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INTERACTIVE APPROACH FOR SOLVING A BI-LEVEL MULTI-OBJECTIVE QUADRATIC PROGRAMMING PROBLEM WITH NEUTROSOPHIC PARAMETERS IN THE OBJECTIVE FUNCTION

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ABSTRACT

In this paper we investigate a hierarchical structure problem that consist from two stages, first stage decision maker and second stage decision maker. Each level has multiple objectives need to reach and achieve them under same constrains. All problems that people faced did not contain clear information most of these problems has vague information and uncertainty, so we introduce in this paper a solution algorithm for the bi-level problem with multiple objective function and with neutrosophic parameter in these objective functions. Our algorithm starts with converting the trapezoidal neutrosophic parameter to crisp then we will use first order taylor method to convert the quadratic form to linear form. After that we will use the weight method to make the problem single objective function at each level and in the final step, we will use the interactive approach to solve the bi-level linear programming problem.

Keywords: *Bi-level programming; linear programming; Neutrosophic set; Trapezoidal neutrosophic number; Multi-objective; Quadratic programming; Interactive approach.*

1. INTRODUCTION

Multilevel programming (MLP) is designed to model the centralized decision-making cases within a hierarchical system. If there are only two levels of an MLP then problem is referred to as the bilevel programming (BLP) [1].

Bilevel Programming Problem (BPP) was first proposed by Bialas [1980], he named it a two-stage programming problem with two levels of decision-makers (DM), the leader or decision-maker of first stage (FLDM) and the decision-maker of second level (SLDM) followers.[2].

The fundamental principle of the methodology of bi-level programming (BLP) is that a first-level decision-maker (FLDM) — the chief — determines his priorities and/or decisions and then asks each hierarchical stage in the organization for their equilibrium, which is determined in isolation; the second-level decision-maker (SLDM) the follower decisions are then submitted and modified by the FLDM with consideration of the overall benefit for the organization. In other terms, although the FLDM optimizes its own advantages individually, the decision may be impacted by the SLDM reaction [3].

Several real-life situations may be modelled as a bilevel problem, and several experiments have been carried out using approximation theory to solve different forms of bilevel problems [4].

The optimum value can be identified via the optimization method, or the best approach. The problems of optimization include the search for optimal or minimal value, or the usage of one criterion or multi-objective. Problems that involve more than one target are called multi-objective optimization (MOO). Throughout real life this category of problem is encountered, such as science, architecture, social sciences, banking, forestry, aviation, transportation, among several others [5].

Multi-objective is a decision-making area that is dealing with problems of mathematical optimization, having more than one objective function at the same stage [4].

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Many decision-making models consider a single objective. However, the majority of real-life decision-making issues are more complicated and involve consideration in the decision-making phase with many competing goals [6].

In real-life cases, deterministic optimization problems are often not identified due to imprecise details and uncertain data [7].

In 1965, Zadeh developed the idea of a fuzzy set called an extension of a classical set or crisp set in which each variable has a degree of association or membership. It is the most successful theoretical approach to vagueness. Unlike standard set theory, fuzzy set theory is represented using membership function where the membership value of each entity belongs to the unit interval [0, 1] such that it can be used in a large variety of domains [8].

Afterwards, Atanassove [6] developed the idea of intuitionist fuzzy set to tackle ambiguous and imprecise knowledge, considering both the function of truth and falsity. But also, intuitionistic fuzzy set does not represent human decision-making process. Because the proper decision is fundamentally a problem of arranging and explicate facts, Smarandache proposed the idea of neutrosophic set theory to tackle ambiguous, imprecise and contradictory knowledge [9].

The neutrosophic set is capable of handling many applications in information systems and decision support systems such as relational database systems, semantic web services, and identification of financial data [10].

The membership functions in the neutrosophic set indicate independently: degree of truthmembership, degree of false membership, and degree of indeterminacy membership [10].

Many researchs addressed the bi-level programming problem with multi-objective [2], [11], [12]. In [12], Emam suggested an interactive method for tackling multi-objective fractional programming problems at two stages. He began by locating the convex hull of his original collection of constraints using the cutting-plane algorithm in the first step of the solution method, then the two-level decision-makers using the Charnes and Cooper transformation to turn the fractional objective functions into linear equivalent functions. The algorithm simplifies the similar problem at the second step by turning it into a different multi-objective decision-making problem and then utilizing the e-constraint approach to overcome it. The theoretical findings are further demonstrated with the aid of a numerical proof.

