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CONNECTEDNESS AND PATH CONNECTEDNESS ON SPACE-TIME TOPOLOGICAL MANIFOLD

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ABSTRACT

We conduct a study on R^4 which is a space- time Topological Manifold. The properties of R^4 like time-path, path-connectedness, sequential connected, weakly connected, irreversibility sequential connected with respect to time, we also study densely connectivity of R^4. Theexistence of homeomorphism between $U\subseteq R^4$-a space time manifold and a Toplogical Manifold T(M) provides a sound mathematical base for the study path connectedness, corona virus in the world manifold.

Keywords: *Topological Manifold, Connectedness, path- connectedness, irreversibility of sequence, time-path, densely connectedness.*

1. INTRODUCTION

We discuss the topological property of R^4, like connectedness, path-connectedness, sequentially connected, irreversibly sequentially connected with respect to time.

In this paper we defined the composition of maps from [0, 1] to M that is $[0\ 1] \rightarrow R^4 \rightarrow M$.

$\Phi o\gamma$

By the properties of R^4 and a continuous map ϕ from $R^4 \rightarrow M$ gives M and also satisfies the properties of R^4 as weakly connected, connectedness, path connectivity, maximal connectedness [4][5].

On complete measure manifold we already introduced the space-time measure manifold [9].

In the last section we defined densely connectedness properties of topological space M which is manifold. This Theorem implies that M is densely (strongly) connected.

We discussed the time-path in R^4 with respect to Causal structure or condition which is $t_1 < t_2$

2. SOME BASIC DEFINITIONS

Definition 2.1: Topological Manifold [13][12][7][8]
An n-dimensional manifold is a topological space M, such that for any $x \in M$, there exists a neighborhood $U \subseteq M$ of x and an open set $V \subseteq R^n$ such that U and V are homeomorphic.

We denote the homeomorphism by $\Phi: U \rightarrow V$ (continuous and bijective)

Core Intuition 1: A manifold is a topological space that locally resembles R^n.

Core Intuition 2: A topological space M is a topological manifold of dimension 'n' which is locally Euclidean i.e., every point has an open neighborhood homeomorphic with an open subset R^n.

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Definition 2.2: Connectedness [12]
A topological space M is said to be connected if the only subsets of M that are both open and closed are itself and the empty set. R^4 be the vector space which always connected with respect to 't'.

Therefore R^4 is connected topological vector space.

Definition 2.3: Path Connected
A topological space M is path connected if given any two points x, y ϵ M, there exists a map γ: [0, 1] →M such that $\gamma[0] = x$, $\gamma(1) = y$. We say that a map a path from x to y.

Definition 2.4: Locally connected
A topological space M is locally connected if every x ϵ M has a connected neighborhood.
A manifold M is locally path connected if every x ϵ M has a path connected neighborhood.

2.5 Some properties

1) If M and N are topological manifold of same dimension and there exists a continuous function from M to N and if M is connected then f(M) C N is connected.
2) Every path connected space is connected.
3) Let X be an open subset of R^n. If X is connected then X is path connected.
4) The product of two path connected space is path connected.
5) Every metric space is path connected.
6) Each curve on topological manifold is path connected.
7) The product of continuous maps is continuous and the product of homeomorphism is a homeomorphism.
8) Every connected and locally path connected is path connected.
9) Every locally connected space need not be locally path connected.

In next section we use word connectedness mean path connectedness*

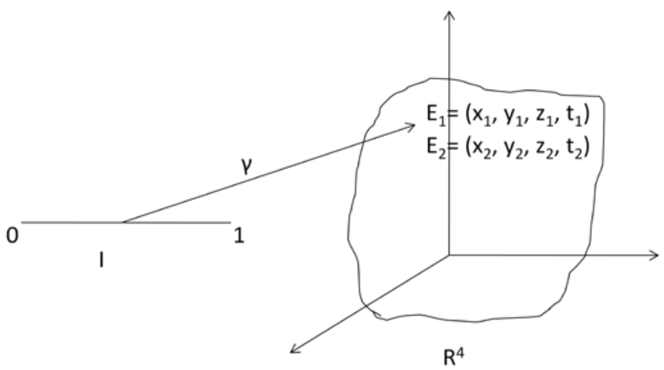
3. CONNECTEDNESS AND PATH CONNECTEDNESS

Let M be a topological of manifold of dimension R^4 be topological vector space of dimension 4 which represents real world. The elements of R^4 are events of the form (x, y, z, t). The (x, y, z) are the Cartesian coordinators and 4^{th} coordinate is time component which is continuous.

