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TRANSLATIONS OF BIPOLAR VALUED MULTI FUZZY SUBSEMIGROUPS OF A SEMIGROUP

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ABSTRACT

In this paper, some translation theorems of bipolar valued multi fuzzy subsemigroups of a semigroup is studied and prove some results on these.

Key Words: *Bipolar valued fuzzy subset, bipolar valued multi fuzzy subset, bipolar valued multi fuzzy subsemigroup and translations.*

INTRODUCTION

In 1965, Zadeh [10] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, soft sets etc. Lee [6] introduced the notion of bipolar-valued fuzzy sets. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar-valued fuzzy subset, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [−1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [7]. Multi fuzzy sets was introduced by Sabu Sebastian, T.V.Ramakrishnan [8]. Bipolar valued fuzzy subgroups of a group, homomorphism in bipolar valued fuzzy subgroups of a group and bipolar valued fuzzy normal subgroups of a group was introduced by M.S.Anitha *et al.*[1, 2, 3]. Bipolar valued multi fuzzy subgroups of a group have defined and introduced by Santhi.V.K and G.Shyamala[9]. The papers were useful for developing the research paper. Indira.R and K.Arjunan [5] defined about using function in bipolar valued multi fuzzy subsemigroups of a semigroup. In this paper, some translation theorems are stated and proved. These theorems will be useful to further research.

1. PRELIMINARIES

Definition 1.1[6]: A bipolar valued fuzzy set (BVFS) A in X is defined as an object of the form A = {< x, $A^+(x)$, $A^-(x)$ >/ $x\in X$}, where $A^+ : X\to [0, 1]$ and $A^- : X\to [-1, 0]$. The positive membership degree $A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar-valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar-valued fuzzy set A.

Example 1.2: A = {< a, 0.8, –0.4 >, < b, 0.4, –0.9 >, < c, 0.2, –0.7 >} is a bipolar valued fuzzy subset of X= {a, b, c}.

Definition 1.3[9]: A bipolar valued multi fuzzy set (BVMFS) A in X is defined as an object of the form A = {< x, $A_1^+(x)$, $A_2^+(x)$, …, $A_n^+(x)$, $A_1^-(x)$, $A_2^-(x)$, …, $A_n^-(x)$ >/ $x\in X$}, where $A_i^+ : X\to[0, 1]$ and $A_i^- : X\to[-1, 0]$ for all i = 1, 2, …, n. The positive membership degrees $A_i^+(x)$ denote the satisfaction degree of an element x to the property corresponding to a bipolar valued multi fuzzy set A and the negative membership degrees $A_i^-(x)$ denote the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued multi fuzzy set A. It is denoted as A = $\langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$.

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Example 1.4: A = {< a, 0.4, 0, 6, 0.4, –0.3, –0.5, –0.5 >, < b, 0.2, 0.6, 0.6, –0.7, –0.4, –0.8 >, < c, 0.4, 0.3, 0.7, –0.4, –0.6, –0.5 >} is a bipolar valued multi fuzzy subset of X = {a, b, c}.

Definition 1.5[5]: Let S be a semigroup. A bipolar valued multi fuzzy subset $A = \langle A_1^+, A_2^+, \ldots, A_i^+, A_1^-, A_2^-, \ldots, A_i^- \rangle$ of S is said to be a bipolar valued multi fuzzy subsemigroup of S (BVMFSSG) if the following conditions are satisfied

(i) $A_i^+(xy) \geq \min\{ A_i^+(x), A_i^+(y) \}$

(ii) $A_i^-(xy) \leq \max\{ A_i^-(x), A_i^-(y) \}$ for all x, y in S and for all i.

Example 1.6: Let S = { 1, –1, i, –i } be a semigroup with respect to the ordinary multiplication. Then A = {< 1, 0.7, 0.8, 0.6, –0.8, –0.7, –0.5 >, < –1, 0.6, 0.7, 0.5, –0.7, –0.6, –0.4 >, < i, 0.4, 0.5, 0.4, –0.6, –0.5, –0.3 >, < –i, 0.4, 0.5, 0.4, –0.6, –0.5, –0.3 > } is a bipolar valued multi fuzzy subsemigroup of S.

