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SOLVING BI-LEVEL LARGE SCALE MULTI-OBJECTIVE QUADRATIC PROGRAMMING PROBLEM WITH NEUTROSOPHIC PARAMETERS IN THE CONSTRAINS USING DECOMPOSITION METHOD

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ABSTRACT

Bi-level large-scale multi-objective is very important topic especially when the problem contains neutrosophic parameters. Most of the practical problems in our world are large-scale problems due to the large number of decision variable that sometimes reach to thousands of decision variables. The presented paper solves bi-level large-scale multi-objective quadratic programming problem with neutrosophic parameter in constrains. The introduced algorithm starts firstly to convert the neutrosophic problem to crisp problem due to simplify the problem then the taylor and weighting method play an important role to convert the problem from bi-level large-scale multi-objective quadratic problem to bi-level large-scale single objective linear programming problem (BLLSPP). As a last step in our algorithm, the decomposition method used to solve BLLSPP and reach the optimal solution for this problem.

Keywords: *Bi-level programming; Large scale; linear programming; Neutrosophic set; Trapezoidal neutrosophic number; Multi-objective; Quadratic programming; Decomposition approach; Block angular structure.*

1. INTRODUCTION

Bilevel programming is with none doubt one among the foremost popular research areas in mathematical Optimization [1].

Bi level interest may be a consequence of various underlying real-world applications from a good range of areas, including economics, logistics, and chemistry, which may be model as hierarchical programs with two decision makers [1].

Bi-level Programming (BLP) is a version of the problem of multi-level programming being defined as a problem of mathematical programming being addresses democratic planning problems with two decision makers (DMs) in a two-level or hierarchical organisation [2].

Multi-objective problems of optimization are a type of complicated problems of optimization, in which many specific objective functions have to be handled at once. Usually there is no approach that optimizes all the multiple target functions simultaneously [3].

The optimization of two or more fractional objective functions that clash in essence is considered as multi-objective fractional programming problem. Multi-objective fractional programming problem is a helpful way of modelling real world decision-making issues with objective functions in the form of fractions such as reducing individual costs / standard costs, optimizing profit / cost, reducing inventory / sales, etc. [4].

In the area of big data, there exists plenty of complicated data in many research fields and real -worldapplications, which raises a variety of optimization problems having multiple objectives and a large numberof decision variables. These types of problems called large scale problems [5].

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espite that most existing MOEAs have been well assessed on the MOPs with a small number of decision variables, their performance degenerates dramatically on MOPs with hundreds or even thousands of decision variables, i.e., the large-scale multi-objective optimization problems (LSMOPs)

Despite that most existing multi-objective evolutionary algorithms (MOPEA) have been well assessed on the multi-objectives problems (MOPs) with a small number of decision variables, the efficiency of these algorithm significantly degenerates for hundreds or even thousands of decision variables on MOPs, i.e. large-scale multi-objective optimisation problems (LSMOPs) [6].

Any of these large-scale real-world challenges also have unique mechanisms which can be used to solve problems. One common special structure is the block-angular structure to the constraints and different forms of decomposition approaches have been developed for linear and nonlinear programming problems with block-angular structure [7]

Neutrosophic logic is a non-classical and modern philosophy created in 1999 by Florentin Smarandache, philosopher and mathematical American. Neutrosophic set theory appeared to be a generalization of both the classical and the fuzzy equivalents [8].

Neutrosophic sets defined by three different degrees, including the degree of truth-membership (T), the degree of indeterminacy-membership (I), and the degree of falsity-membership (F), where T, I, F are regular or non-standard subsets of]-0, 1 +. Neutrosophic decision-makers tend to raise the level of truth-membership and reduce the degree of indeterminacy and membership of falsity [9].

Many researches dealwith bi-level and large scale multi-objective programming problems [3], [10], [11].

In [3] Emam *et al.* tackled a quadratic programming problem with stochastic variables in the constraints (SBLMOLSQPP) on two-level multi-objective large-scale. They dealt with the problem using an algorithm that started to convert the stochastic nature into a deterministic equivalent of this problem, and after that the Taylor series is used to overcome the complexity of the quadratic issue.

