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PYTHAGOREAN THEOREM: A CASE STUDY OF APPLICATION OF PARALLELOGRAM THEOREM

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ABSTRACT

The relationship among base, height and hypotenuse of right handed triangle leads to develop an important formula in mathematics and it is called Pythagoras theorem. There are different types of mathematical methods in geometric and algebraic which will prove this theorem. Proof using similar triangles and Euclid's proof are most important methods used to prove the theorem. In this paper it is tried to show the importance and use of parallelogram theory to develop the Pythagoras theorem.

Keywords: *triangle, parallelogram, Pythagoras.*

1. INTRODUCTION

In mathematics, the **Pythagorean Theorem**, also known as **Pythagoras' theorem**, is a fundamental relation in Euclidean geometry among the three sides of a right triangle. It states that the area of the square whose side is the hypotenuse (the side opposite the right angle) is equal to the sum of the areas of the squares on the other two sides. In secondary level of education, concept of Pythagoras theorem is introduced. This theorem was introduced about 2000 years ago. The beauty of the concept is "When a triangle has a right angle (90°) ...... and squares are made on each of the three sides, then the biggest square has the exact same area as the other two squares put together!"

There exist different types of proof to explain this theorem. Generally, this theorem is proved by using the properties of similarity of triangle. In this paper there is nothing new but it is tried to show the importance of algebraic properties like $(a + b)^2$ (where a and b are variable) and geometry properties of parallelogram which were used to make concept clear for the beginners to understand Pythagoras theorem. Keeping in mind that triangle is right angle triangle, in this paper it is tried to explain the combination of algebraic formula $(a + b)^2$ (where a and b are variable) and geometric properties of parallelogram for the explanation of Pythagoras' theorem.

2. PARALLELOGRAM THEOREM AND ITS APPLICATION IN PYTHAGORAS THEOREM

2.1 *Pythagoras Theorem*

The Pythagorean theorem gives a fundamental relationship between the lengths of the three sides of a right-angled triangle. There are different methods for proof of this mathematical formula.

In a right angle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

It is called "Pythagoras' Theorem" and can be written in one short equation:

$$a^2 + b^2 = c^2$$

Note:

- **c** is the **longest side** of the triangle
- **a** and **b** are the other two sides

The longest side of the triangle is called the "hypotenuse", so the formal definition is -"In a right angled triangle :the square of the hypotenuse is equal to the sum of the squares of the other two sides ".

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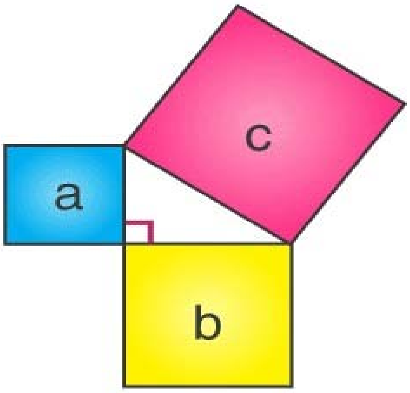

Figure-1: Right angle triangle having length a units, b units, c units as shown in the figure.

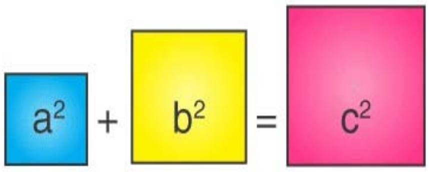

Figure-2: Square having length a units, b units, c units as shown in the figure.

2.2 Parallelogram related theorem

2.2.1 Parallelogram:
A parallelogram is a quadrilateral with both pairs of opposite sides parallel

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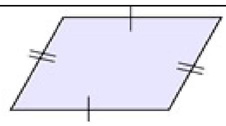
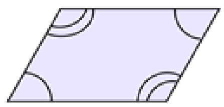
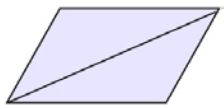
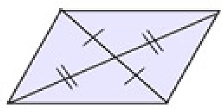
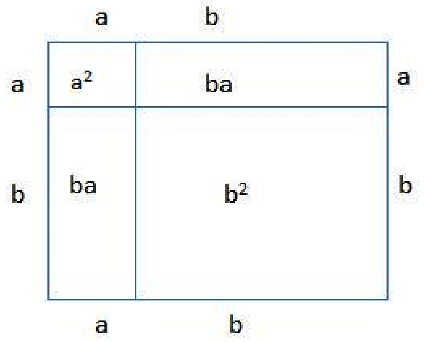
2.2.2 Statement of some important properties/Theorem

Theorem /Statement	Diagram
Theorem: If a quadrilateral is a parallelogram, it has 2 sets of opposite sides congruent.	
Theorem: If a quadrilateral is a parallelogram, it has 2 sets of opposite angles congruent.	
Theorem: If a quadrilateral is a parallelogram, it has consecutive angles which are supplementary.	
Theorem: If a quadrilateral is a parallelogram, it has diagonals which form 2 congruent triangles.	
Theorem: If a quadrilateral is a parallelogram, it has diagonals which bisect each other.	

