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MULTI-OBJECTIVE INTEGER LINEAR PROGRAMMING PROBLEMS: A REVIEW

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ABSTRACT

The objective of this paper to be existing a review of methods that consume remained advanced to explain multi-objective Integer Linear Programming Problems. We effort only on non-interactive particular methods that produce the entire traditional of optimum explanations. The elementary thoughts of these multi-objective problems are introduced along with their solutions. We formerly assessment approximately of the obtainable exact methods.

Keywords: *Multi-Objective Integer Linear Programming, Linear Programming, Integer Linear Programming, Decision making, Non interactive particular method.*

I. INTRODUCTION

Multi-Objective Programming is used in application for several actual problems counting problems in the grounds of manufacturing, mining and economics. In Multi-Objective Programming here are several inconsistent purposes where by refining one neutral will decrease the worth of others, important to a trade-off between solutions. It is assumed that no single explanation will optimise all objectives simultaneously because this would be a trivial case. The main aim of multi-objective programming is to contribution a Decision Making (DM) to indicate a chosen explanation between all the trade-offs. In this situation, it is not essential to produce completely explanations when the DM is difficult in the procedure later approximately explanations may be abolished at each stage. However, in this paper we will focus on non-interactive exact methods that do not involve the DM in direction to produce the complete explanation set.

Multi-Objective Problems take the form of Multi-Objective Linear Programming (MOLP), Multi-Objective Integer Linear Programming (MOIP), and Multi-Objective Mixed Integer (MOMIP) which consume continuous, inaccessible, and equally incessant and separate explanations distinctly. Outstanding to the countryside of MOMIP, here remain numerous changed kinds of difficulties. In this paper we determination effort mainly on bi-objective mixed integer programming problems and the regiment of only exciting supported non-dominated explanations for overall MOMIPs. There are several revisions that contract with MOIP and MOLPs separately, but there is a absence of literature for their combination, MOMIP. Outstanding to the nature of their explanations, MOIP and MOLP cannot be straight used to explain MOMIP. This paper attentions on accumulating and summarising articles published in the English linguistic for MOLP, MOIP and MOMIPs, with extra informs of current progresses later investigations corresponding Ehrgott and Gandibleux [26] and Ruzika and Wiecek [39]. It must be supposed that the algorithms harvest all non-dominated explanations in the objective intergalactic except stated otherwise. There are innumerable explanations as to why the more current algorithms explain in this objective intergalactic in its place of the decision space. The objective intergalactic is naturally much reduced than the conclusion intergalactic because in practically all case there are fewer objects than verdict variables. This simplifies the problem and it becomes less computationally demanding. Effective explanations in the conclusion usual remained also proved to commonly map onto the similar explanation of the outcome usual by Benson [10], important to terminated explanations in the decision usual. Algorithms that explain in the decision intergalactic corresponding Steuer [38], Armand and Malivert [36], Armand [37] and Sayin [40] are tough to smear essentially as computational weights growth significantly as problematic extent growths.

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II. PROPERTIES OF MULTI-OBJECTIVE INTEGER LINEAR PROGRAMMING PROBLEMS

A Multi-Objective Mixed Integer Programming Problematic is expressed as:

$$Max \quad c_x = \left(f_1(x)\, f_2(x) \ldots\ldots f_n(x) \right)$$

$$\text{Subject to } x \in X$$

Where X = $\{Ax \leq b,\ x \geq 0,\ x^c \in \mathbb{R}^n \text{ and } x^I \in \mathbb{Z}^n\}$ is the set of all optimal solutions. The solutions x^c and x^I denote the set of optimal solutions for multi-objective linear and integer problems respectively. A $\in \mathbb{R}^{mxn}$ is an mxn matrix of the m restrictions and n conclusion variables and b $\in \mathbb{R}^m$ is the corresponding rightward lateral. C $\in \mathbb{R}^n$ is a pxn matrix that represents the p $\geq$ 2 neutral occupations to be minimised. The conclusion usual of the explanations is distinct as:

$$Y = \{y \in \mathbb{R}^n : y = c_x,\ x \in X\}$$

Set X and Y are known as the decision space and objective space, respectively.

