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RADIATION EFFECTS ON STEADY HYDROMAGNETIC FLOW OF A VISCOUS FLUID THROUGH A VERTICAL CHANNEL IN A POROUS MEDIUM WITH HEAT GENERATION OR ABSORPTION

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ABSTRACT

This paper investigates the combined effects of radiation and heat generation or absorption on steady hydromagnetic flow of an electrically conducting optically thin fluid through a vertical channel filled with porous medium and non-uniform wall temperatures. Analytical solutions are obtained for the governing coupled dimensionless partial differential equations of velocity and temperature. Numerical results for velocity and temperature are presented graphically and the numerical values of skin-friction and Nusselt number have been tabulated. The effect of different parameters like thermal Grashof number, radiation parameter, porosity parameter and heat generation/absorption parameter on velocity, temperature, skin-friction and Nusselt number are discussed.

Keywords: *Steady hyromagnetic, Vertical channel, Porous medium, Thermal radiation, Heat generation/absorption.*

1- INTRODUCTION:

The study of dynamics and heat transfer of viscous and electrically conducting fluids find applications in various problems like astrophysical and geophysical phenomena. On the other hand, heat generation/absorption process in electrically conducting fluid flows under the influence of magnetic field has important engineering applications. Deka and Bhattacharya [2] have studied the exact solution of unsteady free convective couette flow of a viscous incompressible heat generating/ absorbing fluid confined between two vertical parallel plates in a porous medium. Grief et al. [3] obtained an exact solution for the problem of laminar convective flow in a vertical heated channel within optically thin limit of Cogley et al. [1]. Cookey et al. [4] have studied the combined effects of radiative heat transfer and a transverse magnetic field on steady flow of an electrically conducting optically thin fluid through a horizontal channel filled with porous medium and non- uniform temperature at the walls. Cookey et al. [5] have studied MHD free convection and oscillatory flow of an optically thin fluid bounded by two horizontal porous parallel walls under the influence of an external imposed transverse magnetic field in a porous medium. Cookey and Nwrahari [6] have studied unsteady MHD flow of a radiating fluid over a vertical moving heated porous plate with time - dependent suction. Kumar et al. [7] considered the problem of unsteady MHD periodic flow of viscous fluid through a planar channel in porous medium using perturbation techniques. Kearsley [8] studied problem of steady state couette flow with viscous heating. Makinde and Mhone [9] investigated the effect of thermal radiation on MHD oscillatory flow in a channel filled with saturated porous medium and non-uniform wall temperatures. Makinde and Osalusi [10] considered a MHD steady flow in a channel with at permeable boundaries. Narahari [11] studied an exact analysis of the natural convection in unsteady couette flow of a viscous incompressible fluid confined between two vertical parallel plates in the presence of thermal radiation. Seth et al. [12] have studied unsteady MHD couette flow of a viscous incompressible electrically conducting fluid, in the presence of a transverse magnetic field, between two parallel porous plates.

The objective of the present paper is to investigate the combined effect of thermal radiation and heat generation/absorption on steady hydromagnetic flow of an electrically conducting optically thin fluid through a vertical channel filled with porous medium and non-uniform wall temperatures.

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2- MATHEMATICAL ANALYSIS:

Consider the steady flow of a viscous, incompressible and electrically conducting optically thin fluid bounded by two vertical parallel plates filled with porous medium in the presence of heat generation or absorption and uniform transverse magnetic field of strength B_0. The x' - axis is considered along one of the vertical plate and y' - axis is taken normal to the plate. The temperature of the plate which is at $y'=0$ is maintained at $T'=T_0'$ and the temperature of plate which is at $y'=h$ is maintained at $T'=T_1'$. Also, we assume that the temperatures of both the walls are high enough to induce radiative heat transfer. Then under the usual Boussinesq's approximation, the governing equations for electrically conducting optically thin fluid are as follows:

$$\nu\frac{\partial^2 u'}{\partial y'^2}+g\beta(T'-T_0')-\frac{\nu}{K^*}u'-\frac{\sigma B_0^2}{\rho}u'=0, \tag{1}$$

$$\frac{k}{\rho C_p}\left[\frac{\partial^2 T'}{\partial y'^2}-\frac{1}{k}\frac{\partial q}{\partial y'}-\frac{1}{k}Q_0(T'-T_0')\right]=0, \tag{2}$$