Another research conducted to tackle multi-level multi-objective large-scale issues, In [12] Hawaf AbdAlhakim *et al*. Suggested a modern method to rephrase the objective function relying on the algorithm of decomposition in an integrated model to find an optimal solution to Multi-level Large Scale Quadratic Programming Problem (MLLSQPP), Where objective function can be converted into two distinct linear objective functions and where the constraints have a block angular structure.

Several studies have been conducted in the field of neutrosophic programming problems [8], [9], [13]-[16].

In [15] Mai Mohamed et al proposed integer programming in a neutrosophic environment, taking into account problem coefficients as triangular neutrosophic numbers. Concurrent attention is extended to the degrees of acceptance, indeterminacy and rejection of goals. The Neutrosophic Integer Programming Problem (NIPP) is converted into a linear programming model that employs truth membership (T), indeterminacy membership (I) and falsity membership (F) functions as well as single weighted triangular neutrosophic values.

In [16] Mohamed Abdel-Basset *et al*. developed the neutrosophic linear programming models in which their parameters are described by a trapezoidal neutrosophic number and proposed a solution technique.

This paper is arranged as follows: In Section 2 introduce the problem formulation and some preliminary discussion. In Section 3 discusses a bi-level large-scale multi-objective quadratic programming problem with neutrosophic parameters in the constrains (BLLSMOQPP) and how we will transfer this problem to bi-level programing problem. In Section 4, An algorithm introduced for solving the BLMOQPP with neutrosophic parameters in the constrains. In Section 5, we will apply our algorithm on the numerical example. Lastly, the findings and recommendations for future works are given in Section 6.

2. PROBLEM FORMULATION AND PRELIMINARY DISCUSSION

In this section, we discuss the structure, formulation of the bi-level large-scale multi-objective quadratic programming problem with neutrosophic parameter in constrains and we recall some important definitions related to neutrosophic set, large-scale and bi-level multi-object problem.

2.1 Problem Formulation:

The bi-level multi-objective quadratic programming problem with neutrosophic parameter in the objective functions can represented as follows:

[Upper Level]

$$\underset{\boldsymbol{x_1,x_2}}{\boldsymbol{Max}}\boldsymbol{F_1}(\boldsymbol{x}) = \underset{X_1,X_2}{\text{Max}}(f_{11}(\boldsymbol{x}), \boldsymbol{f_{12}}(\boldsymbol{x}), \ldots, \boldsymbol{f_{1u}}(\boldsymbol{x})) \quad (1)$$

Where x_3, x_4 solves

[Lower Level]

$$\underset{x_3,x_4}{Max}F_2(x) = \underset{X_3,X_4}{\text{Max}}(f_{21}(x), f_{22}(x), \ldots, f_{2n}(x)), \quad (2)$$

Subject to (3)

$$H = \{a_{01}x_1 + a_{02}x_2 + a_{0p}x_p \leq \tilde{b}_0,$$
$$d_1x_1 \leq \tilde{b}_1,$$
$$d_2x_2 \leq \tilde{b}_2,$$
$$d_px_p \leq \tilde{b}_p,$$
$$x_1, \ldots, x_p \geq 0\}.$$

And where

$$f_{ik} = c_{ik}x + \frac{1}{2}x^T L_k^i x \quad , (i = 1,2), (k = 1,2, \ldots, n_i) \quad (4)$$

Let the functions F_1 and F_2 are quadratic objective functions defined on R^n and (L^1, L^2) are $m \times n$ matrices describing the coefficients of the quadratic terms, c_{ik} are $1 \times m$ matrices and trapezoidal neutrosophic numbersin the above problem (1), (2), (4).

Let x_1, x_2, x_3, x_4 be actual vector variables representing the choice of the first level and the second decision level. In addition, the decision-maker at the upper level has x_1, x_2 indicating the option of first level and the decision maker at lower level has x_3, x_4 indicating level choice.