2.3 Algebraic formula $(a+b)^2$:

Formula for $(a+b)^2 = a^2 + 2ab + b^2$ and its geometrical representation will be as follows



2.4 Area of right angle triangle

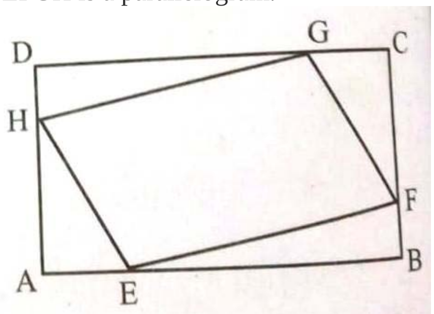
When the angle between any two sides is equal to 90 degrees it is called as right angle triangle. A Right-angled triangle is one of the most important shapes in geometry and is the basics of trigonometry. A right-angled triangle is the one which has 3 sides, "base" "hypotenuse" and "height" with the angle between base and height being 90°.

Area of right angle triangle = (base × height)/2

3. PROOF OF PYTHAGORAS THEOREM

3.1: Application of parallelogram properties:

In rectangle ABCD, the points E, F, G and H are situated on the sides AB, BC, CD and DA respectively in such a way that AE = CG and BF = DH; Prove that EFGH is a parallelogram.



Since, ABCD is a rectangular parallelogram.
AB=DC, AE=CG
Therefore, AB-AE=DC-CG
Therefore, EB=DG
In, triangle ΔDHG and triangle ΔBEF---
DG=EB,
$<GDH=<EBF= 90^0$
DH=FB
So. triangle ΔDHG and triangle Δ BEF are congruent
And so HG=EF
Similarly it can be prove that HE=GF.
Hence, EFGH is a parallelogram.

<table>
<tr>
<td>$(a+b)^2 = a^2 + 2ab + b^2$
In geometrical length of side of parallelogram which is actually square is taken as (a+b) unit and represented in the following diagram.</td>
<td>Let length of each sides of parallelogram ABCD are of length (a+b) units with each angle are 90^o.therefore the value of area of parallelogram ABCD is $(a+b)^2$ square units.</td>
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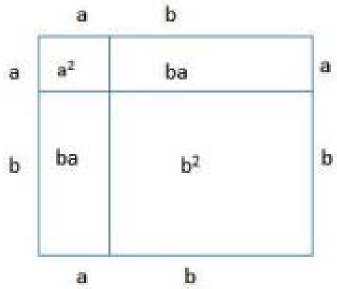
<td>
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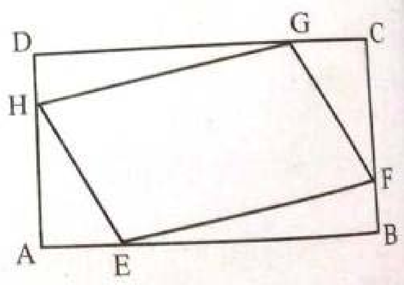
<td>

Let DG=BE=AH=CF= a units
Let DH=AE=BF=CG= b units
So , AB=BC=CD=AD=(a+b) units.
By using the previous theorem ,
EF=FG=GH=HE= c units (Let)
So in sense of area,
ΔAEH +ΔBEF+ΔCGF+ΔGDH +□ EFGH = II□ABCD
$\{(AE \times AH) + (BE\times BF)+(CF\times CG) +(DH\times DG)\}/2 + c^2$
$= 4(a\times b)/2 + c^2$ Square unit</td>
</tr>
<tr>
<td colspan="2">Comparing left and right side of both column we get
$(a+b)^2 = 2ab + c^2$
Or, $a^2 + 2ab + b^2 = 2ab + c^2$
After making cancellation from both sides we get,
Or, $a^2 + b^2 = c^2$</td>
</tr>
</table>

4. CONCLUSION AND FUTURE SCOPE

The Pythagorean Theorem's has given a significant impact in the mathematics. Application of Pythagoras theorem has given idea to solve geometric problems with Algebraic thinking. This paper will grow interest for the secondary level student to understand the importance of parallelogram. A subject becomes very interesting when its application is seen. Co-relation among algebraic equation, parallelogram and triangle will lead to develop more mathematical formula, propositions and theorems in mathematics.

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