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VOLUMETRIC EXPANSION OF RADIATING VISCOUS DARK ENERGY MODEL WITH TIME DEPENDENT GRAVITATIONAL AND COSMOLOGICAL CONSTANTS

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ABSTRACT

In the present investigation the study of homogeneous anisotropic Bianchi type-I cosmological model with time-dependent gravitational and cosmological constants has been presented in the framework of Lyra geometry. Exact solutions of the Einstein field equations are presented towards the law of variation of Hubble parameter that yields a constant value of deceleration parameter. It has been found that the general solution of the average scale factor is a function of time involved the power law function. The analysis of the derived model shows that, for the cosmological constant and gravitational constant are opposite in nature to each other i.e. Cosmological constant decreases whereas the Newtonian gravitational constant increases with time whereas the model become isotropic at late time. Some dynamical and kinematical quantities of the model have been calculated analytically.

Keywords: *Bianchi type-I model; gravitational and cosmological constants; Lyra geometry.*

1. INTRODUCTION

Type-Ia supernovae (SN-Ia) observations reported by Riess *et al.* [1] and Perlmutter *et al.* [2] discovered an accelerated expansion of the universe is due to dark energy (little is known about the nature of dark energy). Another observation like Wilkinson Microwave Anisotropy Probe (WMAP) [3, 4], together with more accurate SN-Ia data [5] point toward that the Universe is almost spatially flat and the dark energy accounting to approximately to 70% of the total content of the Universe also favors a small and positive value of the cosmological constant at the present epoch.

In cosmology, the cosmological constant is look as a matter field with negative pressure (or as a vacuum energy density) that determined an accelerated expansion of the universe. The calculations of quantum field theory predict the value of vacuum to be of the order of 10^{130} s^{-2}, whereas the cosmological constant value is of the order of 10^{-20} s^{-2}; hence, a new thought is prerequisite to explain or determine this problem. The idea of considering the cosmological models as varying vacuum energy density has extensively improved by several authors such as Chen and Wu [6] considered the cosmological constant with proportional relation to scale factor and Hubble's parameter, which depends on adjustable parameters of the quantum field on a curved and expanding background where as Carvalho *et al.* [7] investigated the cosmological consequences of a time-dependent cosmological constant term by introducing the Hubble's parameter and the scale factor of the universe along with some authors Vishwakarma [8, 9]; Berman [10, 11]; Pradhan and Amirhashchi [12]; Chawla *et al.* [13]; Pradhan *et al.* [14, 15]; Mishra *et al.* [16] who have investigated cosmological model in different context of use.As universe is evolving with time due to the coupling which is the Newtonian / Gravitational constant *G* between the geometry of space-time and energy in the general theory of relativity was first assume by Dirac [17]. It has been a subject of considerable interest of cosmologist to study alternative theories of gravitation. The most important among them proposed by Lyra [18] who has suggested a modification of Riemannian geometry, which may also be considered as a modification of Weyl's geometry. Several authors have studied cosmology in Lyra geometry with a constant as well as time-dependent displacement field, which plays the same role as the cosmological constant in the standard general relativity. Halford [19] studied Robertson-Walker

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models in Lyra geometry for a time-dependent gauge function. Singh and Ram [20] discussed the spatially homogeneous Bianchi type-I metric in a different basic form and obtained exact solutions of Einstein field equations in vacuum and in the presence of stiff-matter in the normal gauge when the displacement field is time-dependent. Singh and Singh [21] obtained exact solutions for the spatially homogeneous Bianchi type-I model in the normal gauge for Lyra's geometry. Singh and Singh [22] also discussed FRW models in the Lyra manifold with constant deceleration parameter and are relevant to the study of inflationary cosmology. Chirde and Shekh [23] studied anisotropic and homogenous Bianchi Type VI_0 space-time under the assumption of anisotropy of the fluid within the frame work of Lyra manifold in the presence and absence of magnetism using a special form of deceleration parameter which gives an early deceleration and late time accelerating cosmological models.