In [5] Gunantara, presented two multi-objective optimization approaches, which do not need complex mathematical calculations. These two approaches are the Scalarization and Pareto. There is a dominated answer in the Pareto process, and a non-dominated solution obtained by a constantly updated algorithm. Meanwhile, utilizing weights, the scalarization approach produces multi-objective structures which are turned into a single objective. Scalarization involves three forms of weights which are equivalent weights, rank order centroid weights, and rank-sum weights. Next, the solution using the Pareto approach is a performance indicator component that forms a different MOO which generates a consensus solution which can be shown optimally in the context of Pareto front, whereas the solution using the scalarization approach is a performance indicator component that forms a scalar structure that is integrated into the fitness function.

Sahidul Islam, *et al.* [7] defined the process of geometric programming of neutrosophic goals and implemented a new approach under uncertainty to solve multi-objective non-linear optimization problems. The proposed approach is defined here as an extension of Fuzzy and intuitive Fuzzy Goal geometric programming methodology in which the degree of approval, degree of indeterminacy and degree of rejection are regarded simultaneously.

A.N. El-Hefnawy proposed is twofold: first, certain essential notions such as neutrosophic set, neutrosophic reasoning, neutrosophic calculation, neutrosophic integral and a single valued neutrosophic system (SVNS). Second, the key component is linked to the neutrosophic applications. There is a lot of use in all fields as for example in computer management, software system and decision support framework, relational database structures, semantic cloud infrastructure, financial data set tracking, development of the digital economy and review of declines.

Abdel-Baset *et al.* [14]implemented the neutrosophic LP models in which their parameters are defined by a trapezoidal neutrosophic number and provided a solution technique. The method described was demonstrated with a few numerical samples and through contrast demonstrates their superiority to the state of the art. The conclusion of this paper claimed that the solution presented is easier, more effective and more capable of solving the LP models relative to other approaches.

This paper is arranged as follows: In Section 2 introduce the problem formulation and some preliminary discussion. In Section 3 discusses a bi-level multi-objective quadratic programming problem with neutrosophic parameters in the objective functions (BLMOQPP) and how we will transfer this problem to bi-level programming problem. In Section 4, An algorithm introduced for solving the BLMOQPP with neutrosophic parameters in the objective functions. In Section 5, We will apply our algorithm on the numerical example. Finally, Section 6 provides the conclusions and suggestions for future works.

2. PROBLEM FORMULATION AND PRELIMINARY DISCUSSION

In this section, we write the formulation of the problem and we recall some important definitions related to neutrosophic set and bi-level multi-object problem.

2.1 Problem Formulation:

The bi-level multi-objective quadratic programming problem with neutrosophic parameter in the objective functions can be represented as follows:

[Upper Level]

$$\underset{\boldsymbol{x_1,x_2}}{\boldsymbol{Max F_1}}(\boldsymbol{x}) = \underset{X_1,X_2}{\text{Max}}(f_{11}(\boldsymbol{x}), \boldsymbol{f_{12}}(\boldsymbol{x}), \ldots, \boldsymbol{f_{1u}}(\boldsymbol{x})) \tag{1}$$

Where x_3, x_4 solves

[Lower Level]

$$\underset{x_3,x_4}{Max} F_2(x) = \underset{X_3,X_4}{\text{Max}}(f_{21}(x), f_{22}(x), \ldots, f_{2n}(x)), \tag{2}$$

Subject to

$$\sum_{j=1}^{n} a_{tj}\, x_j \leq b_t\, ; \tag{3}$$

And where

$$f_{ik} = c_{ik}\, x \quad + \frac{1}{2} x^T L_k^i x \qquad , (i = 1,2), (k = 1,2, \ldots, n_i) \tag{4}$$

Let the functions F_1 and F_2 are quadratic objective functions defined on R^n and (L^1, L^2) are $m \times n$ matrices describing the coefficients of the quadratic terms, c_{ik} are $1 \times m$ matrices and trapezoidal neutrosophic numbers in the above problem (1), (2), (4).

