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NEW TOPOLOGICAL STRUCTURE OF NANO TOPOLOGY VIA VAGUE TOPOLOGY

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ABSTRACT

The aim of this paper is to introduce a new hybrid structure namely, Nano Vague Topological Spaces, with a Universe and an equivalence relation R called Indiscernibility relation. The notions of Nano Vague open sets, Nano Vague closed sets; Nano Vague interior and Nano Vague closure are introduced. In section three, the new concepts of Nano vague topology are discussed and in section four, Nano vague Interior and Nano Vague Closure and its properties are discussed.

Keywords: *Nano Vague Topology, Nano Vague Closed sets, Nano Vague interior, Nano Vague closure, Nano Vague Boundary region.*

1. INTRODUCTION

Topological Space is characterized by closed sets, by which topology can be defined. Many topologists were defined new types of closed sets and studied their properties in classical topological spaces. In 1965, Zadeh [7] introduced Fuzzy sets. A Fuzzy set is a mapping from X to the unit interval [0, 1], and A(x) is the membership of x in A. Three years after Zadeh's paper had appeared; Chang [4] introduced the notion of fuzzy topology by using fuzzy set onto general topology. He invented the notion of Fuzzy Topological Space and basic topological properties for Fuzzy Topological Spaces. Later, the idea of fuzzy topology was developed and reached its peak. As a generalization of Fuzzy Topological Space, Intuitionistic Fuzzy sets were defined. In these sets, the membership and non-membership values were considered.

Many researchers were worked on generalization of fuzzy topology and fuzzy sets. Further, as an extension, Vague sets were developed by Gaw and Buchere [5]. Here, there are two aspects of memberships, true membership and false membership. In this work, a new topological space namely, Nano Vague Topological space.

2. PRELIMINARIES

Definition 2.1[5]: A vague set A in the universe of discourse U = $\{x_1, x_2, ... , x_n\}$ is characterized by two membership functions given by:

i. A true membership function t_A: U → [0, 1] and
ii. A false membership function f_A: U → [0, 1],

where $t_A(x)$ is a lower bound on the grade of membership (true membership) of x derived from the evidence for x, $f_A(x)$ is a lower bound on the negation of x derived from the evidence for x (false membership), and $t_A(x) + f_A(x) \leq 1$. The condition $0 \leq t_A(x_i) \leq 1- f_A(x_i)$ should hold for any $x_i \epsilon$ U. Thus the grade of membership in the vague set A is bounded by a sub-interval $[t_A(x), 1- f_A(x)]$ of [0, 1].

This indicates that if the actual grade of membership of x is $\mu(x)$, then, $t_A(x) \leq \mu(x) \leq f_A(x)$.The vague set A is written as $A = \{\langle x, [t_A(x), 1 - f_A(x)]\rangle / x \in X\}$ where the interval $[t_A(x), 1 - f_A(x)]$ is called the vague value of x in A, denoted by $V_A(x)$.

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Definition 2.2[5]: Let A, B be two vague sets in the universe U = {x_1, x_2, …..,x_n}, then the union, intersection, and complement of vague sets are defined as follows:

i) $A \cup B = \{\langle x, [t_A(x) \vee t_B(x), (1 - f_A(x)) \vee (1 - f_B(x))]\rangle | x \in U\}$,

ii) $A \cap B = \{\langle x, [t_A(x) \wedge t_B(x), (1 - f_A(x)) \wedge (1 - f_B(x))]\rangle | x \in U\}$,

iii) $A^c = \{\langle x, [f_A(x), 1 - t_A(x)]\rangle | x \in U\}$

Definition 2.3[5]: Let A, B be two vague sets in the universe of discourse U. If $\forall$ x ∈ U, $t_A(x) \leq t_B(x)$ and $(1 - f_A(x)) \leq (1 - f_B(x))$, then A is called a vague subset of B, denoted by A ⊆ B.

Definition 2.4[6]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let X ⊆ U. Then,

i. The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is defined by $L_R(x) = \cup_{x \in X}$ {R(x): R(x) ⊆ X}, where R(x) denotes the equivalence class determined by x.

ii. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and is defined by $U_R(x) = \cap_{x \in X}$ {R(x): R(x) ∩ X = ϕ}.

iii. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not -X with respect to R and is defined by $B_R(x) = U_R(x) - L_R(x)$.