2. METRIC AND FIELD EQUATIONS

The astronomical observations have revealed that our present universe is homogeneous and isotropic on sufficiently large scales and it is usually described by FLRW universe. But it is believed that, the FLRW universe does not give a correct matter description of the early universe. Moreover, the theoretical argument and the modern experimental data from cosmic microwave background (CMB) and the large structure observations, support the existence of an anisotropic phase which turns into an isotropic one. Therefore, universe having anisotropic background that approach to isotropy at late times are more appropriate for the description of entire evolution of the universe. Hence from the overhead domino effect we consider spatially homogeneous and anisotropic Bianchi type-I space-time of the form

$$ds^2 = dt^2 - a_1^2 dx^2 - a_2^2 dy^2 - a_3^2 dz^2, \tag{1}$$

where the metric potentials a_1, a_2, a_3 are functions of 't'only.

Some kinematical parameters which are important in cosmological observations related to the Bianchi type-I space-time are,
The spatial volume and average scale factor are

$$V = R^3 = a_1 a_2 a_3. \tag{2}$$

The average Hubble parameter,

$$H = \frac{1}{3}\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right). \tag{3}$$

The expansion scalar,

$$\theta = \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}, \tag{4}$$

The anisotropy parameter of the expansion is expressed in terms of average and directional Hubble parameters as,

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i^2}{H}\right)^2, \tag{5}$$

Shear scalar,

$$\sigma^2 = \frac{3}{2}\Delta H^2. \tag{6}$$

The energy momentum tensor T_j^i for perfect fluid distribution is taken as

$$T_j^i = (p+\rho)u^i u_j - p g_j^i, \tag{7}$$

where u^i is the four-velocity vector together with commoving co-ordinates $u^i = (0,0,0,1)$, $u^i u_j = 1,\ v^i v_j = 0$ of the cosmic fluid, p and ρ are the isotropic pressure and energy density of the fluid respectively. The effect to which isotropic pressure p of the perfect fluid reduce by an amount $\xi\theta$ called bulk viscosity, so that the effective pressure of the viscous fluid turns out to be $\bar{p} = p - \xi u^i_{;i} = p - \xi\theta$.

Hence, the energy momentum tensor given in equation (7), with bulk viscous fluid distribution takes the form as

$$T_j^i = (\bar{p}+\rho)u^i u_j - \bar{p} g_j^i, \tag{8}$$

where ξ be the coefficient of bulk viscosity which is a function of time *t*.
Bulk viscosity is very important in cosmology, since it has a greater role in getting accelerated expansion of the universe popularly known as inflationary phase [24, 26].

In co-moving coordinate system, from equation (8), we get

$$T_1^1 = T_2^2 = T_3^3 = -\overline{p} \text{ and } T_4^4 = \rho . \tag{9}$$

The field equations in Lyra geometry for the combined scalar and tensor field are

$$G_{ij} + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_\alpha\phi^\phi = -(8\pi)GT_{ij} + \Lambda \tag{10}$$

where ϕ_i is the displacement field vector defined as $\varphi_i = (0,0,0,\lambda(t))$.

Also from the field equation (10),

$$\left(\frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_\alpha\phi^\phi\right)_{;i} = 0 ,$$

after solving the covariant derivative of above equation, which leads to

$$\frac{3}{2}\phi_i\left[\frac{\partial\phi^j}{\partial x^j} + \phi^l\Gamma_{lj}^j\right] + \frac{3}{2}\phi^j\left[\frac{\partial\phi_i}{\partial\phi^j} + \phi_l\Gamma_{ij}^l\right] - \frac{3}{4}g_i^j\phi_k\left[\frac{\partial\phi^k}{\partial x^j} + \phi^l\Gamma_{lj}^k\right] - \frac{3}{4}\phi^k\left[\frac{\partial\phi_k}{\partial\phi^j} + \phi_l\Gamma_{kj}^l\right] = 0 .$$

Above equation is identically satisfied for $i = 1,2,3$ and for $i = 4$ it reduces to

$$\frac{3}{2}\beta\left[\frac{\partial}{\partial x^4}\left(g^{44}\phi_4\right) + \phi^4\Gamma_{4j}^j\right] + \frac{3}{2}g^{44}\phi_4\left[\frac{\partial\phi_4}{\partial t} - \phi_4\Gamma_{44}^4\right] - \frac{3}{4}g_4^4\phi_4\left[\frac{\partial\phi_4}{\partial x^4} + \phi^4\Gamma_{44}^4\right] - \frac{3}{4}g_4^4 g_4^4\phi_4\left[\frac{\partial\phi_4}{\partial t} - \phi^4\Gamma_{44}^4\right] = 0,$$

which leads to

$$\lambda\dot{\lambda} + \lambda^2\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right) = 0 . \tag{11}$$