Let x_1, x_2, x_3, x_4 be actual vector variables representing the choice of the first level and the second decision level. In addition, the decision-maker at the upper level has x_1, x_2 indicating the option of first level and the decision maker at lower level has x_3, x_4 indicating level choice.

2.2 Preliminaries

In this section, we write the important definitions and preliminaries for our problem:

Definition 1: For any $(x_1, x_2 \in G_1 = \{x_1, x_2 | (x_1, x_2, x_3, \ldots, x_m) \in G\})$ given by first level, if the decision-making variable $(x_3, x_4 \in G_2 = \{x_3, x_4 | \quad (x_1, x_2, x_3, \ldots, x_m) \in G\})$ is the Pareto optimal solution of the second level, then (x_1, x_2, x_3, x_4) is a feasible solution of (BLMONQPP).

Definition 2: If $x^* \in R^m$ is a feasible solution of the (BLMONQPP), no other feasible solution $x \in G$ exists, such that $F_1(x^*) \leq F_1(x)$ so x^* is the Pareto optimal solution of the (BLMOLSQPP).

Definition 3: [15] Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophicset A in X is defined by a function of membership of fact $\mathrm{T_A}$, a function of membership of indeterminacy $\mathrm{I_A}$ and a function of falsity $\mathrm{F_A}$. $\mathrm{F_A}(x), \mathrm{T_A}(x), \mathrm{I_A}(x)$ are real standard or non-standard subset of $]^-0, {}^+1[$. There is no limitation for the submission of $\mathrm{I_A}(x), \mathrm{T_A}(x), \mathrm{I_A}(x)$.

Definition 4: [9] The trapezoidal neutrosophic number $\tilde{Z}$ is a neutrosophic set in R with the following T, I and F membership functions:

$$T_{\tilde{Z}}(x) = \begin{cases} \propto_{\tilde{Z}} \left(\frac{x-z_1}{z_2-z_1}\right) & (z_1 \leq x \leq z_2) \\ \propto_{\tilde{Z}} & (z \leq x \leq z_3) \\ \propto_{\tilde{Z}} & (z_3 \leq x \leq z_4) \\ 0 & otherwise \end{cases} \tag{5}$$

$$I_{\tilde{Z}}(x) = \begin{cases} \frac{(z_2 - x + \theta_{\tilde{Z}}(x - z_1'))}{(Z_2 - Z'_1)} & (z_1' \leq x \leq z_2) \\ \theta_{\tilde{Z}} & (z_2 \leq x \leq z_3) \\ \frac{(x - z_3 + \theta_{\tilde{Z}}(z_4' - x))}{(z_4' - z_3)} & (z_3 \leq x \leq z_4') \\ 1 & otherwise \end{cases} \tag{6}$$

$$F_{\tilde{Z}}(x) = \begin{cases} \frac{(z_2 - x + \beta_{\tilde{Z}}(X - z_1^{''}))}{(z_2 - z_1^{'})} & (z_1^{''} \le x \le z_2) \\ \beta_{\tilde{Z}} & (z_2 \le x \le z_3) \\ \frac{(x - z_3 + \beta_{\tilde{Z}}(z_4^{''} - x))}{(z_4^{''} - z_3)} & (z_3 \le x \le z_4^{''}) \\ 1 & otherwise \end{cases} \tag{7}$$

where $\propto_{\tilde{Z}}$,$\theta_{\tilde{Z}}$ and $\beta_{\tilde{Z}}$ represent the maximum truthiness degree, minimum indeterminacy degree, and minimum falsity degree, sequentially;$\propto_{\tilde{z}}, \theta_{\tilde{z}}$; and $\beta_{\tilde{z}}$ $\in [0, 1]$. Additionally, $z_1^{''} \le z_1 \le z_1^{'} \le z_2 \le z_3 \le z_4^{'} \le z_4 \le z_4^{''}$.

3. SOLUTION CONCEPTS

In this section we will write our solution strategies to solve our problem.

3.1 Ranking Method:

We will use the ranking method to transform the neutrosophic number that exits in the objective functions to crisp number.