H is the large-scale linear constraints set where $b = (\tilde{b}_0, \ldots, \tilde{b}_p)^T$ are trapezoidal neutrosophic numbers, $b = (\tilde{b}_0, \ldots, \tilde{b}_p)^T$ is an $(p + 1)$ vector, and $a_{01}, \ldots, a_{0p}, d_1, \ldots, d_p$ are constants.

2.2 Preliminaries

In this section, we write the important definitions and preliminaries for our problem:

Definition 1: For any $\left(x_1, x_2 \in H_1 = \{x_1, x_2 | (x_1, x_2, x_3, \ldots, x_p) \in H\}\right)$ given by firstlevel, if the decision-making variable $\left(x_3, x_4 \in H_2 = \{x_3, x_4 | \ (x_1, x_2, x_3, \ldots, x_p) \in H\}\right)$ is the Pareto optimal solution of the second level, then (x_1, x_2, x_3, x_4) is a feasible solution of (BLLSMONQPP).

Definition 2: If $x^* \in R^m$ is a feasible solution of the (BLLSMONQPP), no other feasible solution $x \in H$ exists, such that $F_1(x^*) \leq F_1(x)$ so x^* is the Pareto optimal solution of the (BLLSMOQPP).

Definition 3: [17] Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophicset A in X is defined by a function of membership of fact $\mathrm{T_A}$, a function of membership of indeterminacy $\mathrm{I_A}$ and a function of falsity $\mathrm{F_A}$. $\mathrm{F_A}(x), \mathrm{T_A}(x), \mathrm{I_A}(x)$ are real standard or non-standard subset of $]^{-}0, {}^{+}1[$. There is no limitation for the submission of $\mathrm{F_A}(x), \mathrm{T_A}(x), \mathrm{I_A}(x)$.

Definition 4: [16] The trapezoidal neutrosophic number $\tilde{Z}$ is a neutrosophic set in R with the following T, I and F membership functions:

$$T_{\tilde{Z}}(x) = \begin{cases} \propto_{\tilde{Z}} \left(\frac{x-z_1}{z_2-z_1}\right) & (z_1 \leq x \leq z_2) \\ \propto_{\tilde{Z}} & (z \leq x \leq z_3) \\ \propto_{\tilde{Z}} & (z_3 \leq x \leq z_4) \\ 0 & otherwise \end{cases} \quad (5)$$

$$I_{\tilde{Z}}(x) = \begin{cases} \frac{(z_2 - x + \theta_{\tilde{Z}}(x - z_1'))}{(Z_2 - Z'_1)} & (z_1' \leq x \leq z_2) \\ \theta_{\tilde{Z}} & (z_2 \leq x \leq z_3) \\ \frac{(x - z_3 + \theta_{\tilde{Z}}(z_4' - x))}{(z_4' - z_3)} & (z_3 \leq x \leq z_4') \\ 1 & otherwise \end{cases} \quad (6)$$

$$F_{\tilde{Z}}(x) = \begin{cases} \frac{(z_2 - x + \beta_{\tilde{Z}}(X - z_1^{''}))}{(z_2 - z_1^{'})} & (z_1^{''} \le x \le z_2) \\ \beta_{\tilde{Z}} & (z_2 \le x \le z_3) \\ \frac{(x - z_3 + \beta_{\tilde{Z}}(z_4^{''} - x))}{(z_4^{''} - z_3)} & (z_3 \le x \le z_4^{''}) \\ 1 & otherwise \end{cases} \tag{7}$$

where $\propto_{\tilde{Z}}, \theta_{\tilde{Z}}$ and $\beta_{\tilde{Z}}$ represent the maximum truthiness degree, minimum indeterminacy degree, and minimum falsity degree, sequentially;$\propto_{\tilde{Z}}, \theta_{\tilde{Z}}$; and $\beta_{\tilde{Z}}$ $\in [0, 1]$. Additionally, $z_1^{''} \le z_1 \le z_1^{'} \le z_2 \le z_3 \le z_4^{'} \le z_4 \le z_4^{''}$.

3. SOLUTION CONCEPTS

In this section we will discuss our solution strategies and concepts to solve our problem.

3.1 Ranking Method:

We will use the ranking method to transform the neutrosophic number that exits in the objective functions to crisp number.