Definition: 2.5[5]: Let U be the universe, R be an equivalence relation on U and $\tau_R(x)$ = {U, ϕ, $L_R(x)$, $U_R(x)$, $B_R(x)$}, where X⊆U. Then, $\tau_R(x)$ satisfies the following axioms.

i. U and $\phi \in \tau_R(x)$.

ii. The union of the elements of any sub collection of $\tau_R(x)$ is in $\tau_R(x)$.

iii. The intersection of the elements of any finite sub collection of $\tau_R(x)$ is in $\tau_R(x)$.

That is $\tau_R(x)$ forms a topology on U called as the Nano topology on U with respect to X. We call (U, $\tau_R(x)$) as the Nano topological space. The elements of $\tau_R(x)$ are called as Nano open sets.

Definition 2.6[5]: Let U be an initial universe set, V(U) the set of all vague sets on U. A set N is called a vague set over U.

Definition 2.7[5]: Let N_1 and $N_2$are two vague sets over a universe U. If*A* $N_1 \subseteq N_2$, then N_1 is called a vague subset of N_2.

Definition 2.8[5]: Two vague sets N_1 and N_2 over a universe U are said to be vague equal if N_1 is a vague subset of N_2 and N_2 is a vague subset of N_1. This relation is denoted by $N_1 = N_2$.

Property 2.9[5]: If (U, R) is an approximation space and X, Y ⊆ U, then

$L_R(X) \subseteq X \subseteq U_R(X)$

$L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$

$U_R(X \cup Y) = U_R(X) \cup U_R(Y)$

$U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$

$L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$

$L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$

Whenever X ⊆Y, then $L_R(X) \subseteq L_R(Y)$, $U_R(X) \subseteq U_R(Y)$

$U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$.

$L_R L_R(X) = U_R L_R(X) = L_R(X)$

$U_R U_R(X) = L_R U_R(X) = U_R(X)$

3. NANO VAGUE TOPOLOGICAL SPACES

Definition 3.1: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U and let V be a nano vague set with true membership and false membership. Let $\tau_R(X)$ be the collection of Nano Vague sets. Then the nano vague lower approximations, nano vague upper approximations and the nano vague boundary of G denoted by $\underline{V}(G)$, $\overline{V}(G)$, $B_V(G)$ are respectively defined as follows.

The Nano Vague Lower approximation of X with respect to R is $\underline{V}(G) = \{\langle x, \mu\underline{A}(x), \sigma\underline{A}(x)\}\rangle$ / y ∈ $[x]_R$, x ∈ U}.

The Nano Vague Upper approximation of X with respect to R is $\overline{V}(G) = \{\langle x, \mu\overline{A}(x)\}, \sigma\overline{A}(x)\}\rangle$/ y ∈ $[x]_R$, x ∈ U}.

The Nano Vague Boundary region of X with respect to R is $B_V(G) = \overline{V}(G) - \underline{V}(G)$,

where, $\mu\underline{A}(x) = \wedge_{y \in [x]R}\mu_A(y)$, $\sigma\underline{A}(x) = \vee_{y \in [x]R}\sigma_A(y)$.

$\mu\overline{A}(x) = \vee_{y \in [x]R}\mu_A(y)$, $\sigma\overline{A}(x) = \wedge_{y \in [x]R}\sigma_A(y)$.

Definition 3.2: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U and let V be a nano vague set with true membership and false membership. Let $\tau_R(X)$ be the collection of Nano Vague sets. Then, if $\tau_R(X)$ satisfies the following axioms, together with the properties specified in [2.8], then U, $\tau_R(X)$ is called as nano vague topological space.

I. U and $\phi \in \tau_R(X)$.
II. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(x)$.
III. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U is called the Nano vague topology on U with respect to X. Hence, (U, $\tau_R(X)$) is Nano vague topological space, where $\tau_R(X) = \{0, 1, \langle \underline{V}(G), \overline{V}(G), BV(G)\rangle\}$

Definition: 3.3: Let U be the universe and let A and B be two nano vague sets of the form, $A = \langle x, [tA(x), 1 - fA(x)]\rangle / x \in X\}$ and $B = \langle x, [tBx), 1 - fBx)]\rangle / x \in X\}$ respectively. Then the following holds:

(i) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \leq \sigma_B(x)$ $\forall x \in U$.
(ii) $A = B$ iff $A \subseteq B$ and $B \subseteq A$.
(iii) $A^c = \{\langle x, f_A(x), 1 - t_A(x)\rangle / x \in U\}$
(iv) $A \cup B = \{(x, [t_A(x) \vee t_B(x), (1 - f_A(x)) \vee (1 - f_B(x))])|\ x \in U\}$,
(v) $A \cap B = \{(x, [t_A(x) \wedge t_B(x), (1 - f_A(x)) \wedge (1 - f_B(x))])|\ x \in U\}$,
(vi) $0_{NV} = \langle x, 0, 1\rangle$ and $1_{NV} = \langle x, 1, 0\rangle \forall x \in U$.

