International Journal of Mathematical Archive-11(7), 2020, 7-16
Available online through www.ijma.info ISSN 2229 – 5046

STATUS NEIGHBORHOOD DAKSHAYANI INDICES

V. R. KULLI*

Department of Mathematics,
Gulbarga University, Kalaburgi (Gulbarga) - 585106, India.

(Received On: 09-06-20; Revised & Accepted On: 29-06-20)

ABSTRACT

The status of a vertex u in a connected graph G is defined as the sum of the distances between u and all other vertices of G. In this study, we introduce the first, second and third or vertex status neighborhood Dakshayani indices, the first and second hyper status neighborhood Dakshayani indices of a graph and determine exact formulas for some standard graphs and friendship graphs. Also we define the first, second, third or vertex status neighborhood Dakshayani polynomials of a graph and compute exact formulas for some standard graphs and friendship graphs.

Keywords: *status, status neighborhood Dakshayani indices, status neighborhood Dakshayani polynomials, graph.*

Mathematics Subject Classification: *05C05, 05C12, 05C90.*

1. INTRODUCTION

A graph index is a numerical parameter mathematically derived from graph structure. Graph indices are very important on the development of Chemical Graph Theory. Many graph indices were defined by using vertex degree concept, distance concept [1]. Several graph indices have applications in various disciplines of Science and Technology, see [2, 3].

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u in a graph G is the number of vertices adjacent to u. The distance between any two vertices u and v is the length of shortest path containing u and v; and it is denoted by $d(u, v)$. The status $\sigma(u)$ of a vertex u in a connected graph G is the sum of distances of all other vertices from u in G. Let $N(u) = N_G(u) = \{v : uv \in E(G)\}$.

Let $\sigma_d(u) = \sigma(u) + \sum_{v \in N(u)} \sigma(v) = \sum_{v \in N[u]} \sigma(v)$, where $N[u] = N(u) \cup \{v\}$. Then $\sigma_d(u)$ is the status sum of closed neighborhood vertices of u. For other graph terminology and notation, we refer the book [4]. Several status indices of a graph such as first and second status connectivity indices [5], first and second status coindices [6], harmonic status index [7], geometric-arithmetic status index [8], first and second hyper status indices [9], F-status index [10], (a, b)-status index [11], ABC and augmented status indices [12], status Gourava indices [13], multiplicative first and second status indices [14], multiplicative ABC status index [15], multiplicative status indices [16], status neighborhood indices [17] were introduced and studied in the literature.

Recently, the first and second neighborhood Dakshayani indices were introduced by Kulli in [18] and they are defined as

$$ND_1(G) = \sum_{uv \in E(G)} \left[D_G(u) + D_G(v)\right], \qquad ND_2(G) = \sum_{uv \in E(G)} D_G(u) D_G(v),$$

where $D_G(u) = \sum_{v \in N[u]} d_G(v)$.

Recently, some variants of neighborhood Dakshayani indices were introduced and studied such as F-neighborhood Dakshayani index [19], square neighborhood Dakshayani index [20], connectivity neighborhood Dakshayani indices [21], first and second multiplicative neighborhood Dakshayani indices [22], multiplicative ABC, GA neighborhood Dakshayani indices [23].

Corresponding Author: V. R. Kulli*,
Department of Mathematics, Gulbarga University, Kalaburgi (Gulbarga) - 585106, India.

Motivated by the definitions and their applications, we now introduce new distance based topological indices as follows:
The first and second status neighborhood Dakshayani indices of a graph G are defined as

$$SD_1(G)=\sum_{uv\in E(G)}\left[\sigma_d(u)+\sigma_d(v)\right], \qquad SD_2(G)=\sum_{uv\in E(G)}\sigma_d(u)\sigma_d(v).$$

The third or vertex status neighborhood Dakshayani index of a graph is defined as

$$SD_3(G)=\sum_{u\in V(G)}\sigma_d(u)^2.$$

Also we propose the first and second hyper status neighborhood Dakshayani indices of a graph, defined as

$$HSD_1(G)=\sum_{uv\in E(G)}\left[\sigma_d(u)+\sigma_d(v)\right]^2, \qquad HSD_2(G)=\sum_{uv\in E(G)}\left[\sigma_d(u)\sigma_d(v)\right]^2.$$

Recently some hyper indices such as hyper reverse Zagreb indices [24], hyper Revan indices [25] were introduced and studied.

