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C_n-E- SUPER MAGIC LABELING OF SOME GRAPHS

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ABSTRACT

A *graph G possess an H-covering when each edge in E(G) belongs to a subgraph of G isomorphic to H. This graph G is H-magic if there exists a total labelling $f: V(G) \cup E(G) \to \{1,2, \dots p+q\}$ such that for each subgraph H' of G isomorphic to H, $\sum_{v \in V(H')} f(v) + \sum_{e \in E(H')} f(e) = k$ is a constant. An H-E super magic labelling is a bijection $f: V(G) \cup E(G) \to \{1,2, \dots p+q\}$ with $f(E(G)) = \{1,2, \dots q\}$. In this article we explore C_n-E-super magic labeling of some graphs.*

Keywords: *H-covering, H- magic labeling, H-E magic labeling, C_n-E-super magic labeling.*

AMS subject Classification Code: *05C78.*

1. INTRODUCTION

In this whole article we discuss barely finite, simple and undirected graphs. A graph G with vertex set $V(G)$ and the edge set $E(G)$ such that $|V(G)| = p$ and $|E(G)| = q$. A labeling of a graph G is a map that takes graph elements to set of numbers, normally integers.

Several kinds of labeling have been studied and explained by numerous authors and an eminent survey of graph labelings is found in [1]. In 1963, Sedlàček[2] introduced the conception of magic labeling in graphs. A graph G is *magic* if the edges of G can be labelled with a set of numbers$\{1,2, \dots q\}$ so that the summation of labels of all the edges incident with any vertex is the same [3].

A covering of G is a family of subgraphs $H_1, H_2, .. H_h$ such that each edge of E(G) belongs to atleast one of the subgraphs H_i,$1 \le i \le h$. Then,it is known that G admits an$(H_1, H_2, \dots .. H_h)$ covering. When each H_i is isomorphic to the graph H, then G possess an H-covering. Assume G admits an H-covering. A total labelling $f: V(G) \cup E(G) \to \{1,2, \dots |V(G)| + |E(G)|\}$ is known an H-magic labeling of G if there exists a positive integer M (termed the magic constant) such that for every subgraph H' of G isomorphic to H, $\sum_{v \in V(H')} f(v) + \sum_{e \in E(H')} f(e) = M$. A graph which admits such a labeling is known as H-magic. The function f is termed to be H-E super magic labeling if $f(E(G)) = \{1,2, \dots q\}$[6]

The concept of H-magic labeling was introduced by Gutierrez and Llado [4] in 2005.In 2007, Llado and Morgas [5] studied some C_n- supermagic graphs. In 2010, A.A.G Ngurah, A.N.M Salman and I.W.Sudarsana studied H-supermagic labelings of two category of connected graph specifically fans and ladder [7].

Herein, we examine C_n-E-super magic labeling of some graphs.

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2. C_n-E-SUPER MAGIC LABELING OF GRAPHS

In this section we present the C_n-E-super magic labeling of some families of graphs with their super magic sums.

Theorem 2.1: The Wheel W_n for $n \geq 5$ odd, is C_3- E-super magic.

Proof: Denote the vertices of n-cycle of the wheel W_n as $U_1, U_2, \dots . U_n$ and its central vertex by r.

Construct a total labeling ℓ of W_n on $[1,3n+1]$. Set $\ell(rU_1) = 1$

$$\ell(e) = \begin{cases} (n+2) - i & if\ e = rU_i\,,\quad 2 \leq i \leq n \\ (n+i) & if\ e = U_iU_{i+1}\,,\ \ 1 \leq i \leq n-1 \\ 2n & if\ e = U_iU_1,\quad i = n \end{cases}$$

$$\ell(v) = \begin{cases} 2n+1 & if\ v = r \\ 3n+1 & if\ v = U_1 \\ \frac{(4n+1)+i}{2} & if\ v = U_i\,, i\ is\ odd, i > 1 \\ \frac{(5n+1)+i}{2} & if\ v = U_i\,, i\ is\ even \end{cases}$$