In the following type trapezoidal neutrosophic number was presented [16]:
($\tilde{Z} = z^l, z^{m1}, z^{m2}, z^u;\ T_{\tilde{Z}}, I_{\tilde{Z}}, F_{\tilde{Z}}$) where $\tilde{Z}$ is a trapezoidal neutrosophic number and z^l, z^{m1}, z^{m2}, z^u are the lower bound, first and second median value and upper bound for trapezoidal neutrosophic number, respectively. In addition,$F_{\tilde{Z}}, T_{\tilde{Z}}, I_{\tilde{Z}}$ represent the falsity degrees of the trapezoidal number, truthdegrees of the trapezoidal number and finally the indeterminacy degrees of the trapezoidal number.

In case the objective function or the problemis a case of maximization state, at that point the ranking function for this trapezoidal neutrosophic number can be expressed as the following [16]:

$$R(\tilde{Z}) = |\left(\frac{-\frac{1}{3}(3z^l - 9z^u) + 2(Z^{m1} - Z^{m2})}{2}\right) * (T_{\tilde{Z}} - I_{\tilde{Z}} - F_{\tilde{Z}})| \tag{8}$$

However, if the objective functionor the problem is a case of minimization, the ranking method for such a trapezoid is as following [16]

$$R(\tilde{Z}) = |\left(\frac{(Z^l + Z^u) - 3(Z^{m1} + g^{m2})}{-4}\right) * (T_{\tilde{Z}} - I_{\tilde{Z}} - F_{\tilde{Z}})| \tag{9}$$

In case the author works with a symmetric neutrosophic trapezoidal number that has the following form:
$\tilde{Z} = \langle (z^{m1}, z^{m2}); \alpha, \beta \rangle$, where $\alpha = \beta$ and $\alpha, \beta > 0$, The ranking function will then be described as follows for the neutrosophic number [16].

$$R(\tilde{Z}) = \left(\frac{(Z^{m1} + Z^{m2}) + 2(\alpha + \beta)}{2}\right) * T_{\tilde{Z}} - I_{\tilde{Z}} - F_{\tilde{Z}} \tag{10}$$

3.2: Taylor series approach and weighting method:

We will use 1^{st} order Taylor series polynomial to transform the quadratic objectives function to linear objective function throw the following form [17]:

$$K_i(x) \cong F_i^{\wedge}(x) = F_i(x_i^*) + \sum_{j=1}^{n} (x_j - x_{ij}^*) \frac{\partial F_i(x_j^*)}{dx_j}, (j = 1,2,...m), (i = 1,2) \tag{11}$$

Then we will use the weighting method to transform the multi-objective in the first level and second level to single objective function at each level so the problem transferred from (BLMOQPP) to (BLLPP):

3.3: An Interactive Model for the Bi-Level Linear Programming Problem [12]:

We will use the interactive model [12] to solve the BLLPP. The FLDM presents the appropriate, realistic solutions in rank order to the SLDM, and then the SLDM takes the suitable solutions of the FLDM as a constraint to seeking the solutions and eventually achieving the preferred solution of the FLDM.

Finally, according to the following satisfaction test functions, the FLDM determine the favoured solution of SLDM:

The FLDM determines, by means of the following FLDM satisfaction testing function, whether the proposed solution $x_1^F, x_2^F, x_3^S, x_4^S$ is her preferred and acceptable solution or can be modified:

$$\frac{\|F_1(x_1^F, x_2^F, x_3^F) - F_1(x_1^F, x_2^S, x_3^S))\|_2}{\|F_1(x_1^F, x_2^S, x_3^S))\|_2} < \delta^F \tag{12}$$

If δ^F is a fairly small positive constant specified by the FLDM then $x_1^F, x_2^F, x_3^S, x_4^S$is thefavoured solution of the FLDM, Which mean that $x_1^F, x_2^F, x_3^S, x_4^S$ is the optimal solution for (BLLPP).

4. AN ALGORITHM FOR SOLVING THE BLMONQPP

In the following step series, the algorithm for solving the BLMONQPP with neutrosophic parameters in the objective functions illustrated:

Step-1: BLMOQPP with neutrosophic parameters in the objective functions added by decision makers.

