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DISTANCE BASED CONNECTIVITY STATUS NEIGHBORHOOD INDICES OF CERTAIN GRAPHS

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ABSTRACT

Many distance based indices of a graph have been appeared in the literature. The status of a vertex u in a connected graph G is defined as the sum of the distance between u and all other vertices of G. In this paper, we introduce the sum connectivity status neighborhood index, product connectivity status neighborhood index, reciprocal product connectivity status neighborhood index, general first and second status neighborhood indices of a graph and compute their values for some standard graphs, wheel and friendship graphs.

Keywords: *distance, status, sum and product connectivity status neighborhood indices, general status neighborhood indices, wheel graph, friendship graph.*

Mathematics Subject Classification: *05C05, 05C07, 05C12, 05C35.*

1. INTRODUCTION

In this paper, we are concerned with simple graphs. Let G be a connected graph. Let $V(G)$ and $E(G)$ be its vertex and edge sets respectively. The edge between the vertices u and v is denoted by uv. The degree of a vertex u is the number of vertices adjacent to u and is denoted by $d_G(u)$. The distance between two vertices u and v is the length of shortest path connecting u and v. The status $\sigma(u)$ of a vertex u in G is the sum of distances of all other vertices from u in G. Let $N(v) = N_G(v) = \{u{:}uv\in E(G)\}$. Let $\sigma_n(v)=\sum_{u\in N(v)}\sigma(u)$ be the status sum of neighbor vertices. We refer [1] for undefined terms and notations from graph theory.

A graph index is a numerical parameter mathematically derived from the graph structure. Graph indices [2] have applications in various disciplines of Science and Technology [3, 4]. Some of the graph indices may be found in [5, 6, 7, 8].

The first and second status neighborhood indices are introduced by Kulli in [9], and they are defined as

$$SN_1(G)=\sum_{uv\in E(G)}\left[\sigma_n(u)+\sigma_n(v)\right], \qquad SN_2(G)=\sum_{uv\in E(G)}\sigma_n(u)\sigma_n(v).$$

Recently some new status neighborhood indices were studied in [10, 11].

We introduce some connectivity status neighborhood indices as follows:

The sum connectivity status neighborhood index of a graph G is defined as

$$SSN(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{\sigma_n(u)+\sigma_n(v)}}.$$

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The product connectivity status neighborhood index of a graph G is defined as

$$PSN(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{\sigma_n(u)\sigma_n(v)}}.$$

The reciprocal product connectivity status neighborhood index of a graph G is defined as

$$RPSN(G) = \sum_{uv\in E(G)} \sqrt{\sigma_n(u)\sigma_n(v)}.$$

The modified first status neighborhood index of a graph G is defined as

$${}^{m}SN_1(G) = \sum_{uv\in E(G)} \frac{1}{\sigma_n(u)+\sigma_n(v)}.$$

The modified second status neighborhood index of a graph G is defined as

$${}^{m}SN_2(G) = \sum_{uv\in E(G)} \frac{1}{\sigma_n(u)\sigma_n(v)}.$$

We continue these generalizations and introduce the general first and second status neighborhood indices of a graph and they are defined as

$$SN_1^a(G) = \sum_{uv\in E(G)} \left[\sigma_n(u)+\sigma_n(v)\right]^a,$$

$$SN_2^a(G) = \sum_{uv\in E(G)} \left[\sigma_n(u)\sigma_n(v)\right]^a,$$

where a is a real numbers.

Recently, some variants of status indices were studied, for example, in [12, 13, 14, 15, 16, 17, 18, 19, 20, 21].

In this paper, the sum connectivity status neighborhood index, product connectivity status neighborhood index, reciprocal product connectivity status neighborhood index, modified first and second status neighborhood indices, general first and second status neighborhood indices of some standard graphs, wheel and friendship graphs are determined.

2. RESULTS FOR COMPLETE GRAPHS

Theorem 1: The general first status neighborhood index of a complete graph K_n is

$$SN_1^a(K_n) = \frac{n(n-1)}{2}\left[2(n-1)^2\right]^a. \tag{1}$$

Proof: Let K_n be a complete graph with n vertices and $\frac{n(n-1)}{2}$ edges. For any vertex u of K_n, $\sigma(u) = n - 1$. Clearly $\sigma_n(u) = (n - 1)^2$ for any vertex u of K_n. Therefore

$$SN_1^a(K_n) = \sum_{uv\in E(K_n)} \left[\sigma_n(u)+\sigma_n(u)\right]^a = \left[(n-1)^2+(n-1)^2\right]^a \frac{n(n-1)}{2}.$$

$$= \frac{n(n-1)}{2}\left[2(n-1)^2\right]^a.$$

From Theorem 1, we establish the following results.