We also introduce the first, second, third status neighborhood Dakshayani polynomials, first and second hyper status neighborhood Dakshayani polynomials of a graph G and they are defined as:

$$SD_1(G,x)=\sum_{uv\in E(G)}x^{\sigma_d(u)+\sigma_d(v)}.$$

$$SD_2(G,x)=\sum_{uv\in E(G)}x^{\sigma_d(u)\sigma_d(v)}.$$

$$SD_3(G,x)=\sum_{u\in V(G)}x^{\sigma_d(u)^2}.$$

$$HSD_1(G,x)=\sum_{uv\in E(G)}x^{\left[\sigma_d(u)+\sigma_d(v)\right]^2}.$$

$$HSD_2(G,x)=\sum_{uv\in E(G)}x^{\left[\sigma_d(u)\sigma_d(v)\right]^2}.$$

Recently, some different polynomials were studied, for example, in [24, 25].

In this study, the first, second and third status neighborhood Dakashayani indices, first and second hyper status neighborhood Dakshayani indices of complete graphs, complete bipartite graphs, wheel graphs, friendship graphs are computed. Also the status neighborhood Dakshayani polynomials, hyper status neighborhood Dakshayani polynomials of some standard graphs, friendship graphs are determined.

2. RESULTS FOR COMPLETE GRAPHS

Theorem 1: Let K_n be a complete graph with n vertices. Then

(1) $SD_1(K_n)=n^4-2n^3+n.^2$

(2) $SD_2(K_n)=\frac{1}{2}n^3(n-1)^3.$

(3) $SD_3(K_n)=n^3(n-1)^2.$

Proof: Let K_n be a complete graph with n vertices and $\frac{n(n-1)}{2}$ edges. Then $d_{K_n}(u)=n-1$ and $\sigma(u) = n - 1$ for every vertex u in K_n. Hence $\sigma_n(u) = (n - 1)^2$ for every vertex u of K_n. Thus, $\sigma_d(u) = (n - 1) + (n - 1)^2 = n(n - 1)$. Therefore

(1) $$SD_1(K_n)=\sum_{uv\in E(K_n)}\left[\sigma_d(u)+\sigma_d(v)\right]=\frac{n(n-1)}{2}\left[n(n-1)+n(n-1)\right]$$
$$=n^4-2n^3+n^2.$$

(2) $$SD_2(K_n)=\sum_{uv\in E(K_n)}\left[\sigma_d(u)\sigma_d(v)\right]=\frac{n(n-1)}{2}\left[n(n-1)n(n-1)\right]$$
$$=\frac{1}{2}n^3(n-1)^3.$$

(3) $$SD_3(K_n)=\sum_{u\in V(K_n)}\sigma_d(u)^2=n.n^2(n-1)^2=n^3(n-1)^2.$$

Theorem 2: The first and second hyper status neighborhood Dakshayani indices of K_n is

(1) $HSD_1(K_n) = 2n^3(n-1)^3$.

(2) $HSD_2(K_n) = \frac{1}{2}n^5(n-1)^5$.

Proof: If K_n be a complete graph with n vertices, then $\sigma_d(u) = n(n-1)$ for every vertex u of K_n. Thus

(1) $$HSD_1(K_n) = \sum_{uv \in E(K_n)} \left[\sigma_d(u) + \sigma_d(v)\right]^2 = \frac{n(n-1)}{2}\left[n(n-1) + n(n-1)\right]^2$$
$$= 2n^3(n-1)^3.$$

(2) $$HSD_2(K_n) = \sum_{uv \in E(K_n)} \left[\sigma_d(u)\sigma_d(v)\right]^2 = \frac{n(n-1)}{2}\left[n(n-1)n(n-1)\right]^2$$
$$= \frac{1}{2}n^5(n-1)^5.$$

Theorem 3: Let K_n be a complete graph with n vertices. Then

(1) $SD_1(K_n, x) = \frac{n(n-1)}{2}x^{2n(n-1)}$.

(2) $SD_2(K_n, x) = \frac{n(n-1)}{2}x^{n^2(n-1)^2}$.

(3) $SD_3(K_n, x) = nx^{n^2(n-1)^2}$.

(4) $HSD_1(K_n, x) = \frac{n(n-1)}{2}x^{4n^2(n-1)^2}$.

(5) $HSD_2(K_n, x) = \frac{n(n-1)}{2}x^{n^4(n-1)^4}$.