Step-2: In case the BLMONQPP is in the state of maximization, then each neutrosophic parameter in the objective function translated to its corresponding crisp value by equation (8).
But if the objective functions are minimization, then each neutrosophic parameter in the objective function is translated to its corresponding crisp value by equation (9). But in the symmetric trapezoidalcase we will use equation (10).

Step-3: The BLMONQPP is simplified into the equivalent deterministic BLMOQPP.

Step-4: Convert bi level multi-objective quadratic programming to linear by using first order Taylor series approach as bellow:

$$H_i(x) \cong F_i^{\wedge}(x) = F_i(x_i^*) + \sum_{j=1}^{n}(x_j - x_{ij}^*)\frac{\partial F_i(x_i^*)}{dx_j}, (j = 1,2,...n)$$

Step-5: Convert the multi objective problem to single objective by using the weight method.

Step-6: The BLMOQPP simplified into BLLPP.

Step-7: The FLDM finds the individual optimal solution of her problem$x_1^F, x_2^F, x_3^F, x_4^F$.

Step-8: The SLDM defines his problem from the point of view of the FLDM by setting x_1^F, x_2^F to the SLDM constraints.

Step-9: The SLDM finds the optimal solution of her problem $x_1^F, x_2^F, x_3^s, x_4^s$.

Step-10: First Level evaluate the value of δ^Fthen apply the testing function to ensure that the solution is preferred solution for the FLDM.

Step-11: If $\frac{\|F_1(x_1^F,x_2^F,x_3^F)-F_1(x_1^F,x_2^S,x_3^S)\|_2}{\|F_1(x_1^F,x_2^S,x_3^S)\|_2} < \delta^F$, then proceed to step 12; otherwise, proceed to step 7.

Step-12: So, ($x_1^F, x_2^F, x_3^s, x_4^s$.) is the compromised solution for the BLMONQPP

5. NUMERICAL EXAMPLE

In this section, we solve a Bi-level Multi-objective Quadratic Programming Problem (BLMOQPP) with trapezoidal-neutrosophic numbers in the objective function:

[First Level]

$$\max_{X_1,X_2} F_1 \approx ((12,14,2,2)X_1 + (14,15,3,3)X_2^2 + (4,6,1,1)X_4, (4,6,2,2)X_1^2 + (5,7,3,3)X_2^2 + (6,8,2,2)X_3)$$

Where x_3, x_4 solves

[Second Level]

$$\max_{X_3,X_4} F_2 \approx ((6,8,1,1)X_1 + (4,6,2,2)X_3^2 + (10,12,4,4)X_4^2 , (4,8,3,3)X_2 + (5,7,3,3)X_3^2 + (7,9,2,2)X_4^2)$$

Subject to

$$x_1 + x_2 + x_3 + x_4 \le 80$$
$$6x_1 + 4x_2 \le 60$$
$$x_3 + x_4 \le 20$$
$$x_1, x_2, x_3, x_4 \ge 0$$

The order of element for trapezoidal neutrosophic numbers (L, U,α, β) is as follows: lower bound, upper bound,first median value, second median value.

We will convert the neutrosophic number to the crisp number using equation (10) because our neutrosophic number are symmetric.

Now first level and second level transformed from Neutrosophic bi-level multi-objective quadratic programing problem (BLMONQPP) to crisp bi-level multi-objective quadratic programing problem (BLMOQPP) so the crisp model of previous problem will be as follows:

[First Level]

$$\max_{X_1,X_2} F_1 = 6.8\,X_1 + 8.2\,X_2^2 + 2.8X_4 \ , \quad 3.6\,X_1^2 + 4.8X_2^2 + 4.4X_3$$

Where x_3, x_4 solves

[Second Level]

$$\max_{X_3,X_4} F_2 = 3.6\,X_1 + 3.6\,X_3^2 + 8X_4^2 \ , \quad 4.8X_2 + 4.8\,X_3^2 + 4.8X_4^2$$

Subject to

$x_1 + x_2 + x_3 + x_4 \leq 80$
$6x_1 + 4x_2 \leq 60$
$x_3 + x_4 \leq 20$
$x_1, x_2, x_3, x_4 \geq 0$