The following notation is used for $y^1, y^2 \in \mathbb{R}^n$:

$$y^1 \leq y^2 : \Leftrightarrow y^1 \leqq y^2 \text{ and } y^1 \neq y^2$$

$$y^1 \leq y^2 : \Leftrightarrow y_k^1 \leqq y_k^2,\ \forall\ k = 1,\ldots,n$$

The Pareto pinecone is well-defined as:

$$\mathbb{R}^n_{\geq} := \{y \in \mathbb{R}^n : y_k \geq 0,\ k = 1,\ldots,n\}$$

Consider arguments, x* ∈ X. A optimal explanation is called effective or pareto optimal explanation if there exist no x such that Cx ≦ Cx*. The outcome of x*, y* is then called non-dominated. If $y_k^* < Y_k$ for all k, then y_k^* strictly dominates Y_k. Otherwise, y_k^* is weakly non-dominated. A maintained non-dominated explanation is a Pareto explanation that is the optimum explanation to the slanted sum single-objective problematic:

$$\text{Max } (\lambda_1 f_1(x) + \ldots + \lambda n f_n(x))$$

If an existing efficient explanation cannot be found by solving the above problem, it is an unsupported non-dominated solution. Let Y_{ND} signify the usual of non-dominated arguments. For any $y \in Y_{ND}$, y^{conn} denotes a curving mixture of all the non-dominated arguments, without. Here can be three types of non-dominated arguments. They are:

I. Exciting braced if and only if there exists no $y^{conn} \leqq y$

II. Non- exciting supported non-dominated if there exists no $y^{conn} < y$ but there exists $y^{conn} = y$

III. Unsupported non-dominated if there exists $y^{conn} < y$

III. APPROACHES

3.1: Linear Programming:

In MOLP problems, all objectives are linear and must be optimized over a convex polyhedron. MOLP problems are solved as sub-problems for MOIP and MOMIP and all non-dominated solutions of MOLP are supported. MOLP problems are popular and there is a lot of literature that covers finding the efficient set, some of which are covered in this section. Benson [12] generates an external calculation algorithm in the conclusion intergalactic. The main lead of this algorithm is that there is no need for back-pedalling or bookkeeping which is required after explaining in the conclusion intergalactic, as in Benson [11]. This method is later implemented into Benson [13] which introduces a hybrid vector maximization approach which was first introduced by Kuhn and Tucker [17]. Benson includes the distinct simplicial apportioning procedure used by Tuy and Horst [18] into the outcome space using outer approximation. First, a argument $\mathcal{P}$ that deceits in the internal of the efficiency-equivalent polyhedron is designed along with its simplex, S^0 and apex set, V(S^0). At the $\mathcal{K}^{th}$ repetition, the algorithm studies the usual of all apexes, V(S^k) of the present dense polyhedron S^k that covers Y. The algorithm dismisses if respectively component of V(S^k) belongs to Y because S^k = Y. Otherwise, a new polyhedron, $S^{k+1} \subset S^k$ that contains Y, is created by adding a linear equality to S^k. The vertex set V(S^{k+1}) is then computed and the algorithm continues until is S^k = Y satisfied. Later in Benson [14] it remained initiate that this procedure also engendered softly effective arguments in the outcome intergalactic.

Benson and Sun [15] proved that an optimum origin for the linear program LP(w)

$$\text{Max } (w)^T c_x$$

$$\text{Subject to } x \in X,$$

The weight set, $\quad W^0 \equiv \{w \in \mathbb{R}^p | W_j > 0,\ j = 1, 2, \ldots p\}$

can be disintegrated into a limited merger of subsets with a one-to-one correspondence among the weights and well-organized exciting explanations in the conclusion intergalactic. Using this result, Benson and Sun [16] progress a weight set breakdown algorithm. At each step k, a weight w^k is selected and the LP is explained for an optimum exciting point explanation. Each following weight is found by preparing a global tree search method. If no more weights container be initiate then the algorithm axes.