For $1 \leq i \leq n$, let $C_n^{(i)}$ be the 3- cycle of the graph W_n with

$$V\left(C_n^{(i)}\right) = \{U_i: 1 \leq i \leq n\} \cup \{r\}\,, E\left(C_n^{(i)}\right) = \{U_iU_{i+1}: 1 \leq i \leq n\} \cup \{rU_i\}$$

Then

$$k = \sum_{v \in V\left(C_3^{(j)}\right)} \ell(v) + \sum_{e \in E\left(C_3^{(j)}\right)} \ell(e) = [\ell(r) + \ell(U_i)] + [\ell(rU_i) + \ell(U_iU_{i+1})]$$

$$k = \frac{19n+11}{2}$$

Hence ℓ is a $C_3 - E -$ super magic labeling.

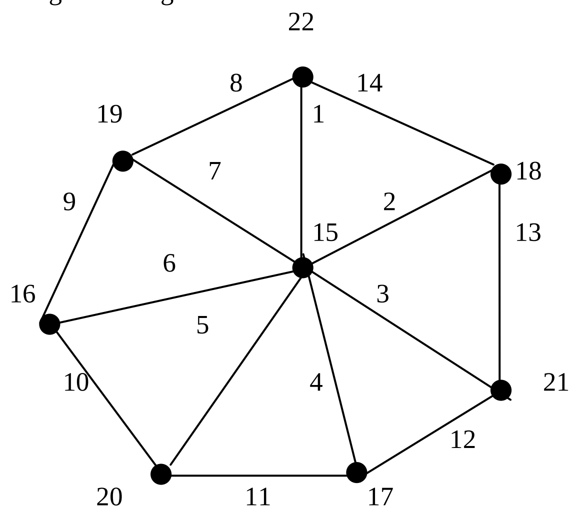

Figure-1: C_3-E- Super magic labeling of W_7, k=72

The Ladder graph L_n is defined as cartesian product $P_2 \times P_n$ of a path on 2 vertices and another path on n vertices.

Theorem 2.2: The Ladder graph $L_n = P_2 \times P_n$ is C_4-E-super magic for all n.

Proof: The vertex set and the edge set of the ladder graph is given as,

$$V = \left\{a_{1.j}, a_{2.j}: 1 \leq j \leq n\right\} \text{ and } E = \left\{a_{1.j}\, a_{1.j+1}, a_{2.j}\, a_{2.j+1}: 1 \leq j \leq n-1\right\} \cup \left\{a_{1.j}\, a_{2.j}: 1 \leq j \leq n\right\}$$

Let $C_n^{(j)}$ be the 4-cycle $a_{1.j}\, a_{1.j+1} a_{2.j+1} a_{2.j}$ for $1 \leq j \leq n-1$.

Construct a total labeling $\ell: V \cup E \rightarrow [1,5n-2]$ as,

$\ell\left(a_{1,j}\right) = 2n + j + 3$ and $\ell\left(a_{2,j}\right) = 5n - j - 1$ for $1 \leq j \leq n$

$\ell\left(a_{1,j}\, a_{2,j}\right) = j$ for $1 \leq j \leq n$

$\ell\left(a_{1,j}\, a_{1,j+1}\right) = 2n - j$ and

$\ell\left(a_{2,j}\, a_{2,j+1}\right) = 3n - j - 1$ for $1 \leq j \leq n$

For $1 \le j \le n-1$
We have

$$k = \sum_{v \in V\left(C_3^{(j)}\right)} \ell(v) + \sum_{e \in E\left(C_3^{(j)}\right)} \ell(e) = \ell(a_{1,j}) + \ell(a_{1,j+1}) + \ell(a_{2,j+1}) + \ell(a_{2,j}) + \ell(a_{1,j}a_{2,j}) + \ell(a_{2,j}a_{2,j+1}) + \ell(a_{2,j+1}a_{1.j+1}) + \ell(a_{1,j+1}a_{1,j})$$

$$k = 19\text{n} + 4$$

Hence ℓ is a $C_4 - E -$ super magic labeling.