In co-moving coordinate system the Lyra geometry field equations (10) together with equation (11) for the metric (1) with the help of (9) are written in the form

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3} + \frac{3}{4}\lambda^2 = -(8\pi)G\overline{p} - \Lambda , \tag{12}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_3}{a_3} + \frac{3}{4}\lambda^2 = -(8\pi)G\overline{p} - \Lambda , \tag{13}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} + \frac{3}{4}\lambda^2 = -(8\pi)G\overline{p} - \Lambda , \tag{14}$$

$$\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} + \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_3}{a_3} - \frac{3}{4}\lambda^2 = (8\pi)G\rho - \Lambda , \tag{15}$$

$$\lambda\dot{\lambda} + \lambda^2\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right) = 0 . \tag{16}$$

Where the overhead dot (.) denotes differentiation with respect to time.

As the field equations (12)–(15) are a coupled system of highly non-linear differential equations with eight unknown parameters.

3. SOLUTION OF FIELD EQUATIONS

Using equations (12) and (13), we get

$$\frac{d}{dt}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right) = -\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right)\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right) . \tag{17}$$

Using equation (2), above equation (17), reduces to

$$\frac{d}{dt}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right) = -\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right)\frac{\dot{V}}{V} , \tag{18}$$

Integrating above equation, we yield

$$\frac{a_1}{b_1} = d_1 \exp\left\{A_1 \int \frac{1}{V} dt\right\}. \tag{19}$$

Similarly using equations (12)-(14), we get

$$\frac{a_2}{a_2} = d_2 \exp\left\{A_2 \int \frac{1}{V} dt\right\}, \tag{20}$$

$$\frac{a_3}{a_3} = d_3 \exp\left\{A_3 \int \frac{1}{V} dt\right\}, \tag{21}$$

where d_1, d_2, d_3 and A_1, A_2, A_3 are constants of integration.

In view of equation (2), we get

$$a_1 = \chi_1 V^{\frac{1}{3}} \exp\left\{A_1 \int \frac{1}{V} dt\right\}, \tag{22}$$

$$a_2 = \chi_2 V^{\frac{1}{3}} \exp\left\{A_2 \int \frac{1}{V} dt\right\}, \tag{23}$$

$$a_3 = \chi_3 V^{\frac{1}{3}} \exp\left\{A_3 \int \frac{1}{V} dt\right\}, \tag{24}$$

where A_i and χ_i, $i = 1, 2, 3$ satisfy the relation $\chi_1\chi_2\chi_3 = 1$ and $A_1 + A_2 + A_3 = 0$.

Now, the metric potentials can be determined as functions of cosmic time if the spatial volume or averagescale factor is known. Thus, the system is initially undetermined and we need additional constraints to close the system. Hence, we consider the following conditions ansatz for Cosmological constant and the scale factor,

i) Consider the case of the phenomenological decay of Λ,
$$\Lambda = H^2. \tag{25}$$
ii) As the model with constant deceleration parameter stand adequately for our present view of different phases of evolution of the Universe, for this viewpoint, we imposed the law of variation of Hubble parameter. The law of variation for Hubble parameter, which was initially proposed by Berman within the context of FLRW space-time in general relativity, yields a constant value of deceleration parameter [27] which is also approximately valid for slowly time varying deceleration parameter of the form

$$q = \frac{-R\ddot{R}}{\dot{R}^2} = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) = kt + n - 1, \tag{26}$$

where $k, n \geq 0$ are positive constants.

Solving above equation (26) for $k = 0$, one obtains two different forms of solutions for the scale factor as

$$V = (nlt + b)^{3/n}, \quad k = 0,\ n > 0, \tag{27}$$

here, l and b are the constants of integration.

Using equation (27), equations (22) to (24) takes the form

$$a_1 = \chi_1 (nlt + b)^{3/n} \exp\left\{\frac{A_1}{l(n-3)}(nlt + b)^{1-\frac{3}{n}}\right\}, \tag{28}$$

$$a_2 = \chi_2 (nlt + b)^{3/n} \exp\left\{\frac{A_2}{l(n-3)}(nlt + b)^{1-\frac{3}{n}}\right\}, \tag{39}$$

$$a_3 = \chi_3 (nlt + b)^{3/n} \exp\left\{\frac{A_3}{l(n-3)}(nlt + b)^{1-\frac{3}{n}}\right\}, \tag{30}$$

From the equations (28) to (30), it is observed that both the metric potentials are the product of exponential and power term. At an initial stage when $t \to 0$, both metric potentials are comes out to be constants and at large time i.e. $t \to \infty$, all are increase indefinitely with the passage of time, which is in complete agreement with the Big-Bang model of the Universe.