Definition 1.7: Let $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ be a bipolar valued multi fuzzy subset of X. Then the following translations are defined as

(i) $?(A) = \langle ?A_1^+, ?A_2^+, \ldots, ?A_n^+, ?A_1^-, ?A_2^-, \ldots, ?A_n^- \rangle$,
where $?A_i^+(x) = \min\{½, A_i^+(x)\}$ and $?A_i^-(x) = \max\{ -½, A_i^-(x) \}$, for all x in X and for all i.

(ii) $!(A) = \langle !A_1^+, !A_2^+, \ldots, !A_n^+, !A_1^-, !A_2^-, \ldots, !A_n^- \rangle$,
where $!A_i^+(x) = \max\{ ½, A_i^+(x) \}$ and $!A_i^-(x) = \min\{ -½, A_i^-(x) \}$, for all x in X.

(iii) (iii) $Q_{\alpha,\beta}(A) = \langle Q_{\alpha,\beta}(A_1)^+, Q_{\alpha,\beta}(A_2)^+, \ldots, Q_{\alpha,\beta}(A_n)^+, Q_{\alpha,\beta}(A_1)^-, Q_{\alpha,\beta}(A_2)^-, \ldots, Q_{\alpha,\beta}(A_n)^- \rangle$,
where $Q_{\alpha,\beta}(A_i)^+(x) = \min\{ \alpha_i, A_i^+(x) \}$ and $Q_{\alpha,\beta}(A_i)^-(x) = \max\{ \beta_i, A_i^-(x) \}$, for all x in X and α_i in [0, 1] and β_i in [–1, 0] and for all i.

(iv) $P_{\alpha,\beta}(A) = \langle P_{\alpha,\beta}(A_1)^+, P_{\alpha,\beta}(A_2)^+, \ldots, P_{\alpha,\beta}(A_n)^+, P_{\alpha,\beta}(A_1)^-, P_{\alpha,\beta}(A_2)^-, \ldots, P_{\alpha,\beta}(A_n)^- \rangle$,
where $P_{\alpha,\beta}(A_i)^+(x) = \max\{ \alpha_i, A_i^+(x) \}$ and $P_{\alpha,\beta}(A_i)^-(x) = \min\{ \beta_i, A_i^-(x) \}$, for all x in X and α_i in [0, 1] and β_i in [–1, 0] and for all i.

(v) $G_{\alpha,\beta}(A) = \langle G_{\alpha,\beta}(A_1)^+, G_{\alpha,\beta}(A_2)^+, \ldots, G_{\alpha,\beta}(A_n)^+, G_{\alpha,\beta}(A_1)^-, G_{\alpha,\beta}(A_2)^-, \ldots, G_{\alpha,\beta}(A_n)^- \rangle$,
where $G_{\alpha,\beta}(A_i)^+(x) = \alpha_i A_i^+(x)$ and $G_{\alpha,\beta}(A_i)^-(x) = -\beta_i A_i^-(x)$, for all x in X and α_i in [0, 1] and β_i in [–1, 0] and for all i.

Theorem 1.8: If $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ and $B = \langle B_1^+, B_2^+, \ldots, B_n^+, B_1^-, B_2^-, \ldots, B_n^- \rangle$ are two bipolar valued multi fuzzy subsemigroups of a semigroup S, then their intersection $A \cap B$ is a bipolar valued multi fuzzy subsemigroup of S.

2. SOME THEOREMS

Theorem 2.1: If $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ is a bipolar valued multi fuzzy subsemigroup of a semigroup S, then $?(A)$ is a bipolar valued multi fuzzy subsemigroup of S.