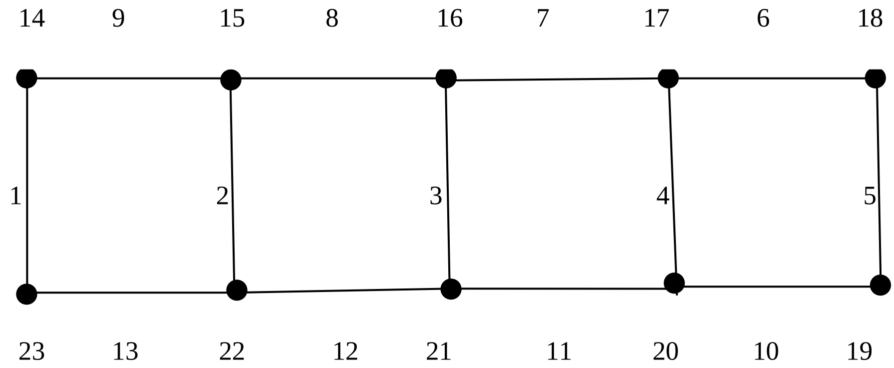

Figure-2: C_4-E-Super magic labeling of ladder $P_2 \times P_5$, k = 99

The Triangular Ladder $L_n, n \ge 2$ is a graph obtained by finishing the ladder $L_n = P_2 \times P_n$ by adding the edges $a_{1,j}a_{2,j+1}$ for $1 \le j \le n-1$.

Theorem 2.3: The Triangular Ladder L_n ,$n \ge 2$ is C_3-E-Super magic.

Proof: The vertex set and the edge set of the Triangular ladder L_n are given by,

$$V = \{a_{1.j}, a_{2.j} : 1 \le j \le n\} \text{ and}$$

$$E = \{a_{1.j}a_{1.j+1}, a_{2.j}a_{2.j+1} : 1 \le j \le n-1\} \cup \{a_{1.j}a_{2.j} : 1 \le j \le n\} \cup \{a_{1,j}a_{2,j+1} : 1 \le j \le n-1\}$$

Further $|V| = n$ and $|E| = 4n - 3$

Let $C_n^{(j)}$ be the 3-cycle $a_{1.j}a_{1.j+1}a_{2.j+1}$ for $1 \le j \le n-1$.

Construct a total labeling $\ell: V \cup E \to [1, 6n-3]$ as,

$\ell(a_{1,j}) = 6n - 2j - 1$ and $\ell(a_{2,j}) = 6n - 2j - 2$ for $1 \le j \le n$

$\ell(a_{1,j}\, a_{2,j}) = 2j - 1$ for $1 \le j \le n$

$\ell(a_{1,j}\, a_{1,j+1}) = 2n + 2j - 2$ for $1 \le j \le n-1$

$\ell(a_{2,j}\, a_{2,j+1}) = (2n+1) + 2j - 2$ for $1 \le j \le n-1$

$\ell(a_{1,j}\, a_{2,j+1}) = 2j$ for $1 \le j \le n-1$

For $1 \le j \le n-1$
We have

$$k = \sum_{v \in V\left(C_n^{(j)}\right)} \ell(v) + \sum_{e \in E\left(C_n^{(j)}\right)} \ell(e) = \ell(a_{1,j}) + \ell(a_{1,j+1}) + \ell(a_{2,j+1}) + \ell(a_{1,j}a_{1,j+1}) + \ell(a_{1,j+1}a_{2,j+1}) + \ell(a_{2,j+1}a_{1.j+1})$$

$$\text{k} = 20\text{n} - 9$$

Hence ℓ is a $C_3 - E -$ super magic labeling.