with the following boundary conditions:

$$\left.\begin{array}{l} u'=0,\quad T'=T_0'\quad \text{at}\quad y'=0,\\ u'=0,\quad T'=T_1'\quad \text{at}\quad y'=h\ , \end{array}\right\} \tag{3}$$

where u' is the axial velocity, T' - temperature, g - acceleration due to gravity, q - radiative heat flux in the y'-direction, β - coefficient of the thermal expansion, ν -kinematic viscosity, σ - electric conductivity, ρ - fluid density, C_p - specific heat capacity at constant pressure, k - thermal conductivity , K^* - permeability of porous medium and Q_0 - heat generation/ absorption constant.

It has been derived by cogley et at. [1] that in the optically thin fluid with relatively low density:

$$\frac{\partial q}{\partial y'}=4\alpha^2(T'-T_0')\ . \tag{4}$$

On introducing the following non-dimensional quantities:

$$\left.\begin{array}{l} y'=hy,\ u'=U_0u,\ \theta=\dfrac{T'-T_0'}{T_1'-T_0'},\ Gr=\dfrac{g\beta h^2(T'-T_0')}{\nu U_0},\\ K^2=\dfrac{h^2}{K^*},M^2=\dfrac{\sigma B_0^2h^2}{\rho\nu},R^2=\dfrac{4\alpha^2h^2}{k},F=\dfrac{Q_0h^2}{k}, \end{array}\right\} \tag{5}$$

where α is the mean radiation absorption coefficient, u - dimensionless velocity, y - dimensionless coordinate axis normal to the plates, θ - dimensionless temperature, Gr - thermal Grashof number, K - porosity parameter, M - magnetic parameter, R - radiation parameter and F - heat generation/ absorption parameter. Using Eqs. (4) and (5), the model is transformed in to the following non-dimensional form of equations:

$$\frac{\partial^2 u}{\partial y^2}+Gr\theta-(K^2+M^2)u=0\ , \tag{6}$$

$$\frac{\partial^2\theta}{\partial y^2}-(R^2+F)\theta=0\ , \tag{7}$$

with boundary condition:

$$\left.\begin{array}{l} u=0, \quad \theta=0 \quad at \quad y=0\ , \\ u=0 \quad \theta=1 \quad at \quad y=1\ . \end{array}\right\} \tag{8}$$

Eqs. (6) and (7) are coupled nonlinear partial differential equations. The solusions in Closed – form are:

$$u(y)=\frac{Gr}{(\lambda^2-H^2)}\left[\frac{Sinh[Hy]}{Sinh[H]}-\frac{Sinh[\lambda y]}{Sinh[\lambda]}\right], \tag{9}$$

$$\theta(y)=\frac{Sinh[Hy]}{Sinh[H]}\ , \tag{10}$$

here $\lambda^2 = K^2 + M^2$ and $H^2 = R^2 + F$.

3- SKIN-FRICTION:

Using Equation (9), the skin-friction or the shear stress at the plate ($y=1$) in non-dimensional form, is given by:

$$\tau=-\left(\frac{\partial u}{\partial y}\right)_{y=1}=-\ \frac{Gr}{(\lambda^2-H^2)}\left[\frac{HCosh[H]}{Sinh[H]}-\frac{\lambda Cosh[\lambda]}{Sinh[\lambda]}\right] \tag{11}$$

4- NUSSELT NUMBER:

Using Equation (10), the rate of heat transfer at the plate ($y=1$) in non-dimensional form, is given by:

$$Nu=-\left(\frac{\partial \theta}{\partial y}\right)_{y=1}=-\ \frac{HCosh[H]}{Sinh[H]} \tag{12}$$

5- RESULTS AND DISCUSSION:

In order to get the physical insight in to problem, the numerical values of the velocity, temperature, skin-friction and Nusselt number are computed for different parameters like thermal Grashof number Gr , porosity parameter K , magnetic parameter M , radiation parameter R and heat generation/ absorption parameter F . The values of main parameters considered are: thermal Grashof number Gr = 2.0, 5.0, 7.0, 10.0; porosity arameter K = 0.3, 0.5, 0.6, 1.0; magnetic arameter M = 1.0, 2.0, 3.0, 4.0; radiation parameter R = 0.5, 1.0, 2.0, 5.0; heat generation/ absorption parameter F = -2.0, -1.0, 0.0, 1.0, 2.0. Figures-1 and 2 present the temperature profiles for different values of heat generation/ absorption parameter and radiation parameter respectively. It is clear from these figures that temperature increases with minimum at the plate ($y=0$) and maximum at the plate ($y=1$). Figure-1 shows that temperature increases for heat generation ($F<0$), and decreases for heat absorption ($F>0$). Figure-2 shows that temperature decreases as the value of radiation parameter R increases. Figures-3, 4, 5, 6 and 7 represent the velocity profiles for different values heat generation/ absorption parameter, radiation parameter, porosity parameter, magnetic parameter and thermal Grashof number respectively. From these figures it is clear that velocity increases for heat generation ($F<0$), and decreases for heat absorption ($F>0$). It is also observed that the velocity decreases with increase of radiation parameter, porosity parameter and magnetic parameter. Figure- 7 shows that velocity increases as the value of thermal Grashof number increases.

The values of Nusselt number and skin-friction coefficient at the plate ($y=1$) are shown in tables - 1 and 2, respectively. Table- 1 shows that the magnitude of the Nusselt number decreases for heat generation ($F<0$) but increases for heat absorption ($F>0$). Further, it increases with increase of radiation parameter. It is clear from table- 2 that coefficient of skin-friction decreases with the increase of porosity parameter, magnetic parameter, radiation parameter, and increases with the increase of thermal Grashof number. Further, the coefficient of skin-friction increases for heat generation ($F < 0$) but decreases for heat absorption ($F > 0$).