Proof: Let K_n be a complete graph with n vertices, then $\sigma_d(u) = n(n-1)$ for every vertex u in K_n. Therefore

(1) $$SD_1(K_n, x) = \sum_{uv \in E(K_n)} x^{\sigma_d(u)+\sigma_d(v)}$$
$$= \frac{n(n-1)}{2}x^{n(n-1)+n(n-1)}$$
$$= \frac{n(n-1)}{2}x^{2n(n-1)}.$$

(2) $$SD_2(K_n, x) = \sum_{uv \in E(K_n)} x^{\sigma_d(u)\sigma_d(v)}$$
$$= \frac{n(n-1)}{2}x^{n(n-1)n(n-1)}$$
$$= \frac{n(n-1)}{2}x^{n^2(n-1)^2}.$$

(3) $$SD_3(K_n, x) = \sum_{u \in V(K_n)} x^{\sigma_d(u)^2} = nx^{n^2(n-1)^2}.$$

(4) $$HSD_1(K_n, x) = \sum_{uv \in E(K_n)} x^{\left[\sigma_d(u)+\sigma_d(v)\right]^2}$$
$$= \frac{n(n-1)}{2}x^{[n(n-1)+n(n-1)]^2}$$
$$= \frac{n(n-1)}{2}x^{4n^2(n-1)^2}.$$

(5) $$HSD_2(K_n, x) = \sum_{uv \in E(K_n)} x^{\left[\sigma_d(u)\sigma_d(v)\right]^2}$$
$$= \frac{n(n-1)}{2}x^{[n(n-1)n(n-1)]^2}$$
$$= \frac{n(n-1)}{2}x^{n^4(n-1)^4}.$$

3. RESULTS FOR COMPLETE BIPARTITE GRAPHS

Theorem 4: Let $K_{p,q}$ be a complete bipartite graph with $p+q$ vertices and pq edges. Then

(1) $SD_1(K_{p,q}) = pq\left[2(p^2+q^2)+(p+q)-2pq-4\right]$.

(2) $SD_2(K_{p,q}) = pq(1+p)(1+q)\left[2(p^2+q^2)-6(p+q)+5pq+4\right]$.

(3) $SD_3(K_{p,q}) = p(1+q)^2(p+2q-2)^2 + q(1+p)^2(q+2p-2)^2$.

Proof: The vertex set of $K_{p,q}$ can be partitioned into two independent set V_1 and V_2 such that $u\in V_1$ and $u\in V_2$ for every edge uv in $K_{p,q}$. Let $K=K_{p,q}$. We have $d_K(u)=q$, $d_K(v)=p$. Then $\sigma(u) = q + 2p - 2$ and $\sigma(v) = p + 2q - 2$.
Thus $\sigma_n(u) = p(q + 2p - 2)$ and $\sigma_n(v) = q(p + 2q - 2)$. Therefore

$$\sigma_d(u)= \sigma(u)+ \sigma_n(u)= (q + 2p - 2) + p(q + 2p - 2) = (1+p)(q + 2p - 2)$$
$$\sigma_d(v)= \sigma(v)+ \sigma_n(v)= (p + 2q - 2) + q(p + 2q - 2) = (1+q)(p + 2q - 2)$$

Hence

(1)
$$\begin{aligned} SD_1(K_{p,q}) &= \sum_{uv\in E(K)} [\sigma(u)+\sigma(v)] \\ &= pq\left[(1+p)(q+2p-2)+(1+q)(p+2q-2)\right] \\ &= pq\left[2(p^2+q^2)+(p+q)+2pq-4\right] \end{aligned}$$

(2)
$$\begin{aligned} SD_2(K_{p,q}) &= \sum_{uv\in E(K)} \sigma(u)\sigma(v) \\ &= pq(1+p)(q+2p-2)(1+q)(p+2q-2) \\ &= pq(1+p)(1+q)\left[2(p^2+q^2)-6(p+q)+5pq+4\right] \end{aligned}$$

(3)
$$\begin{aligned} SD_3(K_{p,q}) &= \sum_{u\in V(K)} \sigma(u)^2 \\ &= p(1+q)^2(p+2q-2)^2 + q(1+p)^2(q+2p-2)^2. \end{aligned}$$

Theorem 5: The first and second hyper status neighborhood Dakshayani indices of $K_{p,q}$ are given by

(1) $HSD_1(K_{p,q}) = pq\left[2(p^2+q^2)+(p+q)+2pq-4\right]^2$.

(2) $HSD_2(K_{p,q}) = pq(1+p)^2(1+q)^2\left[2(p^2+q^2)-6(p+q)+5pq+4\right]^2$.