Definition 3.4: Let (U, $\tau_R(X)$) be a Nano vague Topological Space over X. Then the elements of $\tau_R(X)$ are called Nano vague Open sets.

Definition 3.5: Let (U, $\tau_R(X)$) be a Nano vague Topological Space over X. The complement N^c of any Nano Vague open set N is called a Nano vague Closed set in X if its complement $N^c \in \tau_R(X)$. And the space $(U, \tau_R(X))]^c$ is called the dual Nano vague topological space (U, $\tau_R(X)$).

Example3.6: Let U be the universe. Let $X = \{a_1, a_2, a_3\}$ and $U/R = \{a_1, \{a_2, a_3\}\}$.

Let (U, τ{R} (X)) be a Nano vague Topological Spaces over X, and the nano vague sets G_1, G_2 are defined as follows:

$G_1 = \{\langle a1, (0.3, 0.7)\rangle, \langle a2, (0.4, 0.8)\rangle, \langle a3, (0.5, 0.5)\rangle\}$ and
$G_2 = \{\langle a1, (0.5, 0.8)\rangle, \langle a2, (0.3, 0.5)\rangle, \langle a3, (0.2, 0.4)\rangle\}$.

Now, $\underline{V}(G) = \{\langle u, (0.3, 0.7)\rangle, \langle v, (0.5, 0.8)\rangle\}$, $\overline{V}(G) = \{\langle u, (0.3, 0.7)\rangle, \langle v, (0.4, 0.5)\rangle\}$ and $B_V(G) = \{\langle u, (0.3, 0.7)\rangle, \langle v, (0.4, 0.5)\rangle\}$.
Then $\tau_R(x) = \{0, 1, \{\langle u, (0.3, 0.7)\rangle, \langle v, (0.5, 0.8)\rangle\}, \{\langle u, (0.3, 0.7)\rangle, \langle v, (0.4, 0.5)\rangle\}, \{\langle u, (0.3, 0.7)\rangle, \langle v, (0.4, 0.5)\rangle\}\}$.
Then the dual Nano Vague Topological Space $[(U, \tau_R(x))]^c$ is given by:
$[\tau_R(x)]^c = \{0, 1, \{\langle u, (0.3, 0.7)\rangle, \langle v, (0.2, 0.5)\rangle\}, \{\langle u, (0.3, 0.7)\rangle, \langle v, (0.5, 0.6)\rangle\},$
$\{\langle u, (0.3, 0.7)\rangle, \langle v, (0.5, 0.6)\rangle\}\}$, which is also a Nano Vague Topology on X.

Definition 3.7: Let (U, $\tau_{1R}(X)$) and (U, $\tau_{2R}(X)$) be two Nano vague Topological Spaces over X. If each Nano vague set N $\in \tau_1$ is in τ_2, then τ_2 is called Nano vague topology, finer than τ_1 or τ_1 is Nano Vague topology, weaker than τ_2.

Definition 3.8: Let (U, $\tau_R(X)$) be a Nano vague Topological Space over X and let A be a nano vague set. Then, the Nano Vague interior of A is defined by, NVint(A) = $\vee$\{G: G is a Nano Vague Open Set and G$\subseteq$ A\}.

Clearly, NVint(A) is the largest Nano vague open set that is contained in A.

Definition 3.9: Let (U, $\tau_R(X)$) be a Nano vague Topological Space over X and let A be a nano vague set. Then, the Nano Vague closure of A is defined by, NVcl(A) = $\wedge$\{K:K is a Nano Vague Closed Set and A $\subseteq$ K\}.

Clearly, NVcl(A) is the smallest Nano vague closed set that contains in A.

Preposition 3.10: Let (U, $\tau_R(X)$) be a Nano vague Topological Space over X and let A and B be nano vague sets. Then, the following properties hold for Nano Vague Interior of any Nano Vague Set A.