With the help of equations (28) to (30) spatially homogeneous and anisotropic Bianchi type-I space-time with bulk viscous fluid within the framework of Lyra geometry becomes

$$ds^2 = dt^2 - \chi_1^2 \left(nlt+b\right)^{6/n} \exp\left\{\frac{2A_1}{l(n-3)}\left(nlt+b\right)^{1-\frac{3}{n}}\right\}dx^2$$
$$-\chi_2^2 \left(nlt+b\right)^{6/n} \exp\left\{\frac{2A_2}{l(n-3)}\left(nlt+b\right)^{1-\frac{3}{n}}\right\}dy^2 - \chi_3^2 \left(nlt+b\right)^{6/n} \exp\left\{\frac{2A_3}{l(n-3)}\left(nlt+b\right)^{1-\frac{3}{n}}\right\}dz^2 \tag{31}$$

From the above model (31), it is observed that at $t = 0$, the model is constant but at a specific time $t = t_s = \frac{-b}{nl}$ the metric potential in the model vanishes hence the model represent singular model. Also, there is no such relation between the constants in the model for which the model shows isotropy.

Physical parameters

Cosmological constant of the model,

$$\Lambda = \frac{l}{3}\frac{1}{(nlt+b)}. \tag{32}$$

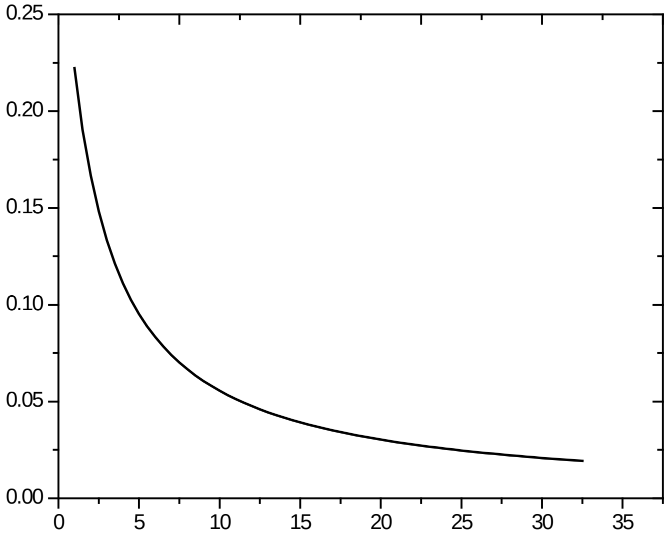

Figure-1: Behavior of cosmological constant of the model versus time with the appropriate choice of constants $n = 0.5,\ l = 1,\ b = 1.$

We observe that, the cosmological constant of the model is positive decreasing function of time and it approaches a small positive value at late time (i.e. at present epoch). A positive value of cosmological constant correspondsto a negative effective mass density (repulsion). Hence, we expect that the universe is with a positive valueof cosmological constant to which the expansion will tend to accelerate whereas in the universe with negative value of cosmological constant the expansion willslow down, stop and reverse.The observations on red-shift of type-Ia supernova suggests that our universe may be an accelerating one with induced due to a positive cosmological constant. Thus, the nature of cosmological constant in our derived model is supported by recent observation of type-Ia and also resembles with the work of Padminin *et al.* [28] as well as the recent investigation via different theoretical models and cosmography tests by Bamba *et al.* [29]. The graphical behavior of the cosmological constant of the model with the appropriate choice of constants versus time is given in fig. (1).