Ehrgott, Puerto and Rodrigues-Chia [27] use the scalarization theorem and single-objective duality theory to progress a original algorithm. This algorithm efforts on applications in system optimization complications. Luc [25] announces two methods to duality, one founded on the duality affiliation between nominal and highest essentials of a set and its accompaniment, and another using polarization among convex polyhedral circles and the epigraph of its sustenance purpose. Developments to standing duality relatives are also discovered. Ehrgott, Lohne and Shao [28] use geometric duality theory to derive a dual variant of the algorithm of Benson [13]. This method concepts the dual stretched image in its place of the primitive image. Once the dual image is attained, geometric duality is used to obtain the primitive image.

Ida [29] uses an exciting ray generation method to consecutively generate competent arguments and rays. This is done by addition difference limitations to the polyhedral optimum region. In the algorithm, objective values for each exciting ray are obtained and tested for effectiveness. A new competent ray is generated if the pair of exciting rays have competent solutions when one of the competent explanations is eliminated in the row process step. All competent exciting rays and argument are obtained when all the rows have been checked.

3.2: Integer Linear Programming:

The foremost modification between MOLP and MOIP problems is that MOIP objectives are isolated, not unceasing. The introduction of integer variables allows for optimum explanations that no longer invention on a line part. This primes to the reality of non-supported competent explanations which are plentiful tougher to find. Some of the methods used in outcome these non-supported are explained here. In bi-criteria problems, it is well known that $2|N|-1$ sub-problems must be explained to generate all non-dominated explanations, where N is the non-dominated set. Primarily $|N|$ sub-problems are explained to generate all argument in N, and then $|N|-1$ more sub-problems are solved to make sure that there is no added non-dominated argument that exist between the ones that have previously been generated. Laumanns, Thiele and Zitzler [30] used an adaptive E-constraint method and showed that problems of higher extents require a bound of $O(|N|^{m-1})$, where m is the number of objectives. Dachert and Klamroth [23] developed an algorithm that needs to solve at most $3|N|-2$ sub-problems for tri-criteria problems.

Przybylski, Gandibleux and Ehrgott [1] comprehensive the two-phase method and smear it to the tri-objective assignment problem. Suitable lower and upper restrictions are calculated and used to update the preliminary search intergalactic from the first phase. Any upper restrictions that are dominated are removed and any non-supported non-dominated argument that is found is interleaved into the updated search intergalactic. The tactic of Dachert and Klamroth [23] use is similar to this, but filter out completed exploration extents.

Due to the issues the classical E-constraint method had with finding weakly competent explanation, Mavrotas [8] introduced an augmented E-constraint method which augments the objective purpose using the one-sided sum of extra slack or surplus variables. This method was enhanced by Mavrotas and Florios [9] precisely for MOIP problems and required scarcer subproblems to be explained and develop a distinction of the augmented E-constraint method called the simple augmented E-constraint method which progresses through using a quickening algorithm with an early exit and a quickening algorithm with lively steps. The algorithm of Ozlen and Azizoglu [31] recursively explains problems with minor objectives using the E-constraint method. The objective functions are minimalized and exploited to generate the varieties for the non-dominated set which are then used to explain for all non-dominated explanations. The algorithm was later enhanced by Ozlen, Burton and MacRae [32]. Kirlik and Sayin [5] find the non-dominated set using a search space of ($\mathcal{P}$-1) dimensions. This algorithm uses rectangles in the search space, with the preliminary rectangle covering the ($\mathcal{P}$-1) dimensional space. Each rectangle is well-defined using lower and upper constraints. These lower and upper constraints are found by minimizing and maximizing each objective function, respectively. The rectangles are separated into smaller disjoint squares and this is recurring pending there are no squares left to exploration.