(i) NVint(A) $\subseteq$ A and NVint(A) = A iff A is a Nano vague open set.
(ii) NVint(NVint(A)) = NVint(A)
(iii) NVint(0) = 0
(iv) NVint(1) = 1
(v) If A $\subseteq$ B, implies NVint(A) $\subseteq$ NVint(B).
(vi) NVint (A$\cap$B) = NVint(A) $\cap$ NVint(B)
(vii) NVint (A$\cup$ B) $\supseteq$ NVint(A) $\cup$ NVint(B)

Proof:

(i): Let A be a nano vague open set in the Nano vague Topological Space (U, $\tau_R(X)$) over X. We know that, always NVint(A) of any set is a subset of the set A. So, NVint(A) ⊆ A. Since A is a nano vague open set, we have, A ⊆ NVint(A). Therefore, NVint(A) = A. Suppose, if NVint(A) = A. Then, since NVint(A) is a nano vague open set, clearly A is also a nano vague open set.

(ii): Since NVint(A) is Nano Vague open, NVint(NVint(A)) = NVint(A).

(iii)-(iv): Since 0 and 1 are Nano Vague open, NVint(0) = 0 and NVint(1) = 1.

(v): A ⊆ B, then 1 - A ⊆ 1 - B, ⇒NVcl (1 - A) ⊆NVcl(1 - B). This implies, NVint(A) ⊆ NVint(B).

(vi)-(vii): Proofs are obvious.

Preposition 3.11: Let (U, $\tau_R(X)$) be a Nano vague Topological Space over X and let A and B be nano vague sets. Then, the following properties hold for Nano Vague Closure of any Nano Vague Set A.

(i) A ⊆NVcl(A) and NVcl(A) = A iff A is a Nano vague closed set.
(ii) NVcl(NVcl(A)) = NVcl(A)
(iii) NVcl(0) = 0
(iv) NVcl(1) = 1
(v) If A ⊆ B, implies NVcl(A) ⊆NVcl(B).
(vi) NVcl(A ∩ B) = NVcl(A) ∩NVcl(B)
(vii) NVcl(A ∪ B) ⊆NVcl(A)∪NVcl(B)

Proof:

(i) - (vii): Proofs are obvious.

Proposition 3.12: Let U be the universe and X ∈ U. Then the following holds:

(i) If $VL_R(x) = VU_R(x) = X$, $\tau_R(X) = \{\phi, U\}$ is the indiscrete nano vague topology on U.
(ii) If $VL_R(x) = \phi$ and $VU_R(x) \neq U$, then $\tau_R(X) = \{\phi, U, VU_R(X)\}$ is the nano vague topology on U.
(iii) If $VL_R(x) \neq \phi$ and $VU_R(x) = U$, then $\tau_R(X) = \{\phi, U, VL_R(X), VB_R(x)\}$ is the nano vague topology on U.
(iv) If $VL_R(x) \neq VU_R(x)$ where $VL_R(x) \neq \phi$ and $VU_R(x) \neq U$ then $\tau_R(X) = \{\phi, U, VL_R(X), VU_R(x), VB_R(x)\}$ is the discrete nano vague topology on U.

Remarks:

(i) In Nano vague Topological space, the boundary region $B_V(G)$ cannot be empty.
(ii) Let $\{\tau_k \mid k \in K\}$ be the family of nano vague topologies on X_i, then $\bigcap_{k \in K}\tau_k$ is a nano vague topology on X.
(iii) Let (U, $\tau_{1R}(x)$) and (U, $\tau_{2R}(x)$) be two Nano vague Topological Spaces over X. Then (U, $\tau_{1R}(x) \cup \tau_{2R}(x)$) need not be a nano vague topology on X.

4. REFERENCES

1. Atanassov. K., Intuitionistic fuzzy Set, Fuzzy set and systems, 20 (1986), 87-96.
2. Biswas R, Vague Groups, Internat. J Comput. Cognition. 4(2) (2006), 20-23.
3. Bustince. H, Burillo. P., Vague sets are intuitionistic fuzzy sets, Fuzzy sets and systems, 79 (1996), 403-405.
4. Chang CL, Fuzzy Topological Spaces, J.Math Anal Appl. 24 (1968), 182-190.
5. Gau. W. L, Buehrer. D. J., Vague sets, IEEE Trans, Systems Man and Cybernet, 23 (2) (1993), 610-614.
6. LellisThivagar. M, Carmel Richard, "Note on Nano topological spaces" (communicated).
7. Zadeh. L. A., Fuzzy Sets, Information and control, 8 (1965), 338-353.

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