Proof: The $K_{p,q}$ is a complete bipartite graph with $p+q$ vertices and pq edges, then $\sigma_d(u) = (1+p)(q + 2p - 2)$ and $\sigma_d(v) = (1+q)(p + 2q - 2)$ for every edge uv of $K_{p,q}$. Thus

(1)
$$\begin{aligned} HSD_1(K_{p,q}) &= \sum_{uv\in E(K)} [\sigma(u)+\sigma(v)]^2 \\ &= pq\left[(1+p)(q+2p-2)+(1+q)(p+2q-2)\right]^2 \\ &= pq\left[2(p^2+q^2)+(p+q)+2pq-4\right]^2. \end{aligned}$$

(2)
$$\begin{aligned} HSD_2(K_{p,q}) &= \sum_{uv\in E(K)} [\sigma(u)\sigma(v)]^2 \\ &= pq\left[(1+p)(q+2p-2)(1+q)(p+2q-2)\right]^2 \\ &= pq(1+p)^2(1+q)^2\left[2(p^2+q^2)-6(p+q)+5pq+4\right]^2. \end{aligned}$$

Theorem 6: Let $K_{p,q}$ be a complete bipartite. Then

(1) $SD_1(K_{p,q},x) = pqx^{2(p^2+q^2)+(p+q)-2pq-4}$.

(2) $SD_2(K_{p,q},x) = pqx^{(1+p)(1+q)\left[2(p^2+q^2)-6(p+q)+5pq+4\right]}$.

(3) $SD_3(K_{p,q},x) = px^{(1+q)^2(p+2q-2)^2} + qx^{(1+p)^2(q+2p-2)^2}$.

(4) $HSD_1(K_{p,q},x) = pqx^{\left[2(p^2+q^2)+(p+q)-2pq-4\right]^2}$.

(5) $HSD_2(K_{p,q},x) = pqx^{(1+p)^2(1+q)^2\left[2(p^2+q^2)-6(p+q)+5pq+4\right]^2}$.

Proof: Let $K_{p,q}$ be a complete bipartite graph. Then $\sigma_n(u) = (1+p)(p + 2q - 2)$ and $\sigma_n(v) = (1+q)(q + 2p - 2)$ for every degree uv of $K_{p,q}$. Thus

(1) $$SD_1(K_{p,q}, x) = \sum_{uv\in E(K)} x^{\sigma_d(u)+\sigma_d(v)} = pqx^{(1+p)(q+2p-2)+(1+q)(p+2q-2)} = pqx^{2(p^2+q^2)+(p+q)+2pq-4}.$$

(2) $$SD_2(K_{p,q}, x) = \sum_{uv\in E(K)} x^{\sigma_d(u)\sigma_d(v)} = pqx^{(1+p)(q+2p-2)(1+q)(p+2q-2)} = pqx^{(1+p)(1+q)\left[2(p^2+q^2)-6(p+q)+5pq+4\right]}.$$

(3) $$SD_3(K_{p,q}, x) = \sum_{u\in V(K)} x^{\sigma_d(u)^2} = px^{(1+q)^2(p+2q-2)^2} + qx^{(1+p)^2(q+2p-2)^2}.$$

(4) $$HSD_1(K_{p,q}, x) = \sum_{uv\in E(K)} x^{\left[\sigma_d(u)+\sigma_d(v)\right]^2} = pqx^{\left[(1+p)(q+2p-2)+(1+q)(p+2q-2)\right]^2} = pqx^{\left[2(p^2+q^2)+(p+q)+2pq-4\right]^2}.$$

(5) $$HSD_2(K_{p,q}, x) = \sum_{uv\in E(K)} x^{\left[\sigma_d(u)\sigma_d(v)\right]^2} = pqx^{\left[(1+p)(q+2p-2)(1+q)(p+2q-2)\right]^2} = pqx^{(1+p)^2(1+q)^2\left[2(p^2+q^2)-6(p+q)+5pq+4\right]^2}.$$

4. RESULTS FOR WHEEL GRAPHS

A wheel W_n is the join of K_1 and C_n. A wheel W_n is shown in Figure 1.

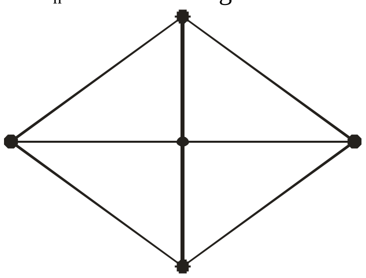

Figure-1: Wheel graph W_4.

This graph W_n has n+1 vertices and $2n$ edges. In W_4, there are two types of status vertices as follows:

$V_1 = \{u \in V(W_n) \mid \sigma(u) = n\}$, $|V_1| = n$.