The displacement vector of the model,

$$\lambda = \frac{A_4}{(nlt+b)^{\frac{3}{n}}}. \tag{33}$$

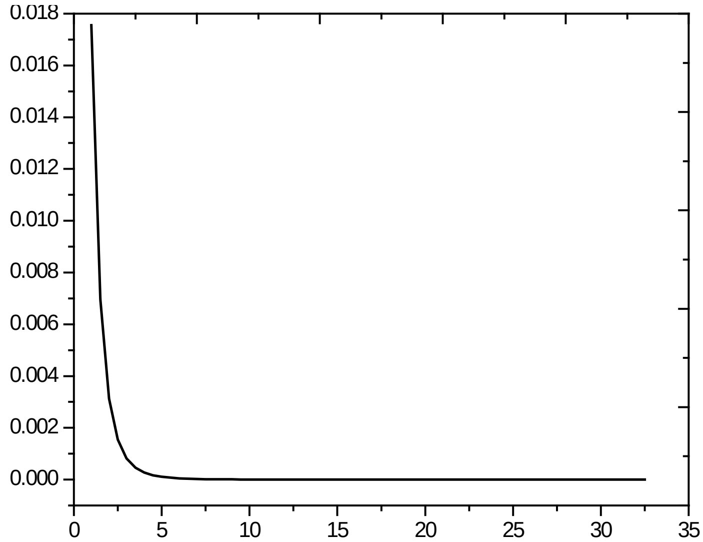

Figure-2: Behavior of displacement vector of the model versus time with the appropriate choice of constants $A_4 = 0.2$, $n = 0.5$, $l = 1$, $b = 1$.

From the above equation (33), it is observed that the displacement vector of the model exist in Lyra geometry is time dependent in relation with $\lambda \propto \left(1/t\right)$. Hence initially it is observed bit at an infinite expansion it vanishes. The behavior is clearly shown in fig. 2.

Energy density of the model,

$$\rho = \frac{A_5}{(nlt+b)^{\frac{3(1+\omega)}{n}}}. \tag{34}$$

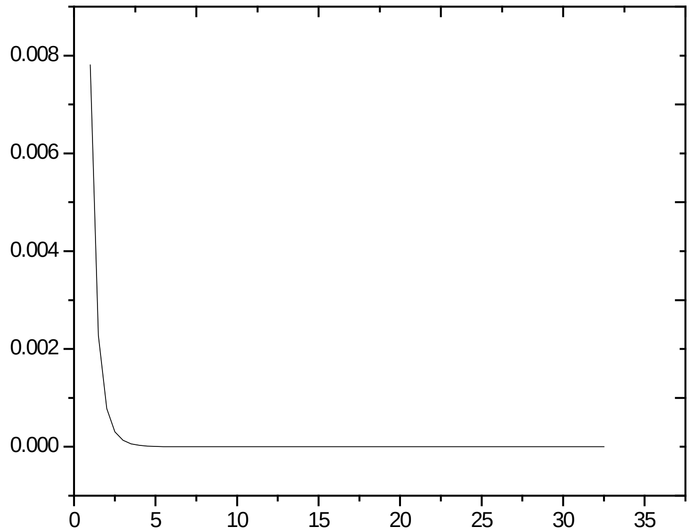

Figure-3: Behavior of energy density of the model versus time with the appropriate choice of constants $A_5 = 0.2$, $n = 0.5$, $l = 1$, $b = 1$.

From the equation (34), we observed that during cosmic evolution, the energy density remains positive having constant at initial $t = 0$ and very large at $t = t_s = \frac{-b}{nl}$. The energy density of the model decreases rapidly when the universe is in accelerating phase and leads to a small positive values which is come closer to zero at cosmic time at late phase (see fig. 3).

Gravitational constant of the model,

$$G = \frac{nl}{24\pi A_5(3\omega - n + 3)}(nlt+b)^{\frac{(3\omega-n+3)}{n}}. \tag{35}$$

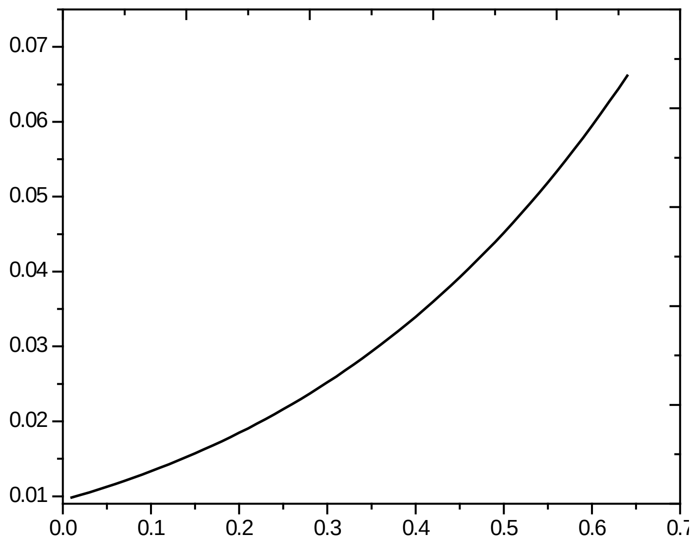

Figure-4: Behavior of gravitational constant of the model versus time with the appropriate choice of constants $A_5 = 0.2$, $n = 0.5$, $l = 1$, $b = 1$, $\omega = 0.33$.