Definition 3.1 Connectedness in R^4
Let R^4 be a topological vector space
Event E_1, E_2 ϵ R^4 are events in R^4. The space of R^4 is said to be connected if there exists a γ: (0, 1) → R^4 such that

$\gamma(0) = E_1 = (x_1, y_1, z_1, t_1)$
$\gamma(1) = E_2 = (x_2, y_2, z_2, t_2)$
with $t_1 < t_2$.

Remark:- Real world is space- time Manifold of Dimension -4, all countries are connected by paths in all ways ,on road, air, water.



Definition 3.2: Sequentially connected
Let R^4 is a non-empty set of events which is a topological vector space R^4 is said to be sequentially connected if there exits a map γ: [0, 1] → R^4 such that

$\gamma(0) = E_1 = (x_1, y_1, z_1, t_1)$
$\gamma(\frac{k}{2^n}) = E_i = (x_i, y_i, z_i, t_i)$, $\frac{k}{2^n}$ ϵ [0,1]
$\gamma(1) = E_n = (x_n, y_n, z_n, t_n)$
with $k \leq 2^n$, I<i<n, nϵN
also $t_1 < t_i < t_n$ ∀ i<nϵN

Definition 3.3: Let R^4 is a topological vector space is said to be irreversibly sequential connected with respect to time if $E_1, E_2, E_3, \epsilon R^4$ then E_1, E_2, E_3 with respect to times t_1, t_2, t_3

Definition 3.4: Time-path in R^4

Let R^4 be a topological vector space with dimension 4 is E_1 & $E_2 \epsilon R^4$ there exists a path γ: $[0,1] \rightarrow R^4$ such that

$\gamma(0) = t_i$ $\gamma(1) = t_j$ $\forall$ $t_i < t_j$

with $E_i = (x_i, y_i, z_i, t_i)$,

$E_j = (x_j, y_j, z_j, t_j)$

such path γ is called time path ,

Any two events are connected by a time path.

Definition 3.5: Weakly Path Connected

Let R^4 be a topological vector space with E_i and $E_j \epsilon R^4$ for some i, j ϵ N, is said to be a weakly connected if there exists a path $\gamma:[0,1] \rightarrow R^4$ such that any two events like $E_i = (x_i, y_i, z_i, t_i)$ and $E_j = (x_i, y_i, z_i, t_j)$ are connected by a time path.

Definition 3.6: Maximally path connected

A topological space R^4 is said to be maximally path connected if there exists a path γ: $[0, 1] \rightarrow R^4$ such that some events $E_i^{'}$ are connected by time path $\forall i \epsilon N$

That is some events of R^4 are internally connected by time paths, each E_i is connected with more than two events $E_j \epsilon R^4$ j=1, 2, 3

Definition 3.7: Densely connected R^4

A topological space R^4 is said to be densely connected if all events E_i of R^4 are connected with all other events E_j by a time path $\gamma:[0,1] \rightarrow R^4$ with

$\gamma(0) = E_1, \gamma(1) = E_n$

$\gamma(\frac{k}{2^n}) = E_k$

for $k \leq 2^n$, $E_1 < E_k < E_n$, and $t_1 < t_k < t_n$

Theorem 3.8: If R^4 space is irreversibly sequential connected with respected to time and a continuous bijective map ϕ: $R^4 \rightarrow M$ then M is irreversibly sequentially connected.