Klein and Hannan [4] propose a successive generation method for verdict all non-dominated explanations in the conclusion intergalactic. This method explains a sequence of gradually more constrained single-objective integer problems. At each step a new constraint is added which eliminates before generated competent arguments. This allows argument which are conquered by the generated non-dominated explanations to be excluded. A distinction of this method is used by Sylva and Crema [21] which successively explains weighted sum problems in its place of single-objective difficulties and future, Sylva and Crema [22] propose another optional that finds a well-dispersed subsection of non-dominated arguments. A progress of the procedure by Sylva and Crema [21] is established by Lokman and Koksalan [24] which decreases the number of extra contractions to be additional at respectively stage. Lemesre, Dhaenens and Talbi [20] propose Parallel Partitioning Method (PPM) to solve bi-objective problems. This method uses three points to determine the entire Pareto anterior. Firstly, the problem is solved for exciting explanations to limit the search intergalactic. Next, the search intergalactic is divided up by searching the competent explanations. Lastly, the explanations found from the previous stage are used to find any other competent explanations.

3.3: Mixed Integer Programming:

MOMIP difficulties are the mixture of MOLP and MOIP difficulties. There are several types of difficulties inside MOMIP himself due to the mixture of incessant and numeral variables. So, for here is no standing procedure that can clarify for mixed 0-1 numeral programs with $\mathcal{P} > 3$ purposes and no overall procedure to invention all non-dominated descriptions. Around is a deficiency of fiction for MOMIP complications nonetheless approximately of the approaches that do occur are protected now. Mavrotas and Diaokoulaki [6] modify the single-objective branch and bound algorithm to find competent explanations in mixed 0-1 MOLP problems in the conclusion intergalactic. Initially all binary variables $x \in \mathbb{R}^n$ are considered free variables relaxed to $x \in [0,1]^n$ and at the following branch of the combinatorial tree an extra binary variable will become fixed pending finally all mixtures are found and the MOLP problem with the fixed binaries are explained. The equivalent non-dominated argument to these nodes is stored and efficient in D_{ex}. Dominated argument are removed from D_{ex} and non-dominated argumentare added. Later, Mavrotas and Diaokoulaki [7] further extend to find the efficient explanations of this problematic using a vector expansion tactic of the branch and bound method. This algorithm was found to be missing some efficient explanations by Vincent [42] and Vincent, Seipp, Ruzika, Przybylski and Gandibleux [2] who then exact the work of Mavrotas and Diakoulaki [7] in the bi-objective instance and explicate the issues of the algorithm. Jozefowiez, Laporte and Semet [34] propose a general multi-objective branch and bound method which does not iteratively solve single-objective problems. The lower and upper boundaries are defined as sets of argument in the objective seats in its place of existence single values. Stidsen, Andersen and Dammann [41] use branch and bound to find all non-dominated explanations for bi-objective mixed integer problem where all integers must be binary and only one of the points may be an incessant. This algorithm first explains the problem with all binary ideals as free variables. Branching is done on the relaxed binary variables and in each node, a six-tuple of values are protected. The algorithm will try to quantity an explanation from the six-tuple pending there are none left to quantity.

Przybylski, Gandibleux and Ehrgott [2] progress some additional possessions for the weight intergalactic for MOMIP and develop their algorithm based on this. The algorithm utilises the bi-objective algorithms of Cohon [19] multi-objective problems into bi-objective problems which can then explained by the bi-objective algorithms. Ozpeynirci and Koksalan [35] utilize some of the properties found in Przybylski et al. [2] to invention all exciting reinforced non-dominated descriptions trendy over-all MOMIPs of slightly objectives. This method introduces dummy argument into the weight intergalactic disintegration. An appropriately small is chosen to guarantee that one of the dummy arguments will minimise the consequential objective function if any weight is close to zero. Adjacent argument is used to determine boundaries of the weight space decomposition and at each iteration, new extreme supported non-dominated argument or convex combinations are initiate pending they have all been acknowledged.

IV. CONCLUSION

In this paper, a new particular method combining the well-known principle of branching in integer linear programming with new efficient censored is designated to generate all integer competent explanation of MOILPs. It can be considered as a general method dedicated to MOLPs with integer as well as zero-one decision variable can be explained by the method. This literature assessment serves as an overview of the research that has been done in explaining problems of MOLP, MOIP and MOMIP. This literature assessment is not a complete review as there is continuing research for each of these problems and time constraints did not allow for a full review for every problem.

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