Corollary 1.1: Let K_n be a complete graph with n vertices. Then

(i) $SSN(K_n) = \dfrac{n}{2\sqrt{2}}.$

(ii) ${}^{m}SN_1(K_n) = \dfrac{n}{4(n-1)}.$

Proof: Put $a = -½, -1$ in equation (1), we obtain the desired results.

Theorem 2: The general second status neighborhood index of a complete graph K_n is

$$SN_2^a\left(K_n\right)=\frac{n(n-1)}{2}(n-1)^{4a}. \tag{2}$$

Proof: Let K_n be a complete graph with n vertices and $\frac{n(n-1)}{2}$ edges. For any vertex u of K_n, $\sigma_n(u) = (n-1)^2$. Thus

$$SN_2^a\left(K_n\right)=\sum_{uv\in E(K_n)}\left[\sigma_n(u)\sigma_n(u)\right]^a=\left[(n-1)^2(n-1)^2\right]^a\frac{n(n-1)}{2}.$$

$$=\frac{n(n-1)}{2}(n-1)^{4a}.$$

We obtain the following results by using Theorem 2.

Corollary 2.1: Let K_n be a complete graph with n vertices. Then

(i) $PSN\left(K_n\right)=\dfrac{n}{2(n-1)}.$

(ii) $RPSN\left(K_n\right)=\dfrac{1}{2}n(n-1)^3.$

(iii) ${}^mSN_2\left(K_n\right)=\dfrac{n}{2(n-1)^3}.$

Proof: Put $a = -½, ½, -1$ in equation (2), we get the desired results.

3. RESULTS FOR COMPLETE BIPARTITE GRAPHS

Theorem 3: The general first status neighborhood index of a complete bipartite graph $K_{p,q}$ is

$$SN_1^a\left(K_{p,q}\right)=pq\left[2\left(p^2+q^2\right)-2\left(p+q\right)+2pq\right]^a. \tag{3}$$

Proof: Let $K_{p,q}$ be a complete graph with $p+q$ vertices and pq edges. For vertex set of $K_{p,q}$ can be partitioned into two independent sets V_1 and V_2 such that $u \in V_1$ and $v \in V_2$ for every edge uv in $K_{p,q}$. Therefore $d_K(u)=q$, $d_K(v)=p$, where $K=K_{p,q}$. Then $\sigma(u)= q + 2p - 2$ and $\sigma(v)= p + 2q - 2$. Thus by calculation, we have $\sigma_n(u)= p(q + 2p - 2)$ and $\sigma_n(v) = q(p + 2q - 2)$. Therefore

$$SN_1^a\left(K_{p,q}\right)=\sum_{uv\in E(K)}\left[\sigma_n(u)+\sigma_n(u)\right]^a=pq\left[p\left(q+2p-2\right)+q\left(p+2q-2\right)\right]^a.$$

$$=pq\left[2\left(p^2+q^2\right)-2\left(p+q\right)+2pq\right]^a.$$

By using Theorem 3, we establish the following results.

Corollary 3.1: Let $K_{p,q}$ be a complete bipartite graph. Then

(i) $SSN\left(K_{p,q}\right)=pq\left[\left(p^2+q^2\right)-2\left(p+q\right)+2pq\right]^{-\frac{1}{2}}.$

(ii) ${}^mSN_1\left(K_{p,q}\right)=\dfrac{pq}{2\left(p^2+q^2\right)-2\left(p+q\right)+2pq}.$

Proof: Put $a = -½ , -1$ in equation (3), we obtain the desired results.

Theorem 4: The general second status neighborhood index of a complete bipartite graph $K_{p,q}$ is

$$SN_2^a\left(K_{p,q}\right)=pq\left[2pq\left(p^2+q^2\right)-6pq\left(p+q\right)+5p^2q^2+4pq\right]^a. \tag{4}$$

Proof: Let $K_{p,q}$ be a complete bipartite graph with $p+q$ vertices and pq edges. By calculation, we obtain $\sigma_n(u) = p(q + 2p - 2)$ and $\sigma_n(v) = q(p + 2q - 2)$. Thus

$$SN_2^a\left(K_{p,q}\right)=\sum_{uv\in E(K)}\sigma_n(u)\sigma_n(v)=pq\left[p\left(q+2p-2\right)q\left(p+2q-2\right)\right]^a$$

$$=pq\left[2pq\left(p^2+q^2\right)-6pq\left(p+q\right)+5p^2q^2+4pq\right]^a.$$

We obtain the following results by using Theorem 4.