Viscous pressure of the model,

$$\bar{p} = \frac{3A_5(n-3\omega-3)}{nl}\left\{\frac{18l(A_2+A_3)+l}{3(nlt+b)^{\frac{(3\omega+3)}{n}}} + \frac{(A_2^2+A_2A_3+A_3^2)}{(nlt+b)^{\frac{(3\omega-n+3)}{n}}} + \frac{(3A_4)}{4(nlt+b)^{\frac{(n^2+9\omega-3n+9)}{3n}}} + \frac{3l^2(9-n)}{(nlt+b)^{\frac{(3\omega+n+3)}{n}}}\right\}. \tag{36}$$

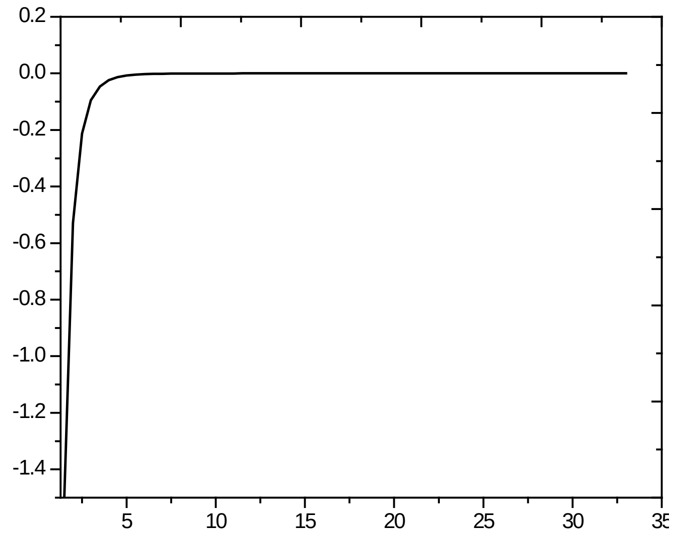

Figure-5: Behavior of viscous pressure of the model versus time with the appropriate choice of constants $A_2 = A_3 = A_4 = A_5 = 0.2$, $n = 0.5$, $l = 1$, $b = 1$, $\omega = 0.33$.

The behavior of viscous pressure of the model versus time with the appropriate choice of constants is shown in fig. 5. In this model, we observe that at the initial epoch $t \to 0$ the viscous pressure of the modelis negative increasingfunction of time and attain a small constant negative valueat later times (see fig. 5). This shows that there is no real visible matter (baryonic matter) in the derive model.

Viscous equation of state parameter of the model,

$$\bar{\omega} = \frac{3(3\omega-n+3)}{nl}\left\{\frac{18l(A_2+A_3)+l}{3(nlt+b)^{\frac{3}{n}}} + \frac{(A_2^2+A_2A_3+A_3^2)}{(nlt+b)^{\frac{(6-n)}{n}}} + \frac{(3A_4)}{4(nlt+b)^{\frac{(n-3)}{3}}} + \frac{3l^2(9-n)}{(nlt+b)}\right\}. \tag{37}$$

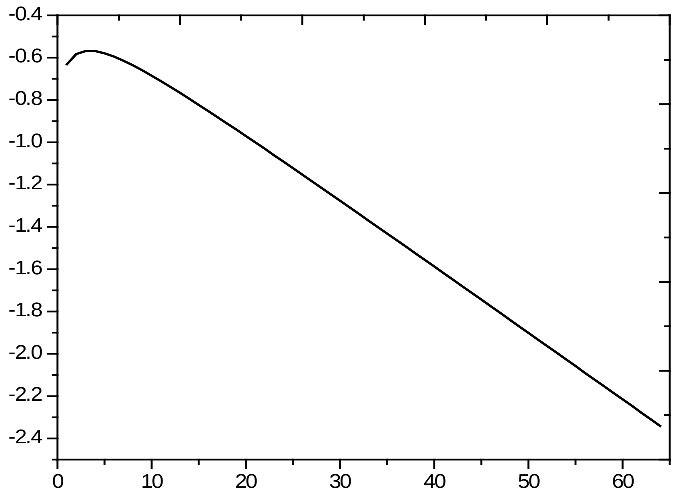

Figure-6: Behavior of viscous equation of state of the model versus time with the appropriate choice of constants $A_2 = A_3 = A_4 = A_5 = 0.2$, $n = 0.5$, $l = 1$, $b = 1$, $\omega = 0.33$.