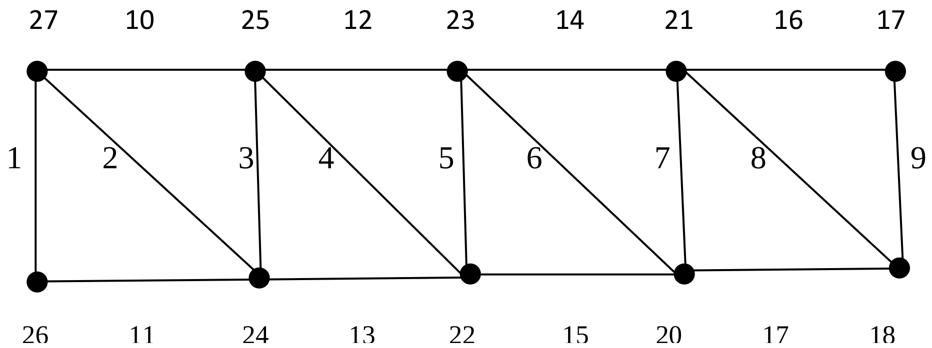

Figure-3: C_3-E-Super magic labeling of triangular ladder L_5, k=91

Triangular snake graph also exhibits C_n-E-super magic labeling.

Theorem 2.4: The Triangular snake graph Δ_n is C_3-E-Super magic for all $n \geq 2$

Proof: Let V be the vertex set and E be the edge set of the Triangular snake graph Δ_n .

$V = \{ub_i : i = 1, .. n+1\} \cup \{u_i : i = 1, \ldots n\}$,

$E = \{e_{1i} : e_{1i} = ub_i u_i : i = 1, .. n\} \cup \{e_{2i} : e_{2i} = u_i ub_{i+1} : i = 1, \ldots n\} \cup \{e_{3i} : e_{3i} = ub_i ub_{i+1} : i = 1, \ldots n\}$

where ub_i are the base vertices and e_{3i} are the base edges.

Consider a total labeling $\ell : V \cup E \to [1, 6n+1]$ as follows:

$\ell(ub_i) = 4n + 2 - i$ for $1 \leq i \leq n+1$

$\ell(u_i) = 4n + 1 + i$ for $1 \leq i \leq n$

$\ell(e_{1i}) = 3n + 1 - i$ for $1 \leq i \leq n$

$\ell(e_{2i}) = n + i$ for $1 \leq i \leq n$

$\ell(e_{3i}) = i$ for $1 \leq i \leq n$

For $1 \leq i \leq n$ and let $C_3^{(i)}$ be the 3-cycle. Now we have,

$k = \sum_{v \in V\left(C_3^{(j)}\right)} \ell(v) + \sum_{e \in E\left(C_3^{(j)}\right)} \ell(e) = \ell(ub_i) + \ell(ub_{i+1}) + \ell(u_i) + \ell(e_{1i}) + \ell(e_{2i}) + \ell(e_{3i})$

$k = 16n + 5$

Hence ℓ is a $C_3 - E -$ super magic labeling.

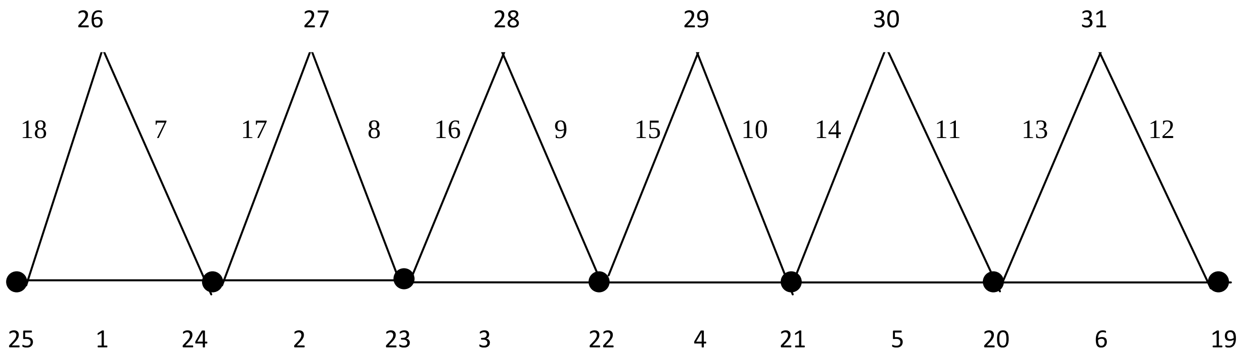

Figure-4: C_3-E-Super magic labeling of triangular snake graph Δ_6, k=101

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