Proof: Let R^4 be a topological space which is irreversibly sequentially connected. Therefore events $E_1, E_2, E_3 \epsilon R^4$ with $E_1 < E_2 < E_3$ with $t_1 < t_2 < t_3$

Therefore $\phi(E_1) = x, \epsilon M, x = (x^1, x^2, \ldots x^n)$

$\phi(E_2) = y, \epsilon M, x = (y^1, y^2, \ldots y^n)$

$\phi(E_3) = z, \epsilon M, x = (z^1, z^2, \ldots z^n)$

As R^4 is irreversibly sequentially connected

Implies $\phi(E_1 < E_2 < E_3) = \phi(E_1) < \phi(E_2) < \phi(E_3)$

$= x < y < z$

Therefore $x < y < z$ for $t_1 < t_2 < t_3$

Therefore x, y, z ϵ M is connected as partial order of x, y, z which M is manifold which is connected sequentially.

Theorem 3.9: Let R^4 be a topological vector space, which is weakly connected and a continous bijective map ϕ: $R^4 \rightarrow M$ then M is also weakly connected.

Proof: Let R^4 be a topological vector space and R^4 when R^4 is weakly connected. Let E_1 and $E_2 \epsilon R^4$ are two events of R^4, then there exists a path γ: $[0, 1] \rightarrow R^4$ Such that

$\gamma(0) = E_1 = (x_1, y_1, z_1, t_1)$

$\gamma(1) = E_2 = (x_2, y_2, z_2, t_2)$

There exists only one time path between E_1 and E_2 with respect to time.

Also ϕ: $R^4 \rightarrow M$ which is homeomorphism (continuous and bijective)

Each element $\phi(E_1) = x$, $\phi(E_2) = y$

For $x = (x^1, x^2, \ldots x^n)$

$y = (y^1, y^2, \ldots y^n)$

Consider a composition map ϕ o γ as

$$[0,1] \xrightarrow{\gamma} R^4 \xrightarrow{\phi} M$$

Φoγ

$$\Phi o\gamma(0) = \phi[\gamma(0)] = \phi[E_1] = x$$
$$\Phi o\gamma(1) = \phi[\gamma(1)] = \phi[E_2] = y$$

This implies $\phi o\gamma(0) = x$
$\Phi o\gamma(1) = y$

This shows a path from x to y in M.

Therefore M is path connected as γ is time path in R^4. Φoγ is composite in M which is continuous and bijective.

Therefore M is weakly connected.

Theorem 3.10: If R^4 is maximally connected with respect to time and a continuous map ϕ: $R^4 \rightarrow M$ is bijective then M is maximally connected.

Proof: Let R^4 be maximally connected vector space this implies there exists a path γ: $[0, 1] \rightarrow R^4$ such that for some events E_i's with respect to Casual condition $t_1 < t_2$ (with respect to time)

Therefore the paths γ: $[0, 1] \rightarrow R^4$ which is continuous and bijective with

$$\gamma(0) = E_1 = (x_1, y_1, z_1, t_1)$$
$$\gamma(\tfrac{1}{2^n}) = E_j = (x_i, y_i, z_i, t_i)$$
$$\gamma(1) = E_n = (x_n, y_n, z_n, t_n)$$

for some $E_j \epsilon R^n$ with $t_1 < t_i \epsilon N \epsilon t_n$ that is each point ϵ[0, 1] has value (events) in R^4

Also there exists a continuous and bijective function (homeomorphic) ϕ: $R^4 \rightarrow M$ such that

$$\phi(E_1) = x_1, \phi(E_i) = x_i$$
$$\phi(E_n) = x_n$$

for n-tuples of $x_1 = (x_1^1, x_1^2 \ldots x_1^n)$
$x_i = (x_i^1, x_i^2 \ldots x_i^n)$
$x_n = (x_n^1, x_n^2 \ldots x_n^n)$

Therefore new consider a composition map ϕoγ as $[0, 1] \xrightarrow{\gamma} R^4 \xrightarrow{\phi} M$

Φoγ

$$\Phi o\gamma(0) = \phi\gamma[0] = \phi(E_1) = x$$
$$\Phi o\gamma(\tfrac{1}{2^i}) = \phi[\gamma\{\tfrac{1}{2^i}\} = \phi(E_i) = x_i \qquad k_i < n\epsilon N$$
$$\Phi o\gamma(1) = \phi\gamma[1] = \phi(E_n) = x_n$$

Therefore E, E_i, $E_n \epsilon R^4$, x, x_i, $x_n \epsilon M$

Therefore the composition map ϕoγ is also continous and bijective.

$$\Phi o\gamma(0) = x, \phi o\gamma(\tfrac{1}{2^i}) = x_i, \phi o\gamma(1) = x_n$$

Shows that there exists a path between any point within $x_i \epsilon M$

Therefore this shows M is manually path connected manifold

Therefore hence if R^4 is maximally connected then M is also maximally connected under a continuous map.