Corollary 4.1: Let $K_{p,q}$ be a complete bipartite graph. Then

(i) $PSN\left(K_{p,q}\right)= pq\left[2pq\left(p^2+q^2\right)-6pq\left(p+q\right)+5p^2q^2+4pq\right]^{-\frac{1}{2}}$.

(ii) $RPSN\left(K_{p,q}\right)= pq\left[2pq\left(p^2+q^2\right)-6pq\left(p+q\right)+5p^2q^2+4pq\right]^{\frac{1}{2}}$.

(iii) ${}^{m}SN_2\left(K_{p,q}\right)= pq\left[2pq\left(p^2+q^2\right)-6pq\left(p+q\right)+5p^2q^2+4pq\right]^{-1}$.

Proof: Put a = –½ , ½ , –1 in equation (4), we obtain the desired results.

4. RESULTS FOR WHEEL GRAPHS

A wheel graph is the join of C_n and K_1 and it is denoted by W_n. This graph has n+1 vertices and 2n edges. A graph W_4 is presented in Figure 1.

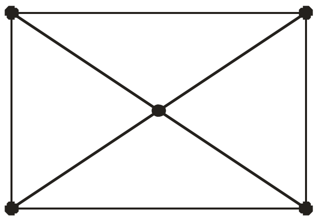

Figure-1: Wheel graph W_4

In a graph W_n, there are two types of edges as follows:

$$E_1=\left\{uv\in E\left(W_n\right) \mid d_{W_n}(u)=d_{W_n}(v)=3\right\}, \qquad |E_1| = n.$$

$$E_2=\left\{uv\in E\left(W_n\right) \mid d_{W_n}(u)=3, d_{W_n}(v)=n\right\}, \qquad |E_2| = n.$$

Therefore, in W_n, there are two types of status edges as follows.

$\sigma(u),\sigma(v) \setminus uv\in E\left(W_n\right)$	(2n – 3, 2n – 3)	(n, 2n – 3)
Number of edges	n	n

Table-1: Status edge partition of W_n

By calculation, we obtain that there are two types of status neighborhood edges as given in Table 2.

$\sigma_n(u)$,$\sigma_n(v)$ $\setminus uv\in E\left(W_n\right)$	(5n – 6, 5n – 6)	5n – 6, n(2n – 3)
Number of vertices	n	n

Table-2: Status neighborhood edge partition of W_n

Theorem 5: The general first status neighborhood index of a wheel graph W_n is given by

$$SN_1^a\left(W_n\right)=n(10n-12)^a+n\left(2n^2+2n-6\right)^a. \qquad (5)$$

Proof: By definition and by using Table 2, we deduce

$$SN_1^a\left(W_n\right)=\sum_{uv\in E\left(W_n\right)}\left[\sigma_n(u)+\sigma_n(v)\right]^a$$

$$=n(5n-6+5n-6)^a+n\left(5n-6+2n^2-3n\right)^a$$

$$=n(10n-12)^a+n\left(2n^2+2n-6\right)^a.$$

From Theorem 5, we establish the following results.

Corollary 5.1: Let W_n be a wheel graph with n+1 vertices and 2n edges. Then

(i) $SSN(W_n) = \dfrac{n}{\sqrt{10n-12}} + \dfrac{n}{\sqrt{2n^2+2n-6}}$

(ii) $^{m}SN_1(W_n) = \dfrac{n}{10n-12} + \dfrac{n}{2n^2+2n-6}$

Proof: Put a = –½, – 1 in equation (5), we obtain the desired results.

Theorem 6: The general second status neighborhood index of a wheel graph W_n is given by

$$SN_2^a(W_n) = n(5n-6)^{2a} + n\left(10n^3 - 27n^2 + 18n\right)^a. \tag{6}$$

Proof: Let W_n be a wheel graph n+1 vertices 2n edges. By definition and by using Table 2, we derive

$$\begin{aligned} SN_2^a(W_n) &= \sum_{uv\in E(W_n)} \left[\sigma_n(u)\sigma_n(v)\right]^a \\ &= n\left[(5n-6)(5n-6)\right]^a + n\left[(5n-6)\left(2n^2-3n\right)\right]^a \\ &= n(5n-6)^{2a} + n\left(10n^3-27n^2+18n\right)^a. \end{aligned}$$

By using Theorem 6, we obtain the following results.