In the following type trapezoidal neutrosophic number was presented [18]:
$(\tilde{Z} = z^l, z^{m1}, z^{m2}, z^u;\ T_{\tilde{Z}}, I_{\tilde{Z}}, F_{\tilde{Z}})$where$\tilde{Z}$is a trapezoidal neutrosophic number and $z^l, z^{m1}, z^{m2}, z^u$are the lower bound, first and second median value and upper bound for trapezoidal neutrosophic number, respectively. In addition,$F_{\tilde{Z}}, T_{\tilde{Z}}, I_{\tilde{Z}}$represent thefalsity degrees of the trapezoidal number, truthdegrees of the trapezoidal number and finally the indeterminacy degrees of the trapezoidal number.

In case the objective function or the problemis a case of maximization state, at that point the ranking function for this trapezoidal neutrosophic number can be expressed as the following[18]:

$$R(\tilde{Z}) = |\left(\frac{-\frac{1}{3}(3z^l - 9z^u) + 2(Z^{m1} - Z^{m2})}{2}\right) * (T_{\tilde{Z}} - I_{\tilde{Z}} - F_{\tilde{Z}})| \tag{8}$$

However, if the objective functionor the problem is a case of minimization, the ranking method for such a trapezoid is as following[18]

$$R(\tilde{Z}) = |\left(\frac{(Z^l + Z^u) - 3(Z^{m1} + g^{m2})}{-4}\right) * (T_{\tilde{Z}} - I_{\tilde{Z}} - F_{\tilde{Z}})| \tag{9}$$

In case the author works with a symmetric neutrosophic trapezoidal number that has the following form:
$\tilde{Z} = \langle (z^{m1}, z^{m2}); \alpha, \beta \rangle$, where $\alpha = \beta$ and $\alpha, \beta > 0$, The ranking function will then be described as follows for the neutrosophic number[18].

$$R(\tilde{Z}) = \left(\frac{(Z^{m1} + Z^{m2}) + 2(\alpha + \beta)}{2}\right) * T_{\tilde{Z}} - I_{\tilde{Z}} - F_{\tilde{Z}} \tag{10}$$

3.2: Taylor series approach and weighting method:

We will use 1^{st} order Taylor series polynomial to transform the quadratic objectives function to linear objective function throw the following form [19]:

$$K_i(x) \cong F_i^{\wedge}(x) = F_i(x_i^*) + \sum_{j=1}^{n} (x_j - x_{ij}^*) \frac{\partial F_i(x_j^*)}{dx_j}, (j = 1,2,...m), (i = 1,2) \tag{11}$$

Then we will use the weighting method to transform the multi-objective in the first level and second level to single objective function at each level so the problem transferred from (BLMOQPP) to (BLLPP):

3.3: Decomposition algorithm for the bi-level large-scale linear programming problem:

The bi-level large-scale linear programming problem is solved by adopting the leader-follower Stackelberg strategy combine with Dantzig and Wolf decomposition method [20]. First, the optimal solution that is acceptable to the FLDM is obtained using the decomposition method to break the large-scale problem into n-sub problems that can be solved directly.

The decomposition technique depends on representing the BLLSLPP in terms of the extreme points of the sets $d_j x_j \le b_j, x_j \ge 0, j = 1,2,..,m$. To do so, the solution space described by each $d_j x_j \le b_j, x_j \ge 0, j = 1,2,..,m$must be bounded and closed.

After that by inserting the upper level decision variable to the lower level for him/her to search for the optimal solution using Dantzig and Wolf decomposition method [20], then the decomposition method break the large scale problem into n-sub problems that can be solved directly and obtain the optimal solution for his/her problem which is the optimal solution to the BILSPP.

The decomposition algorithm terminates in a finite number of iterations, yielding a solution of the large-scale problem. To prove theorem 1 above, the reader is referred to [20].

4. AN ALGORITHM FOR SOLVING THE BLMONQPP

In the following step series, the algorithm for solving the BLLSMOQPP with neutrosophic parameters in constrains illustrated:

Step-1: BLMOQPP with neutrosophic parameters in the objective functions added by decision makers.