$V_2 = \{u \in E(W_n) \mid \sigma(u) = 2n-3\}$, $|V_2| = n$.

By calculation, we find that there are two types of status neighborhood vertices as follows:

$V_1 = \{u \in V(W_n) \mid \sigma_n(u) = n(2n-3)\}$, $|V_1| = n$.

$V_2 = \{u \in E(W_n) \mid \sigma_n(u) = 5n-6\}$, $|V_2| = n$.

By calculation, we obtain that there are two types of status neighborhood Dakshayani vertices as given Table 1.

$\sigma_d(u) \setminus u \in E(W_n)$	$n(2n-2)$	$7n-9$
Number of edges	1	n

Table-1: Status neighborhood Dakshayani vertex partition of W_n

By calculation, we obtain that there are two types of status neighborhood Dakshayani vertices as given Table 1.

$\sigma_d(u)$, $\sigma_d(v)$ \ $uv \in E(W_n)$	$(7n-9, 7n-9)$	$7n-9$, $n(2n-2)$
Number of edges	n	n

Table-2: Status neighborhood Dakshayani edge partition of W_n

Theorem 7: The first, second and third status neighborhood Dakshayani indices of a wheel graph W_n are given by

(1) $SD_1(W_n) = 2n^3 + 19n^2 - 27n.$

(2) $SD_2(W_n) = n(14n^3 + 17n^2 - 108n + 81).$

(3) $SD_3(W_n) = 4n^4 + 41n^3 - 122n^2 + 81n.$

Proof: Let W_n be a wheel graph with n+1 vertices and 2n edges. By definitions and using Table 2, we deduce

(1) $$SD_1(W_n) = \sum_{uv \in E(W_n)} \left[\sigma_d(u) + \sigma_d(v)\right] = n(7n-9+7n-9) + n\left(7n-9+2n^2-2n\right)$$
$$= 2n^3 + 19n^2 - 27n.$$

(2) $$SD_2(W_n) = \sum_{uv \in E(W_n)} \sigma_d(u)\sigma_d(v) = n(7n-9)(7n-9) + n(7n-9)\left(2n^2-2n\right)$$
$$= n\left(14n^3 + 17n^2 - 108n + 81\right).$$

By using definition and Table 1, we derive

(3) $$SD_3(W_n) = \sum_{u \in V(W_n)} \sigma_d(u)^2 = \left(2n^2-2n\right)^2 + n(7n-9)^2$$
$$= 4n^4 + 41n^3 - 122n^2 + 81n.$$

Theorem 8: The first and second hyper status neighborhood Dakshayani indices of a wheel graph W_n are given by

(1) $HSD_1(W_n) = n(14n-18)2\left(2n^2+5n-9\right)^2.$

(2) $HSD_2(W_n) = n(7n-9)^4 + n(7n-9)^2\left(2n^2-2n\right)^2.$

Proof: By definitions and using Table 2, we deduce

(1) $$HSD_1(W_n) = \sum_{uv \in E(W_n)} \left[\sigma_d(u) + \sigma_d(v)\right] = n(7n-9+7n-9)^2 + n\left(7n-9+2n^2-2n\right)^2$$
$$= n(14n-18n)^2 + n\left(2n^2+5n-9\right)^2.$$

(2) $$HSD_2(W_n) = \sum_{uv \in E(W_n)} \left[\sigma_d(u)\sigma_d(v)\right]^2 = n\left[(7n-9)(7n-9)\right]^2 + n\left[(7n-9)\left(2n^2-2n\right)\right]^2$$
$$= n(7n-9)^4 + n(7n-9)^2\left(2n^2-2n\right)^2.$$

Theorem 9: Let W_n be a wheel graph with n+1 vertices and 2n edges. Then

(1) $SD_1(W_n, x) = nx^{14n-18} + nx^{2n^2+5n-9}.$

(2) $SD_2(W_n, x) = nx^{49n^2-126n+81} + nx^{14n^3-32n^2+18n}.$

(3) $SD_3(W_n, x) = x^{4n^4-8n^3+4n^2} + nx^{49n^2-126n+81}.$

(4) $HSD_1(W_n, x) = nx^{(14n-18)^2} + nx^{\left(2n^2+5n-9\right)^2}.$

(5) $HSD_2(W_n, x) = nx^{\left(49n^2-126n+81\right)^2} + nx^{\left(14n^3-32n^2+18n\right)^2}.$

Proof:

(1) By definition and using Table 2, we deduce

$$SD_1(W_n, x) = \sum_{uv \in E(W_n)} x^{\sigma_d(u)+\sigma_d(v)}$$
$$= nx^{7n-9+7n-9} + nx^{7n-9+2n^2-2n}$$
$$= nx^{14n-18} + nx^{2n^2+5n-9}$$

(2) By definition and using Table 2, we derive

$$SD_2(W_n,x)=\sum_{uv\in E(W_n)} x^{\sigma_d(u)\sigma_d(v)}$$
$$=nx^{(7n-9)(7n-9)}+nx^{(7n-9)(2n^2-2n)}$$
$$=nx^{49n^2-126n+81}+nx^{14n^3-32n^2+18n}$$

(3) By definition and by using Table 1, we obtain

$$SD_3(W_n,x)=\sum_{u\in V(W_n)} x^{\sigma_d(u)^2}$$
$$=x^{n^2(2n-2)^2}+nx^{(7n-9)^2}$$
$$=x^{4n^2-8n^3+4n^2}+nx^{49n^2-126n+81}$$

(4) From definition and by using Table 2, we have

$$HSD_1(W_n,x)=\sum_{uv\in E(W_n)} x^{[\sigma_d(u)+\sigma_d(v)]^2}$$
$$=nx^{(7n-9+7n-9)^2}+nx^{(7n-9+2n^2-2n)^2}$$
$$=nx^{(14n-18)^2}+nx^{(2n^2+5n-9)^2}$$

(5) Using definition and Table 2, we deduce

$$HSD_2(W_n,x)=\sum_{uv\in E(W_n)} x^{[\sigma_d(u)\sigma_d(v)]^2}$$
$$=nx^{[(7n-9)(7n-9)]^2}+nx^{[(7n-9)(2n^2-2n)]^2}$$
$$=nx^{(49n^2-126n+81)^2}+nx^{(14n^3-32n^2+18n)^2}$$

5. RESULTS FOR FRIENDSHIP GRAPHS

A friendship graph, denoted by F_n, is a graph obtained by taking $n\geq 2$ copies of C_3 with vertex in common. A Friendship graph F_4 is presented in Figure 2.

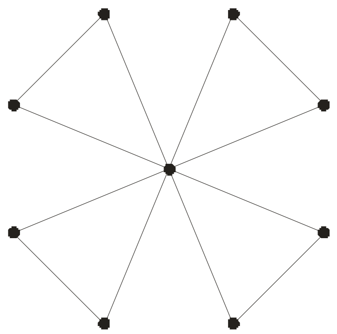

Figure-2: Friendship graph F_4

A graph F_n has $2n+1$ vertices and $3n$ edges. In F_n, there are two types of status vertices as follows:

$V_1 = \{u \in V(F_n) \mid \sigma(u)=2n\}$, $|V_1| = 1$.

$V_2 = \{u \in V(F_n) \mid \sigma(u)=4n-2\}$, $|V_2| = 2n$.

By calculation, there are two types of status neighborhood vertices as follows:

$V_1 = \{u \in V(F_n) \mid \sigma_n(u)=2n(4n-2)\}$, $|V_1| = 1$.

$V_2 = \{u \in V(F_n) \mid \sigma_n(u)=6n-2\}$, $|V_2| = 2n$.

By calculation, we obtain that there are two types of status neighborhood Dakshayani vertices as given in Table 3.

$\sigma_d(u) \setminus u \in E(F_n)$	$2n(4n-1)$	$10n-4$
Number of edges	n	$2n$

Table-3: Status neighborhood Dakshayani vertex partition of F_n.

By calculation, we obtain that there are two types of status neighborhood Dakshayani edges as given in Table 4.

$\sigma_d(u),\ \sigma_d(v) \setminus uv \in E(F_n)$	$(10n-4,\ 10n-4)$	$(10n-4,\ 2n(4n-1))$
Number of edges	n	$2n$

Table-4: Status neighborhood Dakshayani edge partition of F_n.

Theorem 10: The first, second and third status neighborhood Dakshayani indices of a friendship graph F_n are given by

(1) $SD_1(F_n) = 16n^3 + 36n^2 - 16n.$

(2) $SD_2(F_n) = n(160n^3 + 4n^2 - 64n).$

(3) $SD_3(F_n) = 64n^4 + 168n^3 - 156n^2 + 32n.$

Proof: Let F_n be a friendship graph with $2n+1$ vertices and $3n$ edges.