Then we use the first order Taylor series and weight method respectively to transfer the quadratic multi objective quadratic function to linear single objective function so the BLLPP written as following:

[First Level]

$$\underset{x_1,x_2}{\text{Max}} F_1(x) = \underset{x_1,x_2}{\text{Max}}(3.4x_1 + 195x_2 + 2.2x_3 + 1.4x_4 - 1462.5)$$

Where x_3, x_4 solves

[Second Level]

$$\underset{x_3,x_4}{\text{Max}} F_2(x) = \underset{x_3,x_4}{\text{Max}}(1.8x_1 + 2.4x_2 + 96x_3 + 160x_4 - 2560)$$

Subject to

$x_1 + x_2 + x_3 + x_4 \leq 80,$
$6x_1 + 4x_2 \leq 60,$
$x_3 + x_4 \leq 20,$
$x_1, x_2, x_3, x_4 \geq 0$

Now FLDM and SLDM will solve his problem individually by interactive approach as following:

a) FLDM Find Solve his/her problem against the constrain and find individual optimal solution of his problem as follow:

   [First Level]

   $$\underset{x_1,x_2}{\text{Max}} F_1(x) = \underset{x_1,x_2}{\text{Max}}(3.4x_1 + 195x_2 + 2.2x_3 + 1.4x_4 - 1462.5)$$

   Subject to

   $x_1 + x_2 + x_3 + x_4 \leq 80,$
   $6x_1 + 4x_2 \leq 60,$
   $x_3 + x_4 \leq 20,$
   $x_1, x_2, x_3, x_4 \geq 0$

   So the FLDM solution is $(x_1^F, x_2^F, x_3^F x_4^F)$ = (0, 15, 20, 0) and $F_1 = 1506.5$ and $\delta^F = 0.1$ Given by Upper Level/First Level decision maker.

b) SLDM Solve his/her problem against constrain and under point of FLDM view so SLDM will take values of x_1^F, x_2^F as constrain beside the original problem constrains. Now SLDM has six constrains so the new SLDM problem became as following:

   [Second Level]

   $$\underset{x_3,x_4}{\text{Max}} F_2(x) = \underset{x_3,x_4}{\text{Max}}(1.8x_1 + 2.4x_2 + 96x_3 + 160x_4 - 2560)$$

   Subject to

   $x_1 + x_2 + x_3 + x_4 \leq 80,$
   $6x_1 + 4x_2 \leq 60,$
   $x_3 + x_4 \leq 20,$
   $x_1 = 0,$
   $x_2 = 15,$
   $x_1, x_2, x_3, x_4 \geq 0$

   Therefore, SLDM solve his/her problem and the results as follows: $(x_1^F, x_2^F, x_3^S, x_4^S)$ = (0, 15, 20, 0) and $F_2 = 676$

c) In the last step we will use the below first level test function to be ensure that the solution is acceptable for DM:

So $\frac{\|F_1(0,15,20,0)-F_1(0,15,20,0)\|_2}{\|F_1(0,15,20,0)\|_2} = 0$, Therefore the result of the first level test function < .1,

So $(x_1^F, x_2^F, x_3^S, x_4^S)$ = (0, 15, 20, 0) is acceptable solution to the FLDM/Upper Level.

6. CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

This paper introduces a solution algorithm for solving the BLMOQPP with neutrosophic parameters in the objective functions. To minimize the difficulty of the problem, in the first step of the solution algorithm the neutrosophic nature of the problem converted into its corresponding crisp model. In the second step, we used the first order Taylor sequence to transform the quadratic problem type to linear programming problem, then we used the weight approach to convert the multi- function that occurs in each level to a single objective function and in the last step an interactive algorithm is used to arrive at a consensus solution for the BLLPP. Finally, a numerical explanation for the accuracy of the suggested solution algorithm is given.

However, a variety of topics remain subject to potential discussion and can explored by regular, bi-level neutrosophic optimization:

1. Bi-level quadratic large-scale decision-making problems with neutrosophic parameters in both objective functions and constraints.
2. Bi-level quadratic large-scale multi-objective decision-making problems with neutrosophic parameters in both objective functions and constraints with integrity conditions.

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