2n+1$ vertices and $3n$ edges. In H_n , there are three types of edges as given Table 3.

$S_e(u), S_e(v) \setminus uv \in E(H_n)$	$(n(n+2), n+17)$	$(n+17, n+17)$	$(n+17, 3)$
Number of edges	n	n	n

Table-3: Edge partition of H_n

Theorem 7: Let H_n be a helm graph with $2n+1$ vertices and $3n$ edges. Then the (a, b) Kulli-Basava index of a helm graph H_n is

$$N_{a,b}II(H_n)=\left[\{n(n+2)\}^{a}(n+17)^{b}+\{n(n+2)\}^{b}(n+17)^{a}\right]^{a}\times\left[2(n+17)^{a+b}\right]^{n}\times\left[(n+17)^{a}3^{b}+(n+17)^{b}3^{a}\right]^{n}.$$

Proof: Let H_n be a helm graph with $2n+1$ vertices and 3n edges. By using definition and Table 3, we obtain

$$N_{a,b}II(H_n)=\prod_{uv\in E(H_n)}\left[S_e(u)^{a}S_e(v)^{b}+S_e(u)^{b}S_e(v)^{a}\right]$$

$$=\left[\{n(n+2)\}^{a}(n+17)^{b}+\{n(n+2)\}^{b}(n+17)^{a}\right]^{n}$$

$$\times\left[(n+17)^{a}(n+17)^{b}+(n+17)^{b}(n+17)^{a}\right]^{n}\times\left[(n+17)^{a}3^{b}+(n+17)^{b}6^{a}\right]^{n}$$

$$=\left[\{n(n+2)\}^{a}(n+17)^{b}+\{n(n+2)\}^{b}(n+17)^{a}\right]^{n}\times\left[2(n+17)^{a}\right]^{n}$$

$$\times\left[(n+17)^{a}3^{b}+(n+17)^{b}3^{a}\right]^{n}.$$

By using Theorem 7 and observations, we get the following results.

Corollary 7.1: Let H_n be a helm graph with $2n+1$ vertices and $3n$ edges. Then

i) $KB_1II(H_n)=N_{1,0}II(H_n)=(n^2+3n+17)^{n}(2n+34)^{n}(n+20)^{n}$.

(ii) $F_1KBII(H_n)=N_{2,0}II(H_n)=\left[n^2(n+2)^2+(n+17)^2\right]^{n}\times\left[2(n+17)^2\right]^{n}\times\left[(n+17)^2+9\right]^{n}$.

(iii) $KBII_a(H_n)=N_{a,0}II(H_n)=\left[\{n(n+2)\}^{a}+(n+17)^{a}\right]^{n}\times\left[2(n+17)^{a}\right]^{n}\times\left[(n+17)^{a}+3^{a}\right]^{n}$.

(iv) $SDKBII(H_n)=N_{1,-1}II(H_n)=2^{n}\left[\frac{n(n+2)}{n+17}+\frac{n+17}{n(n+2)}\right]^{n}\times\left[\frac{n+17}{3}+\frac{3}{n+17}\right]^{n}$.

(v) $KGO_2II(H_n)=N_{2,1}II(H_n)=2^{n}3^{n}\left[n(n+2)(n^2+3n+17)(n+20)\right]^{n}(n+17)^{5n}$.

Theorem 8: The first multiplicative Kulli-Gourava index of a helm graph H_n is

$$KGO_1II(H_n) = (n^3 + 20n^2 + 37n + 17)^n \times (n+17)^n \times (n+19)^n \times (4n+71)^n .$$

Proof: Let H_n be a helm graph with $2n+1$ vertices and 3n edges. Then

$$KGO_1II(H_n) = \prod_{uv\in E(H_n)} \left[S_e(u)+S_e(v)+S_e(u)S_e(v)\right]$$

$$= \left[n(n+2)+n+17+n(n+2)(n+17)\right]^n \times \left[n+17+n+17+(n+17)(n+17)\right]^n$$

$$\times\left[n+17+3+(n+17)3\right]^n$$

$$= (n^3 + 20n^2 + 37n + 17)^n \times (n+17)^n \times (n+19)^n \times (4n+71)^n .$$

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