Step-2: In case the BLMONQPP is in the state of maximization, then each neutrosophic parameter in the objective function translated to its corresponding crisp value by equation (8). But if the objective functions are minimization, then each neutrosophic parameter in the objective function is translated to its corresponding crisp value by equation (9). But in the symmetrictrapezoidal case we will use equation (10).

Step-3: The BLMONQPP is simplified into the equivalent deterministic BLMOQPP.

Step-4: Convert bi level multi-objective quadratic programming to linear by using first order Taylor series approach as bellow:

$$H_i(x) \cong F_i^{\wedge}(x) = F_i(x_i^*) + \sum_{j=1}^{n}(x_j - x_{ij}^*)\frac{\partial F_i(x_i^*)}{dx_j}, (j = 1,2,\ldots n)$$

Step-5: Convert the multi objective problem to single objective by using the weight method.

Step-6: The BLMOQPP simplified into BLLPP.

Step-7: Start with the upper level problem and convert the master problem in terms of extreme points of the sets $d_j x_j \le b_j$, $x_j \ge 0$, $j = 1,2,3$.

Step-8: Determine the extreme points $X_j = \sum_{k=1}^{k_j} \beta_{jk} \hat{x}_{jk}, j = 1,2,3$ using Balinski's algorithm [21].

Step-9: Set $k = 1$.

Step-10: Compute $z_{jk} - c_{jk} = C_B B^{-1} P_{jk} c_{jk}$

Step-11: If $\overset{*}{z}_{jk} - \overset{*}{c}_{jk} \le 0$, then go to Step 12; otherwise, the optimal solution has been reached, go to Step 17.

Step-12: Determine $\hat{X}_{jk}$ associated with $\overset{*}{min}\{ z_{jk} - \overset{*}{c}_{jk} \}$

Step-13: β_{jk} associated with extreme point $\hat{X}_{jk}$ must enter the solution

Step-14: Determine the leaving variable

Step-15: The new basis is determined by replacing the vector associated with leaving variable with the vector β_{jk} .

Step-16: Set $k = k + 1$, go to step 10.

Step-17: If the SLDM obtain the optimal solution go to Step 20, otherwise go to Step 18

Step-18: Set $(x_1, x_2) = (x_1^F, x_2^F)$ to the SLDM constraints, go to Step 19

Step-19: The SLDM formulate his problem, go to Step 9.

Step-20: $\left(x_1^F, x_2^F, x_3^S, x_4^S\right)$Is as an optimal solution for bi-level large scale linear programming problem, then stop.

5. NUMERICAL EXAMPLE

In this section, we will solve a Bi-level Large-Scale Multi-objective Quadratic Programing Problem (BLMOQPP) with trapezoidalneutrosophic numbers in the constrains:

[First Level]

$$\underset{x_1,x_2}{\text{Max}} F_1(x) = \underset{x_1,x_2}{\text{Max}}(6x_1^2 + 5x_2^2 + x_3 + x_4, 7x_1^2 + 10x_2^2 + x_4)$$

Where x_3, x_4 solves

[Second Level]

$$\underset{x_3,x_4}{\text{Max}} F_2(x) = \underset{x_3,x_4}{\text{Max}}(x_1 + 9x_3^2 + 4x_4^2, x_2 + 9x_3^2 + 6x_4^2)$$

Subject to

$$x_1 + x_2 + x_3 + x_4 \leq (30,42,62,80),$$
$$x_1 + x_2 \leq (18,20,26,30),$$
$$3x_3 + 2x_4 \leq (32,37,46,50),$$
$$x_3 + x_4 \leq (14,16,24,25),$$
$$x_1, x_2, x_3, x_4 \geq 0$$

In this example, we solve a Bi-level Large Scale Multi-objective Quadratic Programing Problem (BLMOQPP) with trapezoidalneutrosophic numbers in the right hand side of constrains.

The order of element for trapezoidal neutrosophic numbers (k^L, M^1, M^2, K^u) is as follows: lower bound, first median, second median and finally upper bound.