Proof: For every x, y in S, for all i, $?A_i^+(xy) = \min\{ ½, A_i^+(xy) \} \geq \min\{ ½, \min\{A_i^+(x), A_i^+(y)\}\} = \min\{ \min\{ ½, A_i^+(x) \}, \min\{ ½, A_i^+(y) \}\} = \min\{ ?A_i^+(x), ?A_i^+(y) \}$. Therefore $?A_i^+(xy) \geq \min\{ ?A_i^+(x), ?A_i^+(y) \}$, for all x and y in S. For every x, y in R, for all i, $?A_i^-(xy) = \max\{ -½, A_i^-(xy) \} \leq \max\{ -½, \max\{ A_i^-(x), A_i^-(y) \}\} = \max\{ \max\{ -½, A_i^-(x) \}, \max\{ -½, A_i^-(y) \}\} = \max\{ ?A_i^-(x), ?A_i^-(y) \}$. Therefore $?A_i^-(xy) \leq \max\{?A_i^-(x), ?A_i^-(y) \}$, for all x and y in S. Hence ?A is a bipolar valued multi fuzzy subsemigroup of S.

Theorem 2.2: If $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ is a bipolar valued multi fuzzy subsemigroup of a semigroup S, then $!(A)$ is a bipolar valued multi fuzzy subsemigroup of S.

Proof: For every x, y in S, for all i, $!A_i^+(xy) = \max\{ ½, A_i^+(xy) \} \geq \max\{ ½, \min\{ A_i^+(x), A_i^+(y) \}\} = \min\{ \max\{ ½, A_i^+(x) \}, \max\{ ½, A_i^+(y)\}\} = \min\{!A_i^+(x), !A_i^+(y) \}$. Therefore $!A_i^+(xy) \geq \min\{!A_i^+(x), !A_i^+(y) \}$, for all x and y in S. For every x, y in S, for all i, $!A_i^-(xy) = \min\{ -½, A_i^-(xy) \} \leq \min\{ -½, \max\{ A_i^-(x), A_i^-(y) \}\} = \max\{ \min\{ -½, A_i^-(x) \}, \min\{ -½, A_i^-(y) \}\} = \max\{ !A_i^-(x), !A_i^-(y) \}$. Therefore $!A_i^-(xy) \leq \max\{ !A_i^-(x), !A_i^-(y) \}$, for all x and y in S. Hence !A is a bipolar valued multi fuzzy subsemigroup of S.

Theorem 2.3: If $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ is a bipolar valued multi fuzzy subsemigroup of a semigroup S, then $Q_{\alpha,\beta}(A)$ is a bipolar valued multi fuzzy subsemigroup of S.

Proof: For every x, y in S, α_i in [0, 1] and for all i, $Q_{\alpha,\beta}(A_i)^+(xy) = \min\{ \alpha_i, A_i^+(xy) \} \geq \min\{ \alpha_i, \min\{ A_i^+(x), A_i^+(y) \}\} = \min\{ \min\{ \alpha_i, A_i^+(x) \}, \min\{ \alpha_i, A_i^+(y) \}\} = \min\{ Q_{\alpha,\beta}(A_i)^+(x), Q_{\alpha,\beta}(A_i)^+(y) \}$. Therefore $Q_{\alpha,\beta}(A_i)^+(xy) \geq \min\{ Q_{\alpha,\beta}(A_i)^+(x), Q_{\alpha,\beta}(A_i)^+(y) \}$, for all x, y in S. For every x, y in S, β_i in [–1, 0] and for all i, $Q_{\alpha,\beta}(A_i)^-(xy) = \max\{ \beta_i, A_i^-(xy) \} \leq \max\{ \beta_i, \max\{ A_i^-(x), A_i^-(y) \}\} = \max\{ \max\{ \beta_i, A_i^-(x) \}, \max\{ \beta_i, A_i^-(y) \}\} = \max\{ Q_{\alpha,\beta}(A_i)^-(x), Q_{\alpha,\beta}(A_i)^-(y) \}$. Therefore $Q_{\alpha,\beta}(A_i)^-(xy) \leq \max\{ Q_{\alpha,\beta}(A_i)^-(x), Q_{\alpha,\beta}(A_i)^-(y) \}$, for all x, y in S. Hence $Q_{\alpha,\beta}(A)$ is a bipolar valued multi fuzzy subsemigroup of S.