(1) From definition and using Table 4, we obtain

$$SD_1(F_n) = \sum_{uv\in E(F_n)} \left[\sigma_d(u) + \sigma_d(v)\right] = n(10n-4+10n-4) + 2n(10n-4+8n^2-2n)$$
$$= 16n^3 + 36n^2 - 16n.$$

(2) From definition and using Table 4, we have

$$SD_2(F_n) = \sum_{uv\in E(F_n)} \sigma_d(u)\sigma_d(v) = n(10n-4)(10n-4) + 2n(10n-4)(8n^2-2n)$$
$$= n(160n^3 - 4n^2 - 64n).$$

(3) Using definition and Table 1, we deduce

$$SD_3(F_n) = \sum_{u\in V(F_n)} \sigma_d(u)^2 = (8n^2-2n)^2 + 2n(10n-4)^2$$
$$= 64n^4 + 168n^3 - 156n^2 + 32n.$$

Theorem 11: The first and second hyper status neighborhood Dakshayani indices of a friendship graph F_n are given by

(1) $HSD_1(F_n) = n(20n-18)^2 + 2n(8n^2+8n-4)^2.$

(2) $HSD_2(F_n) = n(10n-4)^4 + 2n(10n-4)^2(8n^2-2n)^2.$

Proof:

(1) From definition and using Table 4, we derive

$$HSD_1(F_n) = \sum_{uv\in E(F_n)} \left[\sigma_d(u) + \sigma_d(v)\right]^2$$
$$= n(10n-4+10n-4)^2 + 2n(10n-4+8n^2-2n)^2$$
$$= n(20n-8)^2 + 2n(8n^2+8n-4)^2.$$

(2) Using definition and using Table 4, we deduce

$$HSD_2(F_n) = \sum_{uv\in E(F_n)} \left[\sigma_d(u)\sigma_d(v)\right]^2$$
$$= n(10n-4)^2(10n-4)^2 + 2n(10n-4)^2(2n^2-2n)^2$$
$$= n(10n-4)^4 + 2n(10n-4)^2(8n^2-2n)^2.$$

Theorem 12: Let F_n be a friendship graph with $2n+1$ vertices and $3n$ edges. Then

(1) $SD_1(F_n, x) = nx^{20n-8} + 2nx^{8n^2+8n-4}.$

(2) $SD_2(F_n, x) = nx^{100n^2-80n+16} + 2nx^{80n^3-52n^2+8n}.$

(3) $SD_3(F_n, x) = x^{8n^2-2n} + 2nx^{100n^2-80n+16}.$

(4) $HSD_1(F_n, x) = nx^{(20n-18)^2} + 2nx^{(8n^2+8n-4)^2}.$

(5) $HSD_2(F_n, x) = nx^{(100n^2-80n+16)^2} + 2nx^{(80n^3-52n^2+8n)^2}.$

Proof:

(1) Using definition and using Table 4, we obtain

$$SD_1\left(F_n,x\right)=\sum_{uv\in E(F_n)}x^{\sigma_d(u)+\sigma_d(v)}=nx^{10n-4+10n-4}+nx^{10n-4+8n^2-2n}$$

$$=nx^{20n-18}+2nx^{8n^2+8n-4}.$$

(2) From definition, and using Table 4, we deduce

$$SD_2\left(F_n,x\right)=\sum_{uv\in E(F_n)}x^{\sigma_d(u)\sigma_d(v)}$$

$$=nx^{(10n-4)(10n-4)}+2nx^{(10n-4)\left(8n^2-2n\right)}$$

$$=nx^{100n^2-80n+16}+2nx^{80n^3-52n^2+8n}$$

(3) By using definition and Table 3, we have

$$SD_3\left(F_n,x\right)=\sum_{u\in V(F_n)}x^{\sigma_d(u)^2}=x^{8n^2-2n}+2nx^{(10n-4)^2}$$

$$=x^{8n^2-2n}+2nx^{100n^2-80n+16}.$$

(4) From definition and by using Table 4, we deduce

$$HSD_1\left(F_n,x\right)=\sum_{uv\in E(F_n)}x^{\left[\sigma_d(u)+\sigma_d(v)\right]^2}$$

$$=nx^{(10n-4+10n-4)^2}+2nx^{\left(10n-4+8n^2-2n\right)^2}$$

$$=nx^{(20n-8)^2}+2nx^{\left(8n^2+8n-4\right)^2}.$$

(5) Using definition and Table 4, we derive

$$HSD_2\left(F_n,x\right)=\sum_{uv\in E(F_n)}x^{\left[\sigma_d(u)\sigma_d(v)\right]^2}$$