Because the first median and second median are not equal, so this case is non-symmetric and the problem is maximization so the trapezoidal number will be converted to its equivalent crisp number by using equation (8)

The decision makers decide the degree about each value of trapezoidal neutrosophic number is (0.9, 0.3, 0.2) for the first level and the same values for the second level.

Now first level and second level transformed from Neutrosophic bi-level large-scale multi-objective quadratic programing problem (BLMONQPP) to crisp bi-level large-scale multi-objective quadratic programing problem (BLMOQPP) so the crisp model of previous problem will be as follows:

[First Level]

$$\underset{x_1,x_2}{\text{Max}} F_1(x) = \underset{x_1,x_2}{\text{Max}}(6x_1^2 + 5x_2^2 + x_3 + x_4, 7x_1^2 + 10x_2^2 + x_4)$$

Where x_3, x_4 solves

[Second Level]

$$\underset{x_3,x_4}{\text{Max}} F_2(x) = \underset{x_3,x_4}{\text{Max}}(x_1 + 9x_3^2 + 4x_4^2, x_2 + 9x_3^2 + 6x_4^2)$$

Subject to

$$x_1 + x_2 + x_3 + x_4 \leq 36,$$
$$x_1 + x_2 \leq 12,$$
$$3x_3 + 2x_4 \leq 20,$$
$$x_3 + x_4 \leq 9,$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Then we use the first order Taylor series and weight method respectively to transfer the quadratic large-scale multi objective quadratic function to linear large-scale single objective function so the BLLSLPP written as following:

[Upper level]

$$\underset{x_1,x_2}{\text{Max}} F_1(x) = \underset{x_1,x_2}{\text{Max}}(15x_2 + .5x_3 + x_4 - 7.5)$$

[Lower level]

$$\underset{x_3,x_4}{\text{Max}} F_2(x) = \underset{x_3,x_4}{\text{Max}}(.5x_1 + .5x_2 + 10x_4 - 5)$$

Subject to

$$x_1 + x_2 + x_3 + x_4 \leq 36,$$
$$x_1 + x_2 \leq 12,$$
$$3x_3 + 2x_4 \leq 20$$
$$x_3 + x_4 \leq 9$$
$$x_1, x_2, x_3, x_4 \geq 0$$

The first level problem is formulated as:

$$\underset{x_1,x_2}{Max}\ H_1(x) = \underset{x_1,x_2}{Max}(15x_2 + .5x_3 + x_4 - 7.5)$$

Subject to

$$x \in G.$$

After four iterations the first level decision maker optimal solution is obtained:

$$\left(x_1^F, x_2^F, x_3^F, x_4^F\right) = (0,12,0,9).$$

So,

$$F_1 = 181.5$$

Then take the first level decision maker solution and set $(x_1^F, x_2^F) = (0,12)$ to the second level constrain.

The second level decision maker will repeat the same steps as the first level decision maker until the second level decision maker get the optimal solution so:

$\left(x_3{}^s, x_4{}^s\right) = (0,9).$

So, $(x_1{}^F, x_2{}^F, x_3{}^s, x_4{}^s) = (0,12,0,9).$

$F_1 = 181.5,\ F_2 = 91$

6. CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

This research paper proposed an algorithm for solving the BLLSMOQPP with neutrosophic parameters in constrains. The solution algorithm depended on minimizing the complexity of the problem by converting the neutrosophic nature of the problem into its corresponding crisp model. Then first order Taylor series user to overcome the complexity of the quadratic and transfer the quadratic nature to linear problem then weight method used to transfer the multi- objective that occurs in each level to a single objective function. In the last step in our algorithm decomposition algorithm used to get the solution for the large-scale linear programming problem. We Proposed a numerical example to explain and demonstrate our algorithm steps.

However, a variety of topics remain subject to potential discussion and can explored by regular, bi-level neutrosophic optimization:

1. Bi-level quadratic programming problems with neutrosophic parameters in both objective functions and constraints.
2. Bi-level quadratic multi-objective decision-making problems with neutrosophic parameters in both objective functions and constraints with integrity conditions.

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