Corollary 6.1: Let W_n be a wheel graph with n+1 vertices and 2n edges. Then

(i) $PSN(W_n) = \dfrac{n}{5n-6} + \dfrac{n}{\sqrt{10n^3-27n^2+18n}}.$

(ii) $RPSN(W_n) = n(5n-6) + n\sqrt{10n^3-27n^2+18n}.$

(iii) $^{m}SN_2(W_n) = \dfrac{n}{(5n-6)^2} + \dfrac{n}{10n^3-27n^2+18n}.$

Proof: Put a = –½, ½, – 1 in equation (6), we get the desired results.

5. RESULTS FOR FRIENDSHIP GRAPHS

A friendship graph, denoted by F_n, is the graph obtained by taking $n \geq 2$ copies of C_3 with vertex in common. A graph F_n has 2n+1 vertices and 3n edges. A graph F_4 is presented in Figure 2.

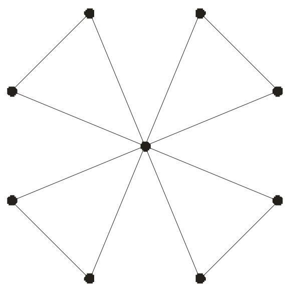

Figure-2: Friendship graph F_4

Let F=F_n. In a graph F_n, we obtain two types of edges as follows:

$E_1 = \{uv \in E(F) \mid d_F(u) = d_F(v) = 2\}$, $\qquad |E_1| = n$.

$E_2 = \{uv \in E(F) \mid d_F(u) = 2, d_F(v) = 2n\}$, $\qquad |E_2| = 2n$.

Therefore in a graph F_n, we find two types of status edges as given in Table 3.

$\sigma(u),\sigma(v) \backslash uv \in E(F_n)$	(4n – 2, 4n – 2)	(2n, 4n – 2)
Number of edges	n	2n

Table-3: Status edge partition of F_n

By calculation, we find that there are two types of status neighborhood edges as given in Table 4.

$\sigma_n(u)$,$\sigma_n(v)$ \ $uv \in E(F_n)$	$(6n-2, 6n-2)$	$(6n-2, 2n(4n-2))$
Number of edges	n	$2n$

Table-4: Status neighborhood edge partition of F_n

Theorem 7: The general first status neighborhood index of a friendship graph F_n is given by

$$SN_1^a(F_n) = n(12n-4)^a + 2n(8n^2+2n-2)^a. \tag{7}$$

Proof: By definition and by using Table 4, we obtain

$$SN_1^a(F_n) = \sum_{uv \in E(F)} \left[\sigma_n(u) + \sigma_n(v)\right]^a$$
$$= n(6n-2+6n-2)^a + 2n(6n-2+8n^2-4n)^a$$
$$= n(12n-4)^a + 2n(8n^2+2n-2)^a.$$

By using Theorem 7, we obtain the following results.

Corollary 7.1; Let F_n be a friendship graph with $2n+1$ vertices and $3n$ edges. Then

(i) $SSN(F_n) = \dfrac{n}{2\sqrt{3n-1}} + \dfrac{2n}{\sqrt{8n^2+2n-2}}.$

(ii) ${}^m SN_1(F_n) = \dfrac{n}{12n-4} + \dfrac{n}{4n^2+n-1}.$

Proof: Put $a = -½, -1$ in equation (7), we get the desired results.

Theorem 8: The general second status neighborhood index of a friendship graph F_n is

$$SN_2^a(F_n) = n(6n-2)^{2a} + 2n\left[4(12n^3-10n^2+2n)\right]^a. \tag{8}$$

Proof: From definition and by using Table 4, we deduce

$$SN_2^a(F_n) = \sum_{uv \in E(F)} \left[\sigma_n(u)\sigma_n(v)\right]^a$$
$$= n\left[(6n-2)(6n-2)\right]^a + 2n\left[(6n-2)(8n^2-4n)\right]^a$$
$$= n(6n-2)^{2a} + 2n\left[4(12n^3-10n^2+2n)\right]^a.$$

We obtain the following results from Theorem 8.

Corollary 8.1: Let F_n be a friendship graph with $2n+1$ vertices and $3n$ edges. Then

(i) $PSN(F_n) = \dfrac{n}{6n-2} + \dfrac{n}{\sqrt{12n^3-10n^2+2n}}.$

(ii) $RPSN(F_n) = n(6n-2) + 4n\sqrt{12n^3-10n^2+2n}.$

(iii) ${}^m SN_2(F_n) = \dfrac{n}{(6n-2)^2} + \dfrac{n}{2(12n^3-10n^2+2n)}.$

Proof: Put $a = -½, ½, -1$ in equation (8), we obtain the desired results.

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