In this model, we observe that at the initial epoch $t \to 0$ the equation of state parameter of the model is negative and a function of time (see fig. 6). This shows that the model filled with dark energy and there is no real visible matter (baryonic matter) in the derive model. Hence, the viscous equation of state parameter of the model starts from negative region, i.e., from the quintessence region $\overline{\omega} > -1$ to a phantom region $\overline{\omega} < -1$ and remains steady in phantom region.

Kinematical parameters

The kinematical properties which are important in cosmology for discussing the geometrical behavior of the universe that are spatial volume,average scale factor,average Hubble parameter, expansion scalar,shear scalarandanisotropic parameter which have the following expressions.

The spatial volume and average scale factor,

$$V = (nlt + b)^{3/n}. \tag{38}$$

It is observed that the spatial volume and average scale factor of the model both are constant at initialtime $t \to 0$, and expands exponentially as time increases and becomes infinitely large at $t \to \infty$ whereas at point $t = t_s = \frac{-b}{nl}$ it is zero.

Average Hubble parameter,

$$H = \frac{l}{(nlt + b)}. \tag{39}$$

Expansion scalar,

$$\theta = \frac{3l}{(nlt + b)}. \tag{40}$$

The average Hubble parameter and expansion scalar of the model both are time dependent with inverse relation of time. Initially they are constant at $t \to 0$, and infinitely large at $t = t_s = \frac{-b}{nl}$. The expansion scalar decreases as time increases. For this model at infinite time $dH/dt \to 0$, which implies the greatest value of the Hubble parameter and the fastest rate expansion of the model. Thus, this model may represent the inflationary era in the earlyand very late time also which shows that the model is expanding with increase of time and the rate of expansion decreases with increase of time.

Anisotropic parameter,

$$\Delta = \frac{(A_1^2 + A_2^2 + A_3^2)}{9l(nlt + b)^{3/n - 1}}. \tag{41}$$

Shear scalar,

$$\sigma^2 = \frac{(A_1^2 + A_2^2 + A_3^2)}{3(nlt + b)^{6/n}}. \tag{42}$$

From equations (41) and (42), the anisotropic parameterand shear scalarboth are the function of time. Initially both are constant and with expansion they are diverse and at $t \to \infty$, the shear and anisotropic parameterboth are insignificant.

The condition of homogeneity and isotropy that is $\lim_{t\to\infty}\frac{\sigma}{\theta}=0$, is satisfied in the present model. The ratio of the shear scalar to the expansion scalar indicates that at the early epoch, the Universe was anisotropic and as time passes, it approaches isotropy.

4. CONCLUSION

The present investigation deals with the study of homogeneous anisotropic Bianchi type-I cosmological model with time-dependent gravitational and cosmological constants in the framework of Lyra geometrytowards the law of variation of Hubble parameter that yields a constant value of deceleration parameter. It has been found that, the general solution of the average scale factor is a time involved the power law function. The analysis of the derived model shows that, for the cosmological constant and gravitational constant are opposite in nature to each other i.e. cosmological constant decreases whereas the Newtonian gravitational constant increases with time whereas the model become isotropic at late time.The cosmological constant of the model is positive decreasing function of time and it approaches a small positive value at late time (i.e. at present epoch) which leads to the expansion will tend to accelerate which resembles with the work of Padminin *et al.* [28] as well as Bamba et al. [29]. The displacement vector of the model exist in Lyra geometry is also time dependent in relation with $\lambda \propto \left(1/t\right)$. During cosmic evolution of the model, the energy density remains positive and having constant value at initial and very large at t_s. The energy density of the model decreases rapidly when the universe is in accelerating phase and leads to a small positive values which is come closer to zero. From the graphical behavior of viscous pressure of the model we observe that at the initial epoch $t \to 0$ the viscous pressure of the model is negative increasing function of time and attain a small constant negative value at later timesin the derive model while the equation of state parameter of the model is a function of time evolved within the range $\overline{\omega} < 0$. This shows that the model only filled with dark energy and there is no real visible matter (baryonic matter) in the derive model.

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