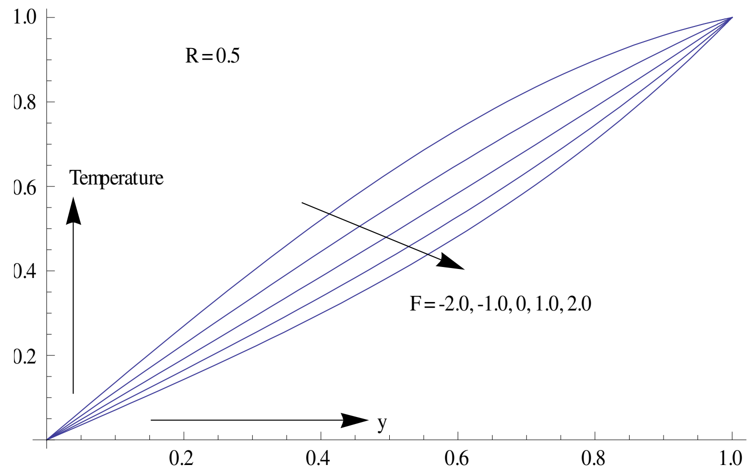

Figure-1: Temperature profiles

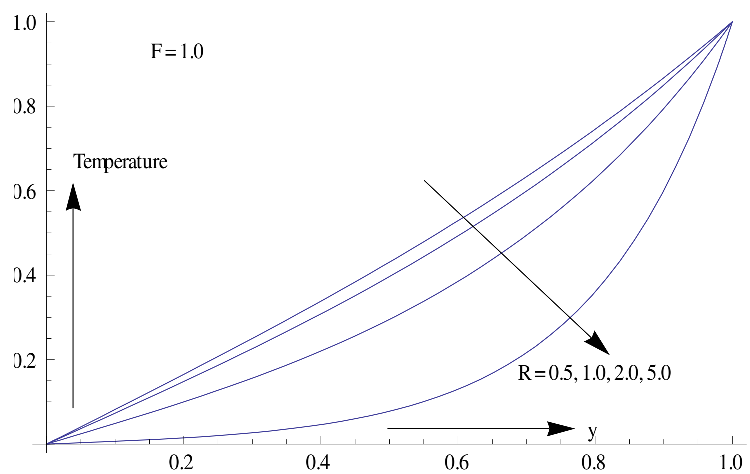

Figure-2: Temperature profiles

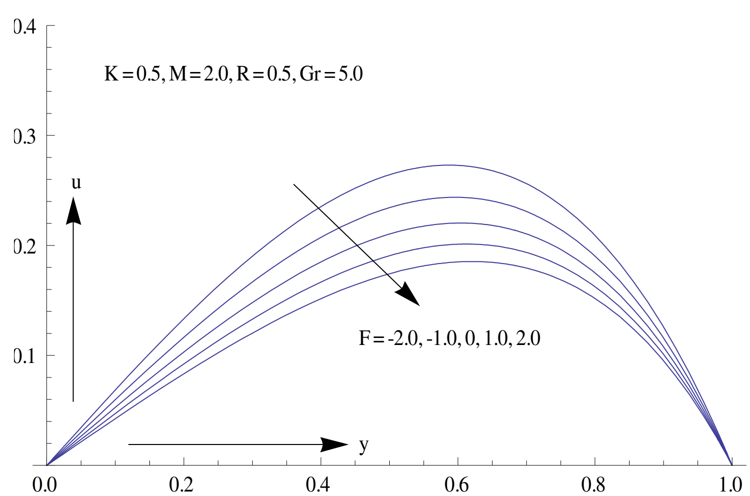

Figure-3: Velocity profiles

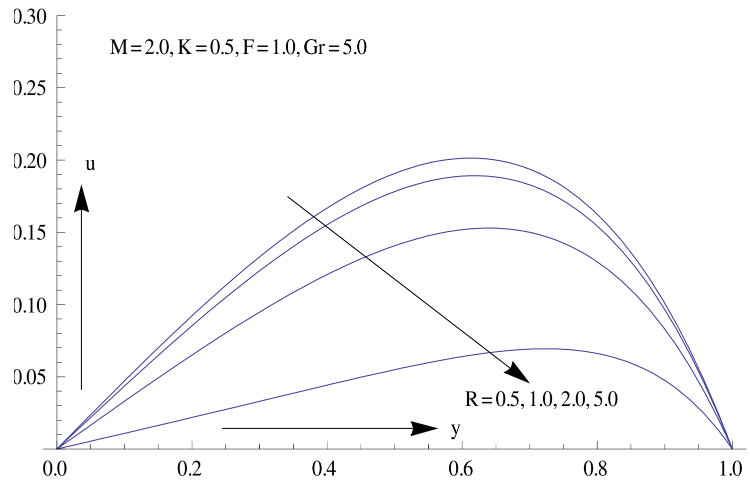

Figure-4: Velocity profiles

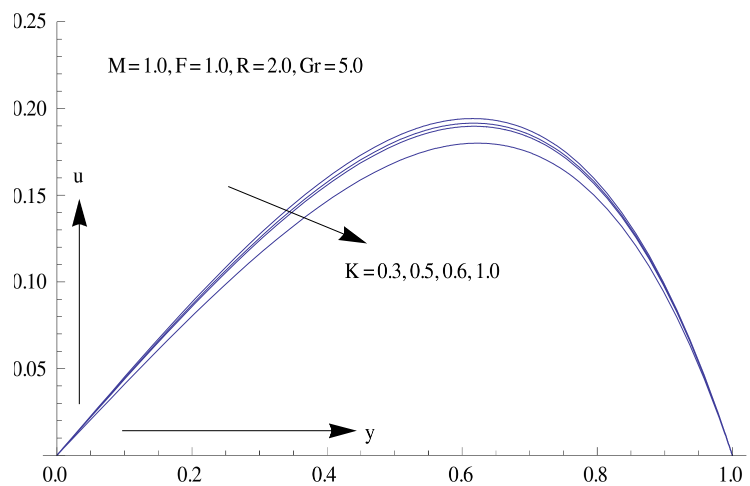

Figure-5: Velocity profiles

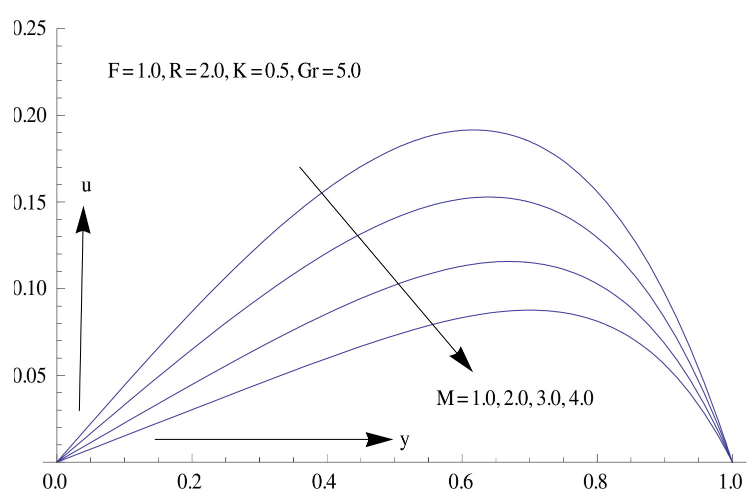

Figure-6: Velocity profiles

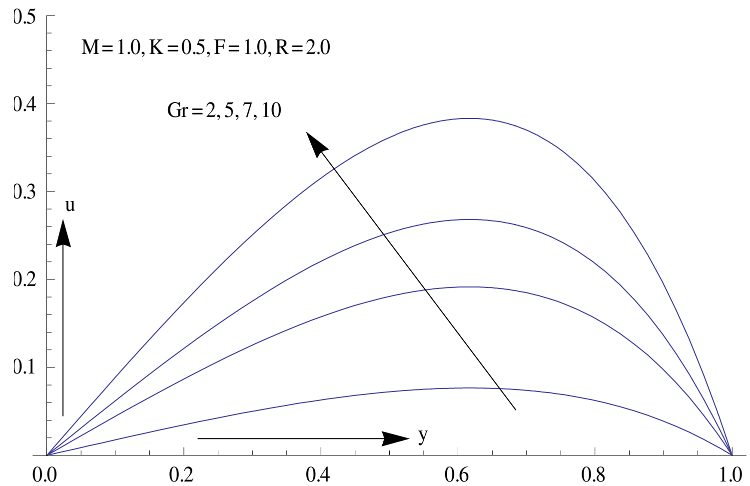

Figure-7: Velocity profiles

Table-1: Nusselt Number

F	R	$\lvert Nu \rvert$
2.0	0.5	1.65719
1.0	0.5	1.38562
0.0	0.5	1.08198
-1.0	0.5	0.73653
-2.0	0.5	0.33485
1.0	1.0	1.59189
1.0	2.0	2.28774

Table-2: Skin-friction

M	K	F	R	Gr	τ
2.0	0.5	2.0	0.5	5.0	1.18059
2.0	0.5	1.0	0.5	5.0	1.23967
2.0	0.5	0.0	0.5	5.0	1.30931
2.0	0.5	-1.0	0.5	5.0	1.39289
2.0	0.5	-2.0	0.5	5.0	1.49547
2.0	0.5	1.0	1.0	5.0	1.19451
2.0	0.5	1.0	2.0	5.0	1.05547
2.0	0.5	1.0	5.0	5.0	0.68275
1.0	0.5	1.0	2.0	5.0	1.20283
1.0	0.6	1.0	2.0	5.0	1.19619
1.0	1.0	1.0	2.0	5.0	1.15975
1.0	0.5	1.0	2.0	5.0	1.20283
2.0	0.5	1.0	2.0	5.0	1.05547
3.0	0.5	1.0	2.0	5.0	0.90300
1.0	0.5	1.0	2.0	2.0	0.48113
1.0	0.5	1.0	2.0	7.0	1.68396
1.0	0.5	1.0	2.0	10.0	2.40566

6- CONCLUSION:

Closed form and numerical solutions have been presented for steady hydromagnetic flow of an optically thin viscous fluid through a vertical channel in a porous medium in the presence of heat generation/absorption and radiation. Some conclusions of the study are as below:

- Temperature increases for heat generation ($F < 0$), and decreases for heat absorption ($F > 0$), i. e. temperature decreases as F increases.
- Temperature decreases as R increases.
- Velocity decreases as R, K and M increase, and increases with the increase of Gr.
- Velocity increases for heat generation ($F < 0$), and decreases for heat absorption ($F > 0$), i. e. Velocity decreases as F increases.
- Coefficient of skin-friction decreases with the increase of R, K and M, and increases with the increase of Gr.
- Coefficient of skin-friction increases for heat generation ($F < 0$), and decreases for heat absorption ($F > 0$).
- The magnitude of the rate of heat transfer (Nusselt number) increases with the increase of F and R.

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