Theorem 2.4: If A = $\langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ is a bipolar valued multi fuzzy subsemigroup of a semigroup S, then $P_{\alpha,\beta}(A)$ is a bipolar valued multi fuzzy subsemigroup of S.

Proof: For every x, y in S, α_i in [0, 1] and for all i, $P_{\alpha,\beta}(A_i)^+(xy) = \max\{ \alpha_i, A_i^+(xy) \} \geq \max\{ \alpha_i, \min\{ A_i^+(x), A_i^+(y) \}\} = \min\{ \max\{ \alpha_i, A_i^+(x) \}, \max\{ \alpha_i, A_i^+(y) \}\} = \min\{ P_{\alpha,\beta}(A_i)^+(x), P_{\alpha,\beta}(A_i)^+(y) \}$. Therefore $P_{\alpha,\beta}(A_i)^+(xy) \geq \min\{ P_{\alpha,\beta}(A_i)^+(x), P_{\alpha,\beta}(A_i)^+(y) \}$, for all x, y in S. For every x, y in S, β_i in [–1, 0] and for all i, $P_{\alpha,\beta}(A_i)^-(xy) = \min\{ \beta_i, A_i^-(xy)\} \leq \min\{\beta_i, \max\{A_i^-(x), A_i^-(y)\}\} = \max\{ \min\{ \beta_i, A_i^-(x) \}, \min\{ \beta_i, A_i^-(y) \}\} = \max\{ P_{\alpha,\beta}(A_i)^-(x), P_{\alpha,\beta}(A_i)^-(y)\}$. Therefore $P_{\alpha,\beta}(A_i)^-(xy) \leq \max\{ P_{\alpha,\beta}(A_i)^-(x), P_{\alpha,\beta}(A_i)^-(y)\}$, for all x, y in S. Hence $P_{\alpha,\beta}(A)$ is a bipolar valued multi fuzzy subsemigroup of S.

Theorem 2.5: If A = $\langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ is a bipolar valued multi fuzzy subsemigroup of a semigroup S, then $G_{\alpha,\beta}(A)$ is a bipolar valued multi fuzzy subsemigroup of S.

Proof: For every x, y in S, α_i in [0, 1] and for all i, $G_{\alpha,\beta}(A_i)^+(xy) = \alpha_i A_i^+(xy) \geq \alpha_i (\min \{A_i^+(x), A_i^+(y) \}) = \min\{ \alpha_i A_i^+(x), \alpha_i A_i^+(y) \} = \min\{ G_{\alpha,\beta}(A_i)^+(x), G_{\alpha,\beta}(A_i)^+(y) \}$. Therefore $G_{\alpha,\beta}(A_i)^+(xy) \geq \min \{ G_{\alpha,\beta}(A_i)^+(x), G_{\alpha,\beta}(A_i)^+(y) \}$, for all x, y in S. For every x, y in S, β_i in [–1, 0] and for all i, $G_{\alpha,\beta}(A_i)^-(xy) = -\beta_i A_i^-(xy) \leq -\beta_i (\max\{ A_i^-(x), A_i^-(y) \}) = \max\{ -\beta_i A_i^-(x), -\beta_i A_i^-(y) \} = \max\{ G_{\alpha,\beta}(A_i)^-(x), G_{\alpha,\beta}(A_i)^-(y) \}$. Therefore $G_{\alpha,\beta}(A_i)^-(xy) \leq \max\{ G_{\alpha,\beta}(A_i)^-(x), G_{\alpha,\beta}(A_i)^-(y) \}$, for all x, y in S. Hence $G_{\alpha,\beta}(A)$ is a bipolar valued multi fuzzy subsemigroup of S.