$$=nx^{[(10n-4)(10n-4)]^2}+2nx^{\left[(10n-4)\left(8n^2-2n\right)\right]^2}$$

$$=nx^{\left(100n^2-80n+16\right)^2}+2nx^{\left(80n^3-52n^2+8n\right)^2}$$

REFERENCES

1. V.R.Kulli, Graph indices, in *Hand Book of Research on Advanced Applications of Graph Theory in Modern Society*, M.Pal, S.Samanta and A.Pal (eds.) IGI GLObal, USA (2020) 66-91.
2. I.Gutman and O.E.Polansky, *Mathematical Concepts in Organic Chemistry*, Springer, Berlin (1986).
3. V.R.Kulli, *Multiplicative Connectivity Indices of Nanostuctures*, LAP LEMBERT Academic Publishing (2018).
4. V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
5. H.S. Ramane and A.S. Yalnaik, Status connectivity indices graphs and its applications to the boiling point of benzenoid hydrocarbons, *Journal of Applied Mathematics and Computing*, 55 (2017) 609-627.
6. H.S.Ramane, A.S. Yalnaik and R. Sharafdini, Status connectivity indices and coindices of graphs and its computation to some distance balance graphs, *AKCE International Journal of Graphs and Combinatorics*, (2018) https://doi.org/10.101bj.akeej.2018.09.002.
7. H.S.Ramane, B. Basavagoud and A.S. Yalnaik, Harmonic status index of graphs, *Bulletin of Mathematical Sciences and Applications*, 17(2016) 24-32.
8. K.P.Narayankar and D.Selvan, Geometric-arithmetic index of graphs, *International Journal of Mathematical Archive*, 8(7) (2017) 230-233.
9. V.R.Kulli, Some new status indices of graphs, *International Journal of Mathematics Trends and Technology*, 65(10) (2019) 70-76.
10. V.R.Kulli, Computation of status indices of graphs, *International Journal of Mathematics Trends and Technology*, 65(12) (2019) 54-61.
11. V.R.Kulli, The (a, b)-status index of graphs, *Annals of Pure and Applied Mathematics* 21(2) (2020) 103-108.
12. V.R.Kulli, Computation of ABC, AG and augmented status indices of graphs, *International Journal of Mathematical Trends and Technology*, 66(1) (2020) 1-7.
13. V.R.Kulli, Status Gourava indices of graphs, *International Journal of Recent Scientific Research*, 10, 1(A) (2020) 36770-36773.
14. V.R.Kulli, Some new multiplicative status indices of graphs, *International Journal of Recent Scientific Research*, 10(10) (2019) 35568-35573.
15. V.R.Kulli, Multiplicative ABC, GA, AG, augmented and harmonic status indices of graphs, *International Journal of Mathematical Archive*, 11(1) (2020) 32-40.

16. V.R.Kulli, Computation of multiplicative status indices of graphs, *International Journal of Mathematical Archive*, 11(4) (2020) 1-6.
17. V.R.Kulli, Computation of status neighborhood indices of graphs, *International Journal of Recent Scientific Research*, 11(4) (2020) 38079-38085.
18. V.R.Kulli, Neighborhood Dakshayani indices, *International Journal of Mathematical Archive,* 10(7) (2019) 23-31.
19. V.R.Kulli, Neighborhood Dakshayani indices of nanostructures, *International Journal of Current Research in Science and Technology,* 5(7) (2019) 1-12.
20. V.R.Kulli, F_1-Neighborhood and square neighborhood Dakshayani indices of some nanostructures, *International Journal of Engineering Sciences and Research Technology*, 8(8) (2019) 126-137.
21. V.R.Kulli, Connectivity neighborhood Dakshayani indices of POPAM dendrimers, *Annals of Pure and Applied Mathematics*, 20(1) (2019) 49-54.
22. V.R.Kulli, Some multiplicative neighborhood Dakshayani indices of certain nanostructures, *International Journal of Mathematics and its Applications,* 7(4) (2019) 209-217.
23. V.R.Kulli, Multiplicative *ABC*, *GA* and *AG* neighborhood Dakshayani indices dendrimers, *International Journal of Fuzzy Mathematical Archive*, 17(2) (2019) 77-82.
24. V.R.Kulli, Reverse Zagreb and reverse hyper-Zagreb indices and their polynomials of rhombus silicate networks, *Annals of Pure and Applied Mathematics,* 16(1) (2018) 47-51.
25. V.R.Kulli, Hyper Revan indices and their polynomials of silicate networks, *International Journal of Current Research in Science and Technology*, 4(3) (2018) 17-21.

Source of support: Nil, Conflict of interest: None Declared.