Theorem 3.11: If R^4 is a densely connected topological manifold space, with a continuous and bijective map then M is also densely (strongly) connected.

Proof: Let R^4 be a densely connected topological vector space.

All events E_i of R^4 are connected with all other events E_j bt space time path. γ: $[0, 1] \rightarrow R^4$ is homeomorphism with

$$\gamma(0) = E_1 \ \forall \ I < k < n\epsilon N$$
$$\gamma(\tfrac{k}{2^n}) = E_k \text{ for } k \leq 2^n \text{ and } E_1 < E_k < E_n \text{ and } t_1 < t_k < t_n$$

Also there exists a continuous bijective

Therefore map $\phi: R^4 \rightarrow M$ each events of R^4 is n-tuples

$\Phi(E_1) = x_1 \quad \phi(E_k) = x_k, \quad \phi(x_n) = x_n$

$1 \leq k < n$

Where $x_1 = (x_1^1, x_1^2 \ldots x_1^n)$

$x_k = (x_k^1, x_k^2 \ldots x_k^n) \quad k < n$

$x_n = (x_n^1, x_n^2 \ldots x_n^n)$

consider a composition map ϕoγ

$[0, 1] \overset{\gamma}{\rightarrow} R^4 \overset{\phi}{\rightarrow} M$

Φoγ

which is also homeomorphism

$\phi o\gamma[0] = \phi[\gamma(0)] = \phi(E_1) = x_1 = (x_1^1, x_1^2 \ldots x_1^n)$

$\phi o\gamma[\frac{k}{2^n}] = \phi[\gamma(\frac{k}{2^n})] = \phi(E_k) = x_k = (x_k^1, x_k^2 \ldots x_k^n)$

$\phi o\gamma[1] = \phi[\gamma(1)] = \phi(E_n) = x_n = (x_n^1, x_n^2 \ldots x_n^n)$

Therefore each point of M connected path.

Therefore there exists a path from each point to all other points ϵM

Therefore this multiple connected paths in M is a dense path which is connected.

Therefore M is densely path connected space.

Remark:

1) Real world is space- time Manifold of Dimension -4, all countries are connected by paths in all ways, on road, air, water due to all such types of connectivity spreads virus **like corona (covid-19)**
2) To prevents such virus we have to use cut points and punctured points [5] as disconnect the real world manifold. To maintain social distance, Lock down, corotine, Home corontine, institute corontine are also forms punctured space in the real world.
3) The Role of lock down and curfew in society plays an important role in society as punctured points in real world manifold. (details in next section/paper).
4) If all paths are disconnects from country to country, state to state, district to district, village to village, street to street, and Home to Home , which will reduces the corona virus spread lot.
5) Also as per holy faith vedic tradition records of aurveda burnning Homa, yadnya, /(YAGYA) , creates puncture space which was non virus space, wherever jwala has reached all types of virus dies space becomes holy.

CONCLUSION

Real world is space- time Manifold of Dimension -4, all countries are connected by paths in all ways, on road, air, water due to all such types of connectivity spreads virus **like corona (covid-19)**. prevents such virus we have to use cut points and punctured points [5] as disconnect the real world manifold. To maintain social distance, Lock down, quarantine, Home quarantine, institute quarantine are also forms punctured space in the real world. The Role of lock down and curfew in society plays an important role in society as punctured points in real world manifold.(details in next section/paper). If all paths are disconnects from country to country, state to state, district to district, village to village, street to street, and Home to Home , which will reduces the corona virus spread lot.

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