Theorem 2.6. If $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ and $B = \langle B_1^+, B_2^+, \ldots, B_n^+, B_1^-, B_2^-, \ldots, B_n^- \rangle$ are bipolar valued multi fuzzy subsemigroups of a semigroup S, then $!(A \cap B) = !(A) \cap !(B)$ is also a bipolar valued multi fuzzy subsemigroup of S.

Proof: The proof follows from the Theorems 1.8 and 2.2.

Theorem 2.7: If $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ and $B = \langle B_1^+, B_2^+, \ldots, B_n^+, B_1^-, B_2^-, \ldots, B_n^- \rangle$ are bipolar valued multi fuzzy subsemigroups of a semigroup S, then $?(A \cap B) = ?(A) \cap ?(B)$ is also a bipolar valued multi fuzzy subsemigroup of S.

Proof: The proof follows from the Theorems 1.8 and 2.1.

Theorem 2.8: If A= $\langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ is a bipolar valued multi fuzzy subsemigroup of a semigroup S, then $!(?(A)) = ?(!(A)) = \langle ½, ½, \ldots, ½, -½, -½, \ldots, -½ \rangle$ is also a bipolar valued multi fuzzy subsemigroup of S.

Proof: For every x in S and for all i, $?A_i^+(x) = \min\{ ½, A_i^+(x) \} \leq ½$ and $!A_i^+(x) = \max \{ ½, A_i^+(x) \} \geq ½$, so $!(?(A_i^+)) = ?(!(A_i^+)) = ½$. And $?A_i^-(x) = \max\{ -½, A_i^-(x) \} \geq -½$ and $!A_i^-(x) = \min\{ -½, A_i^-(x) \} \leq -½$, so $!(?(A_i^-)) = ?(!(A_i^-)) = -½$. Hence $!(?(A)) = ?(!(A)) = \langle ½, ½, \ldots, ½, -½, -½, \ldots, -½ \rangle$ is a bipolar valued multi fuzzy subsemigroup of S.

Theorem 2.9: If $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ and $B = \langle B_1^+, B_2^+, \ldots, B_n^+, B_1^-, B_2^-, \ldots, B_n^- \rangle$ are bipolar valued multi fuzzy subsemigroups of a semigroup S, then $P_{\alpha,\beta}(A \cap B) = P_{\alpha,\beta}(A) \cap P_{\alpha,\beta}(B)$ is also a bipolar valued multi fuzzy subsemigroup of S.

Proof: The proof follows from the Theorems 1.8 and 2.4.

Theorem 2.10: If $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ and $B = \langle B_1^+, B_2^+, \ldots, B_n^+, B_1^-, B_2^-, \ldots, B_n^- \rangle$ are bipolar valued multi fuzzy subsemigroups of a semigroup S, then $Q_{\alpha,\beta}(A \cap B) = Q_{\alpha,\beta}(A) \cap Q_{\alpha,\beta}(B)$ is also a bipolar valued multi fuzzy subsemigroup of S.

Proof.: The proof follows from the Theorems 1.8 and 2.3.

Theorem 2.11: If A= $\langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ is a bipolar valued multi fuzzy subsemigroup of a semigroup S, then $P_{\alpha,\beta}(Q_{\alpha,\beta}(A)) = Q_{\alpha,\beta}(P_{\alpha,\beta}(A)) = \langle \alpha_1, \alpha_2, \ldots, \alpha_n, \beta_1, \beta_2, \ldots, \beta_n \rangle$ is also a bipolar valued multi fuzzy subsemigroup of S.

Proof: The proof follows from the Theorems 2.3 and 2.4.

Theorem 2.12: If A = $\langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ and B = $\langle B_1^+, B_2^+, \ldots, B_n^+, B_1^-, B_2^-, \ldots, B_n^- \rangle$ are bipolar valued multi fuzzy subsemigroups of a semigroup S, then $G_{\alpha,\beta}(A \cap B) = G_{\alpha,\beta}(A) \cap G_{\alpha,\beta}(B)$ is also a bipolar valued multi fuzzy subsemigroup of S.

Proof: The proof follows from the Theorems 1.8 and 2.5.

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